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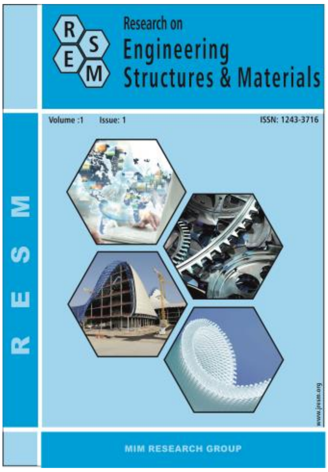

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Research article

Free Vibration and Bending of Functionally Graded Beams Resting on Elastic Foundation

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Abstract

In this paper, free vibration and static bending analysis of functionally graded (FG) beams resting on Winkler foundation are investigated within Euler-Bernoulli beam theory and Timoshenko beam theory. Material properties of the beam change in the thickness direction according to power-law distributions. In deriving of the governing equations, the Hamilton's principle is used. In the solution of the governing equations, the Navier-type method is used for simply-supported beams. In order to establish the accuracy of the present formulation and results, the natural frequencies are obtained, and compared with the published results available in the literature. Good agreement is observed. In the numerical results, the influences the different material distributions, foundation stiffness, and shear deformation on the bending and free vibration behavior of FG beams are presented.

1. Introduction

Functionally graded materials (FGMs) are a new generation of composites where the volume fraction of the FGM constituents vary gradually, giving a non-uniform microstructure with continuously graded macro properties such as elasticity modulus, density, heat conductivity, etc.. Typically, in an FGM, one face of a structural component is ceramic that can resist severe thermal loading and the other face is metal which has excellent structural strength. FGMs consisting of heat-resisting ceramic and fracture-resisting metal can improve the properties of thermal barrier systems because cracking and delamination, which are often observed in conventional layered composites, are reduced by proper smooth transition of material properties. Since the concept of FGMs has been introduced in 1980s, these new kinds of materials have been employed in many engineering application fields, such as aircrafts, space vehicles, defense industries, electronics and biomedical sectors, to eliminate stress concentrations, to relax residual stresses, and to enhance bonding strength. In the literature, the mechanical behavior of FG beams resting on elastic foundation are as follows; Ying et al. [1] presented exact solutions for bending and free vibration of FG beams resting on a Winkler-Pasternak elastic foundation based on the two-dimensional theory of elasticity. Omidi et al. [2] studied the dynamic stability of simple supported FG beams resting on a continuous elastic foundation. with piezoelectric layers subjected to periodic axial compressive load. Pradhan and Murmu [3] investigated thermo-mechanical vibration

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analysis of FG beams and FG sandwich beams resting on Winkler foundation and two-parameter elastic foundation by using the modified differential quadrature method. Fallah and Aghdam [4] studied large amplitude free vibration and post-buckling analysis of FG beams rest on nonlinear elastic foundation subjected to axial force. Akbaş [5] investigated static analysis of cracked FGM beam resting on Winkler foundation by using finite element method. Mohanty et al. [6] investigated the dynamic stability of FG ordinary beam and FG sandwich beam on Winkler's elastic foundation using finite element method. The dynamic response of FG beams with an open edge crack resting on an elastic foundation subjected to a transverse load moving at a constant speed is studied by Yan et al. [7]. Fallah and Aghdam [8] studied thermo-mechanical buckling and nonlinear free vibration analysis of FG beams on nonlinear elastic foundation with Von Karman strain-displacement relation. Esfahani et Al. [9] examined thermal buckling and post-buckling analysis of FG Timoshenko beams resting on a non-linear elastic foundation by using Generalized Differential quadrature Method with considered von-Karman's strain-displacement relations. Post-buckling and nonlinear vibration analysis of geometrically imperfect beams made of functionally graded materials resting on nonlinear elastic foundation subjected to axial force are studied by Yaghoobi and Torabi [10]. Arefi [11] studied the nonlinear responses of a FG beam resting on a nonlinear foundation. Duy et al. [12] examined free vibration of FG beams on an elastic foundation and spring supports. Shen and Wang [13] investigated the large amplitude vibration, nonlinear bending and thermal post-buckling of FG beams resting on an elastic foundation in thermal environments. Li and Shao [14] studied nonlinear bending problem of FG cantilever beams resting on a Winkler elastic foundation under distributed load are discussed. Esfahani et al. [15] studied Small amplitude vibrations of a FG beam resting on nonlinear hardening elastic foundation under in-plane thermal loading in the pre-buckling and post-buckling regimes. Kanani et al. [16] investigated large amplitude free and forced vibration of FG beam resting on nonlinear elastic foundation containing shearing layer and cubic nonlinearity based on Euler-Bernoulli beam theory and von Karman geometric nonlinearity. Babilio [17] examined the nonlinear dynamics of axially graded beams rest on a linear viscoelastic foundation under axial time-dependent excitation is studied. Gan and Kien [18] studied a finite element procedure for the large deflection analysis of FG beams resting on a two-parameter elastic foundation. Sina et al. [19] investigated the vibration of FG beams by using analytical method. Şimşek and Reddy[20] studied bending and vibration analysis of FG microbeams based on modified coupled stress theory. Şimşek and Yurtçu [21] investigated bending and buckling of FG nanobeams based on nonlocal elasticity theory. Akbaş [22] studied wave propagation analysis of edge cracked beams resting on elastic foundation.

The distinctive feature of this study is free vibration and static bending behaviour of simple supported FG beams resting on Winkler foundation are investigated by using analytical solution within Euler-Bernoulli beam theory and Timoshenko beam theory. Material properties of the beam change in the thickness direction according to power-law distributions. In deriving of the governing equations, the Hamilton's principle is used. In the solution of the governing equations, the Navier-type method is used for simply-supported beams. In the solution of the governing equations, the Navier-type method is used for simply-supported beams. In the study, the effects of foundation stiffness and different material distributions on the natural frequencies and the bending responses of the functionally graded beams are investigated in detail. Also, the difference between Euler-Bernoulli beam theory and Timoshenko beam theory on the FG beam is discussed. Some of the present results are compared with the previously published results to establish the validity of the present formulation.

2. Theory and Formulations

Consider a simple supported FG beam of length L, width b, thickness h, subjected to uniform distributed load q_0, resting on Winkler foundation with spring constant k_w as shown in Figure 1.

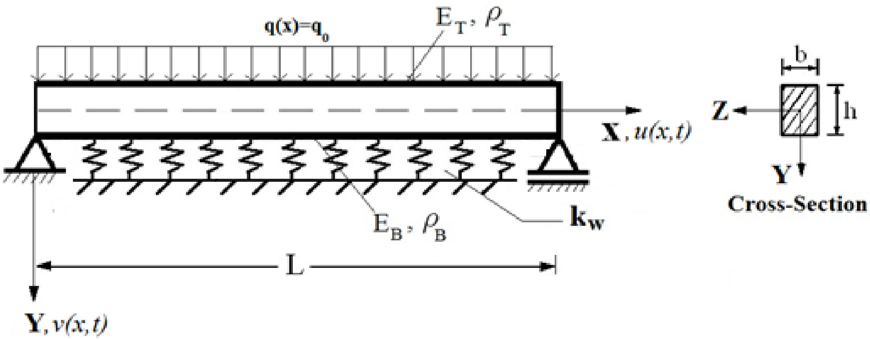

Fig. 1. A simple supported FG beam subjected to uniform distributed load resting on Winkler foundation and cross-section.

In this study, it is assumed that the material properties of the FG beam *P*, i.e., Young's modulus *E*, Poisson's ratio ν, shear modulus *G* and mass density ρ vary continuously in the thickness direction (*Y* axis) according to a power-law form.

$$P(Y) = (P_T - P_B)\left(-\frac{Y}{h} + \frac{1}{2}\right)^n + P_B \tag{1}$$

where P_T and P_B are the material properties of the top and the bottom surfaces of the beam. It is clear from Eq. (1) that when $Y=-h/2$, $P= P_T$, and when $Y=h/2$, $P= P_B$. where *n* is the non-negative power-law exponent which dictates the material variation profile through the thickness of the beam.

2.1. ***Governing equation of free vibration and static bending of FG beams***

Acoording to the coordinate system *(X,Y,Z)* shown in figure 1, based on Timoshenko beam theory, the axial and the transverse displacement field are expressed as

$$u(X,Y,t) = u_0(X,t) - Y\emptyset(x,t) \tag{2}$$

$$v(X,Y,t) = v_0(X,t) \tag{3}$$

$$w(X,Y,t) = 0 \tag{4}$$

where u, v, w are x, y and z components of the displacements, respectively. $\emptyset$ is the total bending rotation. Also, u_0 and v_0 are the axial and the transverse displacements in the mid-plane, *t* indicates time. By using Eqs. (2)-(4), the components of the strain are expressed as

$$\varepsilon_{xx} = \frac{\partial u}{\partial X} = \frac{\partial u_0(X,t)}{\partial X} - Y\frac{\partial \emptyset(x,t)}{\partial X} \tag{5a}$$

$$\varepsilon_{yy} = \varepsilon_{zz} = Y\frac{\nu(Y)\partial^2 v_0(X,t)}{\partial X^2} \tag{5b}$$

$$\varepsilon_{xz} = \varepsilon_{yz} = 0 \tag{5c}$$

$$2\varepsilon_{xy} = \gamma_{xy} = \frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \emptyset \tag{5d}$$

Because the transversal surfaces of the beam is free of stress, then

$$\sigma_{zz} = \sigma_{yy} = 0 \tag{6}$$

According to Hooke's law, constitutive equations of the FGM beam are as follows:

$$\sigma_{xx} = E^*(Y)\varepsilon_{xx} = \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\left[\frac{\partial u_0(X,t)}{\partial X} - Y\frac{\partial \emptyset(x,t)}{\partial X}\right] \tag{7a}$$

$$\sigma_{xy} = k_s G(Y)\gamma_{xy} = \frac{E(Y)}{2(1+\nu(Y))}\left[\frac{\partial \mathrm{v}}{\partial \mathrm{x}} - \emptyset\right] \tag{7b}$$

Where σ_{xx}, σ_{xy} and k_s are normal stresses, shear stresses and the shear correction factor., respectively. When the total bending rotation $\emptyset = {\partial \mathrm{v}}/{\partial \mathrm{x}}$, the beam model reduces to Euler-Bernoulli beam model.

In deriving of the governing equations, the Hamilton's principle is used;

$$\delta \int_0^t [T - (U_i - U_e)]dt = 0 \tag{8}$$

where U_i, T, U_e are the strain energy, the kinetic energy and the potential energy of the external load, respectively. The first variation of the strain energy (U_i) is expressed as

$$\delta \int_0^t U_i dt = \int_0^t \int_V (\sigma_{xx}\,\delta\varepsilon_{xx} + \sigma_{xy}\,\delta\gamma_{xy})dV dt + \int_0^t \int_0^L (k_w w\,\delta w)dxdt \tag{9a}$$

$$= \int_0^t \int_0^L \left[N_{XX}\frac{\partial \delta u_0}{\partial x} - M_Z\left(\frac{\partial^2 \delta w}{\partial x^2} - \frac{\partial \delta\gamma_{xy}}{\partial x}\right) + Q_Y\,\delta\gamma_{xy} + \int_0^t \int_0^L (k_w w\,\delta w\right] dxdt \tag{9b}$$

where N_{XX}, M_Z and Q_Y stress resultants, and expressed as follows:

$$N_{XX} = \int_A \sigma_{xx}\mathrm{dA} = \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\mathrm{dA}\frac{\partial u}{\partial x} - \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}Y\,\mathrm{dA}\left(\frac{\partial^2 \delta w}{\partial x^2} - \frac{\partial \delta\gamma_{xy}}{\partial x}\right) \tag{10a}$$

$$M_Z = \int_A y\sigma_{xx}\mathrm{dA} = \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}Y\,\mathrm{dA}\frac{\partial u}{\partial x} - \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}Y^2\,\mathrm{dA}\left(\frac{\partial^2 \delta w}{\partial x^2} - \frac{\partial \delta\gamma_{xy}}{\partial x}\right) \tag{10b}$$

$$Q_Y = \int_A \sigma_{xz}\mathrm{dA} = k_s\int_A \frac{E(Y)}{2(1+\nu(Y))}\mathrm{dA}\,\gamma_{xy} \tag{10c}$$

The first variation of the kinetic energy (T) is expressed as

$$\delta \int_0^t T dt = \int_0^t \int_0^L \left[J_1 \left(\frac{\partial u_0}{\partial t} \frac{\partial \delta u_0}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial w}{\partial t} \right) - J_2 \frac{\partial u_0}{\partial t} \left(\frac{\partial^2 \delta w}{\partial x \partial t} - \frac{\partial \delta \gamma_{xy}}{\partial t} \right) - J_2 \frac{\partial \delta u_0}{\partial t} \left(\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial \gamma_{xy}}{\partial t} \right) + J_3 \left(\frac{\partial^2 w}{\partial x \partial t} - \frac{\partial \delta \gamma_{xy}}{\partial t} \right) \left(\frac{\partial^2 \delta w}{\partial x \partial t} - \frac{\partial \delta \gamma_{xy}}{\partial t} \right) \right] dx\, dt \tag{11}$$

where

$$J_1 = \int_A \rho(Y) \mathrm{dA} \tag{12a}$$

$$J_2 = \int_A \rho(Y) Y \mathrm{dA} \tag{12b}$$

$$J_3 = \int_A \rho(Y) Y^2 \mathrm{dA} \tag{12c}$$

The first variation of the potential energy (U_e) of the external load is expressed as

$$\delta \int_0^t U_e dt = \int_{\mathbf{0}}^{\boldsymbol{t}} \int_{\mathbf{0}}^{\mathrm{L}} q_0\, \delta w \;\, dxdt \tag{13}$$

Substituting Eqs. (9), (11) and (13) into Eq. (8), and then using integrating by parts, the governing equations of the problem can be obtained as follows;

$$\frac{\partial N_{XX}}{\partial x} = J_1 \frac{\partial^2 u_0}{\partial \mathrm{t}^2} - J_2 \frac{\partial^3 w}{\partial x \partial t^2} + J_2 \frac{\partial^2 \gamma_{xy}}{\partial t^2} \tag{14a}$$

$$k_w w + \frac{\partial^2 M_Z}{\partial \mathrm{x}^2} + q_0 = J_1 \frac{\partial^2 \mathrm{w}}{\partial \mathrm{t}^2} + J_2 \frac{\partial^3 u_0}{\partial x \partial t^2} - J_3 \frac{\partial^4 w}{\partial x^2 \partial t^2} + J_3 \frac{\partial^3 \gamma_{xy}}{\partial x \partial t^2} \tag{14b}$$

$$\frac{\partial M_Z}{\partial x} - Q_Y = J_2 \frac{\partial^2 u_0}{\partial t^2} - J_3 \frac{\partial^3 w}{\partial x \partial t^2} + J_3 \frac{\partial^2 \gamma_{xy}}{\partial t^2} \tag{14c}$$

The boundary conditions at the beam ends are as follows;

N_{XX}=0 or u=0 at x=0,L (15a)

$$\frac{\partial M_Z}{\partial x} - J_2 \frac{\partial^2 u_0}{\partial t^2} + J_3 \frac{\partial^3 w}{\partial x \partial t^2} - J_3 \frac{\partial^2 \gamma_{xy}}{\partial t^2} = 0 \quad \text{or} \quad w=0 \quad \text{at } x=0, L \tag{15b}$$

M_Z=0 or $\frac{\partial w}{\partial x}$=0 at x=0,L (15c)

Substituting Eq. (10) into Eq. (14), the governing equations of the problem can be expressed as follows;

$$\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,\mathrm{dA}\,\frac{\partial^2 u_0}{\partial x^2}+\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,Y\,\mathrm{dA}\left(\frac{\partial^2\gamma_{xy}}{\partial x^2}-\frac{\partial^3 w}{\partial x^3}\right)-I_A\frac{\partial^2 u_0}{\partial t^2}+$$

$$I_B\frac{\partial^3 w}{\partial x\partial t^2}-I_B\frac{\partial^2\gamma_{xy}}{\partial t^2}=0 \tag{16a}$$

$$k_w w+\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,Y\,\mathrm{dA}\,\frac{\partial^2 u_0}{\partial x^2}+\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,Y^2\,\mathrm{dA}\left(\frac{\partial^3\gamma_{xy}}{\partial x^3}-\frac{\partial^4 w}{\partial x^4}\right)-I_A\frac{\partial^2 w}{\partial t^2}-$$

$$I_B\frac{\partial^3 u_0}{\partial x\partial t^2}+I_D\frac{\partial^4 w}{\partial x^2\partial t^2}-I_D\frac{\partial^3\gamma_{xy}}{\partial x\partial t^2}+q=0 \tag{16b}$$

$$\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,Y\,\mathrm{dA}\,\frac{\partial^2 u_0}{\partial x^2}+\int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))}\,Y^2\,\mathrm{dA}\left(\frac{\partial^2\gamma_{xy}}{\partial x^2}-\frac{\partial^3 w}{\partial x^3}\right)\frac{\partial^2\gamma_{xy}}{\partial x^2}-$$

$$k_s\int_A \frac{E(Y)}{2(1+\nu(Y))}\,\mathrm{dA}\,\gamma_{xy}-I_B\frac{\partial^2 u_0}{\partial t^2}+I_D\frac{\partial^3 w}{\partial x\partial t^2}-I_D\frac{\partial^2\gamma_{xy}}{\partial t^2}=0 \tag{16c}$$

2.2 Naiver solution of the problem

The governing equations of the problem are solved for free vibration and static bending of a simply-supported beam by using Navier method for functionally graded materials [19,20,21]. In the Navier solution, the displacement fields are expressed as follows:

$$u(x,t)=\sum_{n=1}^{N} A_n e^{i\beta_n t}\cos(kx) \tag{17a}$$

$$v(x,t)=\sum_{n=1}^{N} B_n e^{i\beta_n t}\sin(kx) \tag{17b}$$

$$\gamma_{xy}(x,t)=\sum_{n=1}^{N} C_n e^{i\beta_n t}\cos(kx) \tag{17c}$$

where β_n is the natural frequency, (*An, Bn, Cn*) are the unknown constants, $k=n\pi/L$ and $i=\sqrt{-1}$. When the *Cn*=0, the governing equations of the beam reduces to Euler-Bernoulli beam theory.

In the static bending case, time and its derivatives are zero in the governing equations. According to the Navier solution, the uniform distributed load q_0 is defined as follows;

$$q_0(x) = \sum_{n=1}^{N} \frac{4q_0}{n\pi} \sin(kx) \qquad n=1,3,5,... \tag{18}$$

Substituting Eqs. (17) and Eq. (18) into Eqs. (16), and then using matrix procedure, the algebraic equations can be expressed for static bending case as follows;

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} A_n \\ B_n \\ C_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{4q_0}{n\pi} \\ 0 \end{Bmatrix} \tag{19}$$

where

$$K_{11} = k^2 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} dA, \tag{20a}$$

$$K_{12} = K_{21} = -k^3 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} Y dA, \tag{20b}$$

$$K_{13} = K_{31} = k^2 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} Y dA, \tag{20c}$$

$$K_{22} = k_w + k^4 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} Y^2 dA \tag{20d}$$

$$K_{23} = K_{32} = -k^3 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} Y^2 dA, \tag{20e}$$

$$K_{33} = k^2 \int_A \frac{E(Y)(1-\nu(Y))}{(1+\nu(Y))(1-2\nu(Y))} Y^2 \, dA + k_s \int_A \frac{E(Y)}{2(1+\nu(Y))} \, dA \tag{20f}$$

In the free vibration problem, the algebraic equations can be expressed as follows;

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - (\beta_n)^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \right) \begin{Bmatrix} A_n \\ B_n \\ C_n \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$M_{11} = J_1, \tag{22a}$$

$$M_{12} = M_{21} = -k \, J_2, \tag{22b}$$

$$M_{13} = M_{31} = J_2, \tag{22c}$$

$$M_{22} = J_1 + k^2 J_3, \tag{22d}$$

$$M_{23} = M_{32} = -k J_3, \tag{22e}$$

$$M_{33} = J_3, \tag{22f}$$

In Eq. (21), the algebraic equation is an eigenvalue problem. With solving the Eq. (21), the fundamental frequency can be obtained.

4. Numerical Results

In the numerical examples, the natural frequencies and the static bending deflections of the FG beams are calculated and presented in figures for different material distributions and , foundation stiffness. Also, the difference between Euler-Bernoulli beam theory and Timoshenko beam theory on the FG beam is discussed. The beam considered in numerical examples is made of Aluminum (Al; E=70 GPa, ν=0.3, ρ=2702 kg/m^3) and Silicon Carbide (SiC; E=427GPa, ν=0.17, ρ=3100 kg/m^3). The top surface material of the FG beam is Aluminum, the bottom surface material of the FG beam is Silicon Carbide. When the power index n=0, the beam material is reduced to full Aluminum (homogeneous Aluminum) according to Eq. 1. Unless otherwise stated, it is assumed that the width of the beam is b=0.1m and height of the beam is h=0.1m in the numerical results. The shear correction factor is taken as k_s=5/6. Unless otherwise stated, it is assumed that the width of the beam is b=0.1m, height of the beam is h=0.1 m.

In order to establish the accuracy of the present formulation and the computer program developed by the author, the results obtained from the present study are compared with the available results in the literature. For this purpose, the non-dimensional fundamental frequencies of a FG simple supported beam are calculated and compared with those of Sina et al. [19] for different material distributions (n) and the slenderness ratio (L/h) according to Timoshenko beam theory in Table 1. The material parameter used in Sina et al. [19] are;

Aluminum :E_u=70 GPa, ρ_u=2700 kg/m^3, ν_u=0,23

Alumina : E_B=380 GPa, ρ_B=3800 kg/m^3, ν_B=0,23

The following non-dimensional frequency parameter used in Sina et al. [19] are as follows;

$$\beta = \omega L^2 \sqrt{\frac{I_A}{h^2 A_{XX}}} \tag{23}$$

Table 1 Comparison study for the non-dimensional fundamental frequency of the FG simply supported beam for different material (n) distributions and slenderness ratio (L/h).

	Non-dimensional fundamental frequency β			
L/h	***n*=0**		***n*=0.3**	
	Sina et al. [19]	Present	Sina et al. [19]	Present
10	2.797	2.797	2.695	2.695
30	2.843	2.843	2.737	2.737
100	2.848	2.848	2.742	2.742

As seen from Table 1, the present results are in good agreement with that the results of Sina et al. [19].

In figure 2, the effect of material distribution (the power-law exponent) and beam theories on the maximum the vertical displacements of the FG beam is shown for different slenderness ratio (L/h) for Winkler spring constant k_w=10^8 N/m and distributed load q=10^8 N/m.

It is seen from Fig. 2 that increase in the material power law index (n) causes decrease in the vertical deflections for all values of the slenderness ratio (L/h): Because when the material power law index (n) increase, the material of the beam get close to Silicon Carbide (bottom side material) according to Eq. 1 and it is known from the physical properties of the Silicon Carbide and Aluminum (top side material) that the Young modulus of Silicon Carbide is approximately six times greater than that of Aluminum. With increase in the power-law index, the bending rigidity of the beam increases. As a result, the strength of the material increases. Also, it is seen from Fig. 2 that increase in the material power law index (n), the curve has an asymptote. With the material power law index (n) increases, the functionally graded material beam is reduced to the homogeneous Silicon Carbide beam according to Eq. 1. Also, it is clearly seen from Fig. 2 that, with decrease in the ratio L/h, the difference between the results of Euler Bernoulli beam theory and Timoshenko beam theory differs considerably.

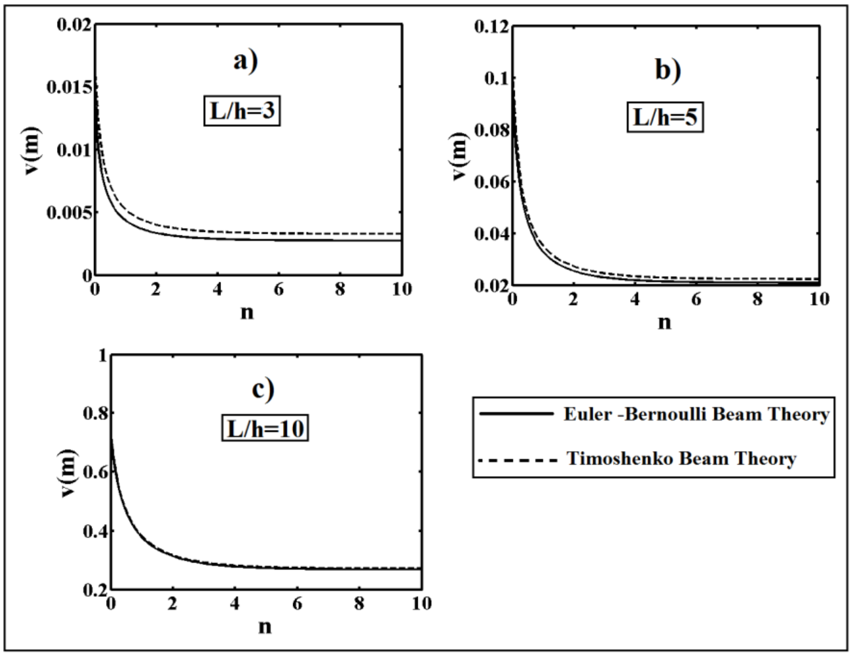

Fig. 2. The effect of material distribution (n) on the maximum the vertical displacements of the FG beam for different slenderness ratio (L/h), a) L/h=3, a) L/h=5 and a) L/h=10.

Figure 3 shows that the effect of Winkler spring constant (k_w) on the maximum the vertical displacements of the FG beam for different slenderness ratio (L/h) for the power-law exponent n=0.3 and distributed load q=10^8 N/m.

It is seen from Fig. 3 that the value of the Winkler parameter (k_w) play important role on the static response of the beam. With increase in the Winkler parameter(k_w), the displacements of the FG beam decreases. Because, with increasing the Winkler parameter(k_w), the beam gets more stiffer. Also, it is observed Fig. 3a that the the difference between the results of Euler Bernoulli beam theory and Timoshenko beam theory increase with decrease the Winkler parameter (k_w).

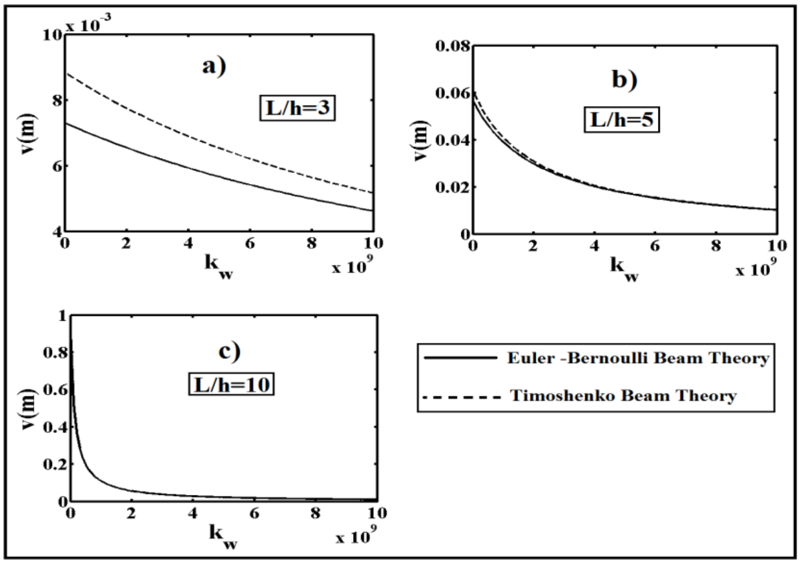

Fig. 3. The effect of Winkler spring constant (k_w) on the maximum the vertical displacements of the FG beam for different slenderness ratio (L/h), a) L/h=3, a) L/h=5 and a) L/h=10.

In figure 4, the effect of material distribution (the power-law exponent) on the fundamental frequency (ω) of the FG simply supported beam is shown for different slenderness ratio (L/h) for Winkler spring constant k_w=10^8 N/m.

It is seen from Fig. 4 that increase in the material power law index (n) causes increase in the fundamental frequency (ω) of the beam. It is noted before that an increase in the power-law index causes a increase in the rigidity of the beam. Also, it is clearly seen from Fig. 4 that, with increase in the ratio L/h, the difference between the results of Euler Bernoulli beam theory and Timoshenko beam theory on the fundamental frequency coincide with each other.

In Figure 5, the effect of Winkler spring constant (k_w) on the fundamental frequency of the FG beam for different slenderness ratio (L/h) for the power-law exponent n=0.3.

It is seen from Fig. 5 that with increase in the Winkler parameter (k_w), the fundamental frequency increases. Because, with increasing the Winkler parameter (k_w), the beam gets more stiffer. It is observed Fig. 5 that the the difference between the results of Euler Bernoulli beam theory and Timoshenko beam theory on the fundamental frequency decrease with increase the Winkler parameter (k_w).

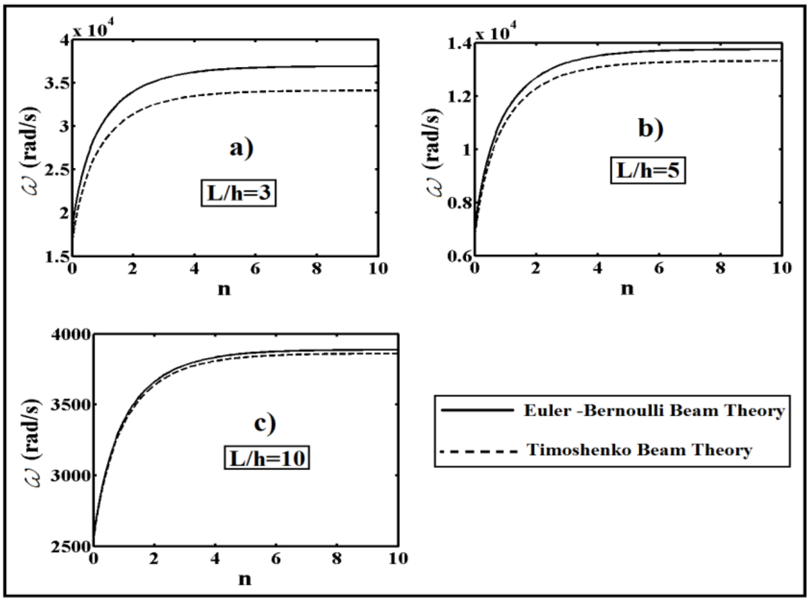

Fig. 4. The effect of material distribution (n) on the maximum the fundamental frequency (ω) of the FG beam for different slenderness ratio (L/h), a) L/h=3, a) L/h=5 and a) L/h=10.

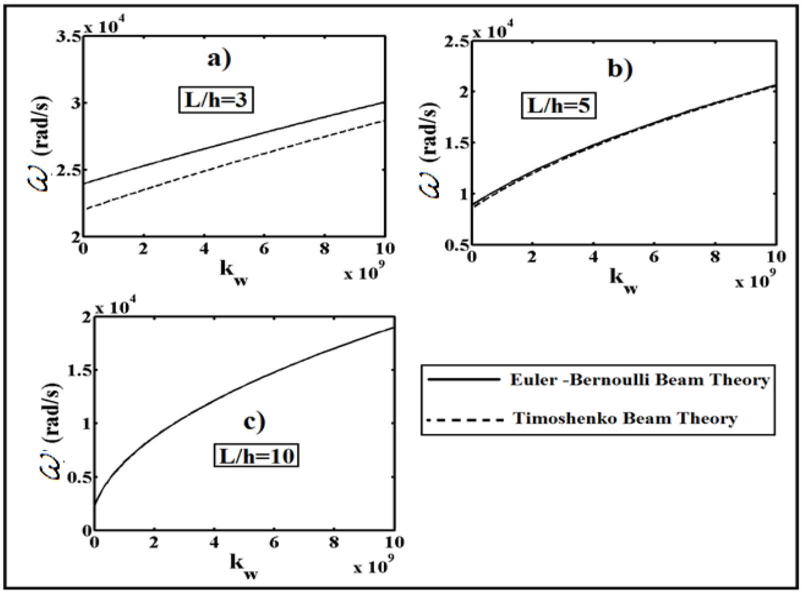

Fig. 5. The effect of Winkler spring constant (k_w) on the fundamental frequency (ω) of the FG beam for different slenderness ratio (L/h), a) L/h=3, a) L/h=5 and a) L/h=10.

4. Conclusions

Free vibration and static bending analysis of FG beams resting on Winkler foundation are investigated within Euler-Bernoulli beam theory and Timoshenko beam theory. Material properties of the beam change in the thickness direction according to power-law distributions. In deriving of the governing equations, the Hamilton's principle is used. In the solution of the governing equations, the Navier-type method is used for simply-supported beams.

Numerical results show that the material distribution and foundation parameter play an important role on static and vibration of the FG beam. It was seen from the investigations that the difference between the results of Euler-Bernoulli beam theory and Timoshenko beam theory increases considerably with decreases in the slenderness ratio. Therefore, for small slenderness of beam, Timoshenko beam theory must be used instead of Euler-Bernoulli beam theory because of the effect of the shear stresses on the deformation. Also, by choosing a suitable material distribution (power-law exponent (n)), it can be reduce the negative influence of the stresses and displacements in the design of FG structures.

References

[1] Ying J., Lü C.F. and Chen W.Q. Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations. Composite Structures, 2008; 84: 209-219. http://dx.doi.org/10.1016/j.compstruct.2007.07.004

[2] Omidi N, Karami Khorramabadi M and Niknejad, A. Dynamic stability of functionally graded beams with piezoelectric layers located on a continuous elastic foundation. Journal of Solid Mechanics, 2009; 1: 130-136.

[3] Pradhan S.C. and Murmu T. Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method . Journal of Sound and Vibration, 2009;321:342-362. http://dx.doi.org/10.1016/j.jsv.2008.09.018

[4] Fallah A and Aghdam M.M. Nonlinear free vibration and post-buckling analysis of functionally graded beams on nonlinear elastic foundation. European Journal of Mechanics, A/Solids, 2011;30:571-583. http://dx.doi.org/10.1016/j.euromechsol.2011.01.005

[5] Akbaş Ş.D. Static analysis of a functionally graded beam with edge cracks on elastic foundation, *Proceedings of the 9 th International Fracture Conference, Istanbul, Turkey,* 2011;70-80.

[6] Mohanty S.C., Dash R.R. and Rout, T. Parametric instability of a functionally graded Timoshenko beam on Winkler's elastic foundation. Nuclear Engineering and Design, 2011;241: 2698-2715. http://dx.doi.org/10.1016/j.nucengdes.2011.05.040

[7] Yan T, Kitipornchai S, Yang J and He X.Q. Dynamic behaviour of edge-cracked shear deformable functionally graded beams on an elastic foundation under a moving load. Composite Structures, 2011;93: 2992-3001. http://dx.doi.org/10.1016/j.compstruct.2011.05.003

[8] Fallah A. And Aghdam M.M. Thermo-mechanical buckling and nonlinear free vibration analysis of functionally graded beams on nonlinear elastic foundation. Composites Part B: Engineering, 2012;43: 1523-1530. http://dx.doi.org/10.1016/j.compositesb.2011.08.041

[9] Esfahani S.E., Kiani Y. and Eslami M.R. Non-linear thermal stability analysis of temperature dependent FGM beams supported on non-linear hardening elastic foundations. International Journal of Mechanical Sciences, 2013; 69:10-20. http://dx.doi.org/10.1016/j.ijmecsci.2013.01.007

[10] Yaghoobi H. and Torabi M. Post-buckling and nonlinear free vibration analysis of geometrically imperfect functionally graded beams resting on nonlinear elastic foundation. Applied Mathematical Modelling, 2013; 37: 8324-8340. http://dx.doi.org/10.1016/j.apm.2013.03.037

[11] Arefi M. Nonlinear analysis of a functionally graded beam resting on the elastic nonlinear foundation. Journal of Theoretical and Applied Mechanics, 2014; 44: 71-82. http://dx.doi.org/10.2478/jtam-2014-0011

[12] Duy H.T, Van T.N. and Noh H.C. Eigen analysis of functionally graded beams with variable cross-section resting on elastic supports and elastic foundation. Structural Engineering and Mechanics, 2014;52: 1033-1049. http://dx.doi.org/10.12989/sem.2014.52.5.1033

[13] Shen HS and Wang ZX. Nonlinear analysis of shear deformable FGM beams resting on elastic foundations in thermal environments. International Journal of Mechanical Sciences, 2014; 81: 195-206. http://dx.doi.org/10.1016/j.ijmecsci.2014.02.020

[14] Li Q.L. and Shao Q.H. Non-linear analysis of a FGM cantilever beam supported on a Winkler elastic foundation. Applied Mechanics and Materials, 2014; 602-605:131-134. http://dx.doi.org/10.4028/www.scientific.net/AMM.602-605.131

[15] Esfahani S.E., Kiani Y., Komijani M. And Eslami M.R. Vibration of a temperature-dependent thermally pre/postbuckled FGM beam over a nonlinear hardening elastic foundation. Journal of Applied Mechanics, Transactions ASME, 2014; 81:Article number 011004.

[16] Kanani A.S., Niknam H., Ohadi A.R. and Aghdam M.M. Effect of nonlinear elastic foundation on large amplitude free and forced vibration of functionally graded beam. Composite Structures, 2014; 115: 60-68. http://dx.doi.org/10.1016/j.compstruct.2014.04.003

[17] Babilio E. Dynamics of functionally graded beams on viscoelastic foundation. International Journal of Structural Stability and Dynamics, 2014;14: Article number 1440014. http://dx.doi.org/10.1142/S0219455414400148

[18] Gan B.S. and Kien N.D. Large deflection analysis of functionally graded beams resting on a two-parameter elastic foundation. Journal of Asian Architecture and Building Engineering, 2014;13: 649-656. http://dx.doi.org/10.3130/jaabe.13.649

[19] Sina S.A., Navazi H.M. and Haddadpour H. An analytical method for free vibration analysis of functionally graded beams. Materials and Design, 2009;30: 741-747. http://dx.doi.org/10.1016/j.matdes.2008.05.015

[20] Şimşek M. and Reddy J.N. Bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. International Journal of Engineering Science, 2013; 64: 37-53. http://dx.doi.org/10.1016/j.ijengsci.2012.12.002

[21] Şimşek M. and Yurtçu H.H. Analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory. Composite Structures, 2013; 97: 378-386. http://dx.doi.org/10.1016/j.compstruct.2012.10.038

[22] Akbaş Ş.D. Wave propagation analysis of edge cracked beams resting on elastic foundation. International Journal of Engineering and Applied Sciences, 2014; 6: 40-52.