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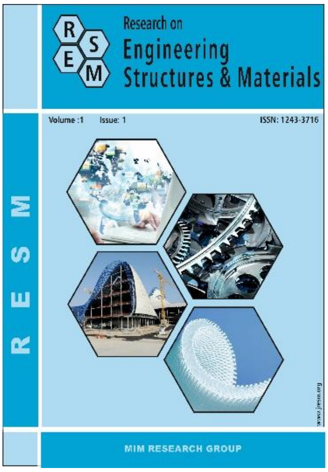

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Research Article

Free vibrations of fluid conveying pipe with intermediate support

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Abstract

In this study, linear vibration of fluid carrying pipe with intermediate support was discussed. Supports located at the ends of the pipe were clamped supports. A support was located in the o0middle section show the features of a simple support. It was accepted that the fluid velocity varied harmonically by an average speed. The equation of motion and limit conditions of the system were obtained by using Hamilton principle. The solutions were obtained using the Multiple Scale Method, which is one of the Perturbation Methods. The first term in the perturbation series causes the linear problem. Exact natural frequencies were calculated by the solution of the linear problem for the different positions of the support at the center (η), different longitudinal stiffness (v_b), different pipe coefficient (v_f), different rate of fullness (β) and natural frequencies depending on velocity of the fluid (v0) were calculated exactly. The obtained results were shown in graphics.

1. Introduction

Vibrations of fluid-carrying continuums are under study for a long time in terms of their significance. They have several fields of application ranging from oil pipelines carrying fluid to natural gas pipelines, water pipes, the pipes that carry hazardous fluids in chemical plants, fire extinction equipment, sewers and underwater pipelines for conducting fluid. Such systems are considered as problems of fluid-carrying strip pipes, fluid-carrying flexible pipes and fluid-carrying pipes. Studies have been conducted on dynamics of fluid-carrying pipes where friction of fluid is neglected [1]. Ulsoy et al. [2] studied principal developments on vibrations and stability of strips. Pakdemirli and Batan [3] achieved an approximate analytical solution for strips that accelerate axially with multiple time-scale method (perturbation technique [4]). Pakdemirli et al. [5] studied transverse vibrations of axially-moving strips. Pakdemirli et al. [6] used two different approaches in their studies. In the first approach, they used discretization-perturbation method. The second approach was used in continuums with axial movement for the first time in the said study and it introduces some advantages. This approach neglects the necessity to write the equations in a new form and define an orthogonal base system. In their study where they compared the results of two methods for nonlinear cable vibration, they showed that branching and stability analysis were different for each method and the behavior of the actual system was better represented by direct-perturbation method. Pakdemirli and Ulsoy [7] conducted stability analysis of strips with axial motion, comparing direct-perturbation and discretization-perturbation methods. Velocity is considered as fixed and analyses were made on this basis in most of the studies mentioned above. Mulcahy [8] examined the vibrations arising from the fluid effect in nuclear power plant reactors and then the natural

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frequencies of vibrations caused by the flow between 2 nested pipes [9]. Chen et al. [10] studied the vibration of pipes under the flow effect. Lee and Mote [11] studied the changes in frequency values of fluid-carrying pipes depending on speed. Paidoussis and Semler [12] studied the nonlinear dynamics of bent pipes on the assumption of smaller mass on the free end. Ridvan and Boyaci [13] studied transversal vibrations of the pipe in cases where the speed of fluid depends on time. Özkaya and Pakdemirli [14] examined the behavior of transmission from the strip to the beam, assuming that the beam coefficient is very small and studied the vibrations of such axially-moving beams. Özkaya [15] studied the beams that carry concentrated mass. Wang et al. [16] studied vibrations caused by fluids in Euler-Bernoulli beams. Öz and his coworkers [17] and [18] studied the natural frequency of tensioned pipes for different limit conditions. Wang et al. [19] designed nonlinear fluid load model for elastic cylinder. Modarres and Païdoussis [20] conducted a dynamic analysis of pipes supported at both ends, which carry fluids with a weak nonlinearity using the Galerkin method. In that study, the velocity of the fluid is considered to be fixed. Wang et al. [21] studied the effect of geometric defects on fluid-carrying pipes, using the Galerkin approach. Nikolić and Rajković [22] analyzed bifurcation points of fluid-carrying pipes supported at both ends using Lyapunov-Schmidt reduction and singularity theory. Enz [23] studied simple supported straight pipe using perturbation analysis with multiple time-scaled method and measurements made by Coriolis flowmeters were compared. Ritto et al. [24] studied fluid-carrying Euler-Bernoulli pipe by means of finite elements method. Dai et al. [25] studied vortex-induced vibrations of pipes carrying pulsed fluids, using multiscale method. Kheiri et al. [26] studied dynamic stability of fluid-carrying pipes supported by bows on its ends. Chen et al. [27] studied using Galerkin method the nonlinear vibrations of fluid-carrying viscoelastic pipes at about critical velocity. Li et al. [28] analyzed by means of matrix transfer method the vibration of fluid-carrying systems. Kheiri and Païdoussis [29] used generalized Hamilton principle to get the motion equations of fluid-carrying pipes. Yang et al. [30] studied the stability of transversal vibrations of beams modelled as viscoelastic pipes. Ghayesh et al. [31] studied nonlinear plenary dynamics of fluid-carrying bent pipes. Kesimli et al. [32] studied nonlinear vibrations of spring supported string by means of multiple scaled method. Zhang and Chen [33] studied external and internal resonances of fluid-carrying pipes around the critical velocity. Modarres and Païdoussis [34] studied oscillations of fluid-carrying pipe with a mass on its end. Banerjee [35] studied free vibrations of the beam that has a mass-spring system on its end. Yi-Min et al. [36] calculated natural frequencies of the simply-supported fluid-carrying system using the Ferrari method that is used for solving quartic equations. Lee et al. [37] conducted the dynamic analysis of the beam that bears a mass-spring system with embedded and simple support on its ends, using finite element method. Bağdatli et al. [38] Studied dynamics of axially accelerating beams with multiple supports. Ghayesh et al. [39] studied three-dimensional dynamics of the fluid-carrying bent pipe that has a spring on its middle section and a mass on its end. Ni et al. [40] calculated natural frequencies of fluid-carrying pipes with different limit conditions, using differential transfer method (DTM). Li et al. [41] identified free vibrations of fluid-carrying pipes with multiple support, using dynamic stiffness method. Bağdatli et al. [42] Investigated to dynamics of intermediate support beams.

This study examines the fluid-carrying pipe with multiple support. The situation where the pipe is attached to the ground by embedded supports on its ends and bears a simple support on its middle section is studied. Fluid velocity is assumed to change harmonically around an average velocity ($v(t)=v_0+\varepsilon v_1 \sin\Omega t$). Taking into consideration the nonlinear effects caused by extension of pipes, motion equations and limit conditions are found by Hamilton's principle. Dependence on material or geometric structure was eliminated by nondimensionalizing the equations of motion. Approximate solutions were found by using the multiple time-scaled method, which is one of the perturbation methods. Natural frequencies were precisely calculated for different positions, different pipe coefficient and different rates of fullness and fluid velocity values of the support on the middle part.

2. Equation of Motion

In this section, equations of motion will be worked out for the fluid-carrying pipe supported at its two ends and a middle point as specified in Fig. 1. Hamilton's principle will be used to find equations of motion. Euler-Bernoulli pipe where turning inertia and shear stresses are neglected was used in working out the equation of motion. It was assumed that sizes of sections do not change during motion.

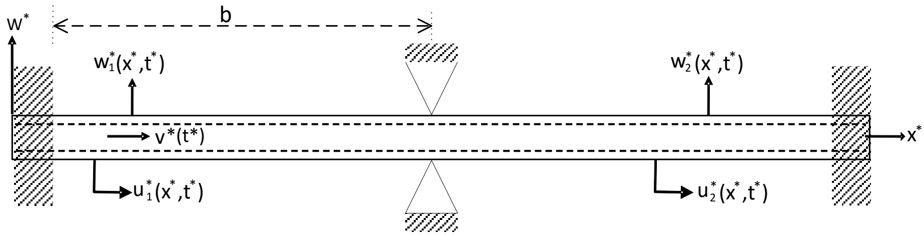

Fig. 1 Pipe carrying fluid with intermediate support

In the fluid-carrying pipe with three supports shown in Fig. 1, u^* (x^*,t^*) shows displacement in the direction of x*; v^* (t^*) shows the velocity of fluid in the direction of x* (independent of x*); $w_1^*(x^*,t^*)$ and w_2^* (x^*,t^*) w^* show displacement at the right and left of the support, respectively, at a middle location in the direction of w*. Expressions of kinetic energy and elastic potential energy may be written as follows, respectively.

$$
\begin{aligned}
T = {} & \frac{1}{2}\rho_f A_f \int_0^b \left[\left(\dot{w}_1^* + w_1^{*\prime} v^* \right)^2 + \left(v^* + \dot{u}_1^* + u_1^{*\prime} v^* \right)^2 \right] dx + \frac{1}{2}\rho_p A_p \int_0^b \dot{w}_1^{*2} dx \\
& + \frac{1}{2}\rho_f A_f \int_b^L \left[\left(\dot{w}_2^* + w_2^{*\prime} v^* \right)^2 + \left(v^* + \dot{u}_2^* + u_2^{*\prime} v^* \right)^2 \right] dx + \frac{1}{2}\rho_p A \int_b^L \dot{w}_2^{*2} dx
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
V = {} & \frac{1}{2}E_p A_p \int_0^b \left(u_1^{*\prime} + \frac{1}{2} w_1^{*\prime 2} \right)^2 dx + P\int_0^b \left(u_1^{*\prime} + \frac{1}{2} w_1^{*\prime 2} \right) dx + \frac{1}{2}E_p I_p \int_0^b w_1^{*\prime\prime} dx + \\
& \frac{1}{2}E_p A_p \int_b^L \left(u_2^{*\prime} + \frac{1}{2} w_2^{*\prime 2} \right)^2 dx + P\int_b^L \left(u_2^{*\prime} + \frac{1}{2} w_2^{*\prime 2} \right) dx + \frac{1}{2}E_p I_p \int_b^L w_2^{*\prime\prime} dx
\end{aligned} \tag{2}
$$

In the Eq. (2) above, first integrals are about the transformation, the second integral (P) is about axial stress and the third is about deflection. In the Eqs. (1-2), b stands for the distance of the support at the middle part to the starting point. ρ_p, ρ_f show the density of the pipe and fluid, A_p, A_f the section area of the pipe and the fluid, $w_{1,2}^*$ the transversal displacement of each part of the pipe among the supports, E_p, E_f the elasticity module of the pipe and fluid, respectively, $u_{1,2}^*$ the axial displacement of each part of the pipe among the supports, $(\dot{\ })$ the derivative on the basis of time and $(\ )'$ the derivative on the basis of x. The Lagrangian, kinetic and potential energy of the system are different. According to Hamilton's Principle, variation of the integral of Lagrangian on time is zero.

$$
\delta \int_{t_1}^{t_2} (T - V) dt = 0 \tag{3}
$$

In order to make the solutions independent of the material and geometric structure used and more general, the equations have to be nondimensionalized. The non-dimensional parameters required for transformations are defined as follows.

$$w_{1,2} = \frac{w_{1,2}^{*}}{L} \quad u_{1,2} = \frac{u_{1,2}^{*}}{L} \quad x = \frac{x^{*}}{L} \quad \eta = \frac{b}{L} \quad m_f = \rho_f A_f \quad m_p = \rho_p A_p \quad \beta = \frac{m_f}{m_f + m_p} \tag{4}$$

$$t = \sqrt{\frac{P}{(m_f + m_p)L^2}}\, t^{*} \quad v = \frac{v^{*}}{\sqrt{\frac{P}{(m_f + m_p)}}} \quad v_f^2 = \frac{E_p I_p}{PL^2} \quad v_b^2 = \frac{E_p A_p}{P} \tag{5}$$

Where v_b represents longitudinal stiffness, v_f pipe coefficient, v^* velocity of the fluid and β the rate of fullness. Nondimensionalized motion Eqs. (7-8) can be written as follows:

$$\ddot{w}_1 + 2\sqrt{\beta} v \dot{w}_1' + (v^2 - 1) w_1'' + \sqrt{\beta}\dot{v} w_1' + v_f^2 w_1^{IV} = \frac{1}{2} v_b^2 w_1'' \left(\frac{1}{2}\int_0^{\eta} w_1'^2 dx + \frac{1}{2}\int_{\eta}^{1} w_2'^2 dx \right) \tag{6}$$

$$\ddot{w}_2 + 2\sqrt{\beta} v \dot{w}_2' + (v^2 - 1) w_2'' + \sqrt{\beta}\dot{v} w_2' + v_f^2 w_2^{IV} = \frac{1}{2} v_b^2 w_2'' \left(\frac{1}{2}\int_0^{\eta} w_1'^2 dx + \frac{1}{2}\int_{\eta}^{1} w_2'^2 dx \right) \tag{7}$$

3. Perturbation Analysis

If our equation of motion is modified by transformation of degree $\sqrt{\varepsilon}$ this leads to nonlinear expressions at the degree ε.

$$\sqrt{\varepsilon} y_1 = w_1 \qquad \sqrt{\varepsilon} y_2 = w_2 \tag{8}$$

Change of velocity can be expressed on the basis of time and space and derivatives as an Eqs. (9-13).

$$v = v_0 + \varepsilon v_1 \sin\Omega t \qquad \dot{v} = \varepsilon v_1 \Omega \cos\Omega t \qquad v^2 = v_0^2 + 2\varepsilon v_0 v_1 \sin\Omega t \tag{9}$$

$$T_0 = t, \qquad T_1 = \varepsilon t \tag{10}$$

$$\frac{\partial}{\partial t} = D_0 + \varepsilon D_1 \qquad \frac{\partial^2}{\partial t^2} = D_0^2 + 2\varepsilon D_0 D_1 \tag{11}$$

$$y_1(x,t,\varepsilon) = y_{11}(x,t,\varepsilon) + \varepsilon y_{12}(x,T_0,T_1) \tag{12}$$

$$y_2(x,t,\varepsilon) = y_{21}(x,t,\varepsilon) + \varepsilon y_{22}(x,T_0,T_1) \tag{13}$$

Therefore, our equation of motion is like (14-15) for the regions I and II.

$$\begin{aligned} &\left(D_0^2 y_{11} + 2\varepsilon D_0 D_1 y_{11} + \varepsilon D_0^2 y_{12}\right) + 2\sqrt{\beta}\left(D_0 v_0 y_{11}' + \varepsilon D_1 v_0 y_{11}' + \varepsilon D_0 v_0 y_{12}' + \varepsilon D_0 v_1 \sin\Omega t\, y_{11}'\right) \\ &+ \left(v_0^2 y_{11}'' + 2\varepsilon v_0 v_1 \sin\Omega t\, y_{11}'' + \varepsilon v_0^2 y_{12}'' - y_{11}'' - \varepsilon y_{12}''\right) + \sqrt{\beta}\varepsilon v_1 \Omega \cos\Omega t\, y_{11}' \\ &+ v_f^2 y_{11}^{IV} + \varepsilon v_f^2 y_{12}^{IV} = \frac{1}{2}\varepsilon v_b^2 y_{11}'' \left[\frac{1}{2}\int_0^{\eta} y_{11}'^2 dx + \frac{1}{2}\int_{\eta}^{1} y_{21}'^2 dx \right] \end{aligned} \tag{14}$$

$$\left(D_0^{\,2}y_{21}+2\varepsilon D_0D_1y_{21}+\varepsilon D_0^{\,2}y_{22}\right)+2\sqrt{\beta}\left(D_0v_0y_{21}'+\varepsilon D_1v_0y_{21}'+\varepsilon D_0v_0y_{22}'+\varepsilon D_0v_1\sin\Omega t\,y_{21}'\right)$$
$$+\left(v_0^2y_{21}''+2\varepsilon v_0v_1\sin\Omega t\,y_{21}''+\varepsilon v_0^{\,2}y_{22}''-y_{21}''-\varepsilon y_{22}''\right)+\sqrt{\beta}\varepsilon v_1\Omega\cos\Omega t\,y_{21}' \tag{15}$$
$$+v_f^2y_{21}^{IV}+\varepsilon v_f^2y_{22}^{IV}=\frac{1}{2}\varepsilon v_b^2y_{21}''\left[\frac{1}{2}\int_0^{\eta}y_{11}'^2dx+\frac{1}{2}\int_{\eta}^{1}y_{21}'^2dx\right]$$

If the terms of degree 1 and ε are written separately, the equations of motion and limit conditions are found as follows.

Order (1):

$$D_0^{\,2}y_{11}+2\sqrt{\beta}D_0v_0y_{11}'+\left(v_0^2-1\right)y_{11}''+v_f^2y_{11}^{IV}=0 \tag{16}$$

$$D_0^{\,2}y_{21}+2\sqrt{\beta}D_0v_0y_{21}'+\left(v_0^2-1\right)y_{21}''+v_f^2y_{21}^{IV}=0 \tag{17}$$

Boundary Conditions:

$$y_{11}(0)=y_{21}(1)=0 \qquad y_{11}'(0)=y_{21}'(1)=0 \tag{18}$$

$$y_{11}(\eta)=y_{21}(\eta) \qquad y_{11}'(\eta)=y_{21}'(\eta) \qquad y_{11}''(\eta)=y_{21}''(\eta) \tag{19}$$

Order (ε):

$$\begin{aligned}&D_0^{\,2}y_{12}+2\sqrt{\beta}D_0v_0y_{12}'+\left(v_0^{\,2}-1\right)y_{12}''+v_f^2y_{12}^{IV}\\&=-2D_0D_1y_{11}-2\sqrt{\beta}D_1v_0y_{11}'-2\sqrt{\beta}D_0v_1\sin\Omega t\,y_{11}'\\&-\sqrt{\beta}v_1\Omega\cos\Omega t\,y_{11}'-2v_0v_1\sin\Omega t\,y_{11}''+\frac{1}{2}v_b^2y_{11}''\left[\frac{1}{2}\int_0^{\eta}y_{11}'^2dx+\frac{1}{2}\int_{\eta}^{1}y_{21}'^2dx\right]\end{aligned} \tag{20}$$

$$\begin{aligned}&D_0^{\,2}y_{22}+2\sqrt{\beta}D_0v_0y_{22}'+\left(v_0^{\,2}-1\right)y_{22}''+v_f^2y_{22}^{IV}\\&=-2D_0D_1y_{21}-2\sqrt{\beta}D_1v_0y_{21}'-2\sqrt{\beta}D_0v_1\sin\Omega t\,y_{21}'\\&-\sqrt{\beta}v_1\Omega\cos\Omega t\,y_{21}'-2v_0v_1\sin\Omega t\,y_{21}''+\frac{1}{2}v_b^2y_{21}''\left[\frac{1}{2}\int_0^{\eta}y_{11}'^2dx+\frac{1}{2}\int_{\eta}^{1}y_{21}'^2dx\right]\end{aligned} \tag{21}$$

Boundary Conditions:

$$y_{12}(0)=y_{22}(1)=0 \qquad y_{12}'(0)=y_{22}'(1)=0 \tag{22}$$

$$y_{12}(\eta)=y_{22}(\eta) \qquad y_{12}'(\eta)=y_{22}'(\eta) \qquad y_{12}''(\eta)=y_{22}''(\eta) \tag{23}$$

The equations at the degree of (1) (16-17) make up the linear equation of motion and the linear problem of limit conditions. We can suggest the Eqs. (24-25) for solution of the equation of linear motion.

$$y_{11}=A(T_1)e^{i\omega T_0}Y_1(x)+\overline{A}(T_1)e^{-i\omega T_0}\overline{Y}_1(x) \tag{24}$$

$$y_{21}=A(T_1)e^{i\omega T_0}Y_2(x)+\overline{A}(T_1)e^{-i\omega T_0}\overline{Y}_2(x) \tag{25}$$

If the solutions we suggest are written to their specific places, the Eqs. (26-27) is found.

$$v_f^2 Y_1^{IV} + (v_0^2 - 1)Y_1'' + 2\sqrt{\beta} v_0 Y_1' - \omega^2 Y_1 = 0 \tag{26}$$

$$v_f^2 Y_2^{IV} + (v_0^2 - 1)Y_2'' + 2\sqrt{\beta} v_0 Y_2' - \omega^2 Y_2 = 0 \tag{27}$$

This equation is a fourth degree, linear, ordinary differential equation. We can use the Eqs. (28-29) for solution of the Eqs. (26-27).

$$Y_1 = c_1 e^{i\gamma_1 x} + c_2 e^{i\gamma_2 x} + c_3 e^{i\gamma_3 x} + c_4 e^{i\gamma_4 x} \tag{28}$$

$$Y_2 = c_5 e^{i\gamma_5 x} + c_6 e^{i\gamma_6 x} + c_7 e^{i\gamma_7 x} + c_8 e^{i\gamma_8 x} \tag{29}$$

When we write the Eqs. (26-27) to its place in the equation of motion and equation of limit condition, the expressions (30-32) are found.

$$v_f^2 \gamma_n^{IV} + (1 - v_0^2)\gamma_n'' - 2\sqrt{\beta} v_0 \gamma_n' - \omega^2 \gamma_n = 0 \tag{30}$$

$$Y_1(0) = Y_2(1) = 0 \qquad Y_1'(0) = Y_2'(1) = 0 \tag{31}$$

$$Y_1(\eta) = Y_2(\eta) \qquad Y_1'(\eta) = Y_2'(\eta) \qquad Y_1''(\eta) = Y_2''(\eta) \tag{32}$$

Natural frequency values depending on the changes in the velocity of fluid were found for the values of η, v_f and β in the figures above. Fig. 2 shows the graph that indicates the change of v_f for the values η=0.1 and β=0.50. When the v_f value increases, the first three natural frequency values also increase. On the other hand, when the velocity of the fluid increases, the natural frequencies decrease. Fig. 3 shows the graph that indicates β change for the values η=0.1 and v_f=0.50. An increase in the value of β does not cause any change in first three natural frequency values (v_0=0), then decreases those values. When the velocity of the fluid increases, natural frequencies decrease. Fig. 4 shows the frequency values that indicate change of v_f for the values η=0.3 and β=0.50. When the v_f value increases, the values of natural frequencies increase. When the velocity of fluids increases, natural frequencies decrease. Fig. 5 shows the graph that indicates change of β for the values η=0.3 and β=0.50. When the β value increases, the values of natural frequencies increase. When the velocity of fluid increases, natural frequencies decrease. Fig. 6 shows frequency values that show the change of v_f pipe coefficient for the values η=0.5 and β=0.50. When the v_f value increases, the values of natural frequencies increase. When the velocity of fluid increases, natural frequencies decrease. Fig. 7 shows the natural frequency values that indicate change of β for the values η=0.5 and β=0.50. When the β value increases when the support is on the midpoint, the values of natural frequencies decrease. Fig. 8 shows natural frequency values that show the change of the position η for the values β =0.50 and v_f=0.50. When the location of support approaches the midpoint, frequency values increase in mode 1 and mode 3. The highest frequency value occurs in η=0.3 while the lowest frequency value occurs in η=0.1 for mode 2. For each case, when the velocity of fluid increases, the natural frequency values decrease. Fig. 9, Fig. 10 and Fig. 11 show mode structure graphs for different locations of the support (η=0.1-0.3-0.5) on the middle section for v_f=0.5, β=0.5 and v_0=2. Critical non-dimensional fluid velocity of the system was given in table 1 for β=0.5, v_f=0.5 and different η locations.

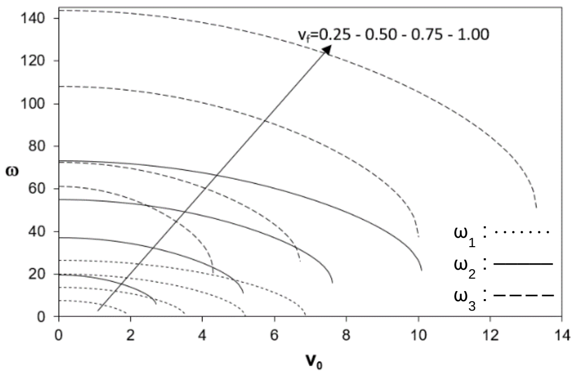

Fig. 2 Variation of the first three modes with fluid velocity for η =0.1, β =0.5 parameters and for different v_f values.

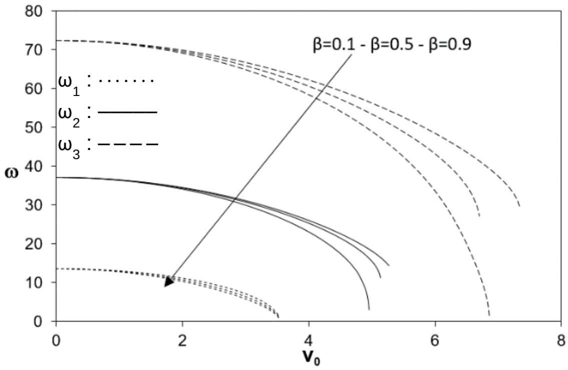

Fig. 3 Variation of the first three modes with fluid velocity for η =0.1, v_f=0.5 parameters and for different β values.

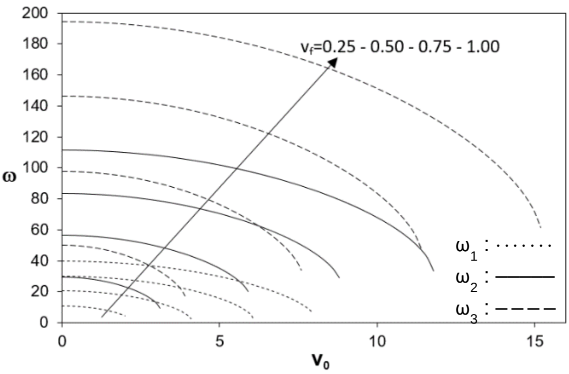

Fig. 4 Variation of the first three modes with fluid velocity for η =0.3, β =0.5 parameters and for different v_f values.

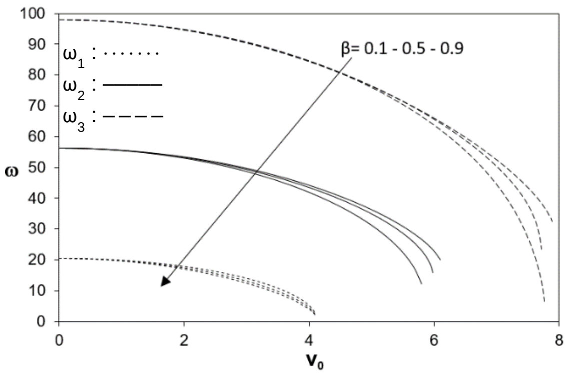

Fig. 5 Variation of the first three modes with fluid velocity for η =0.3, v_f=0.5 parameters and for different β values.

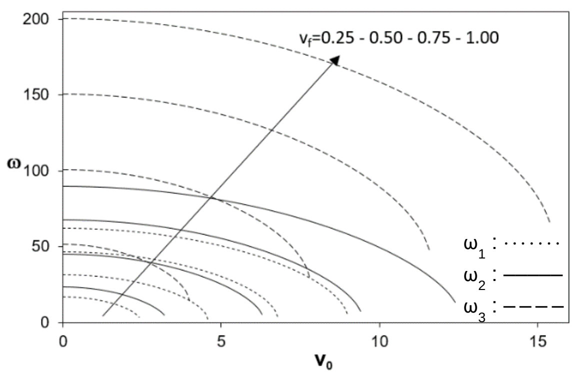

Fig. 6 Variation of the first three modes with fluid velocity for η =0.5, β =0.5 parameters and for different v_f values.

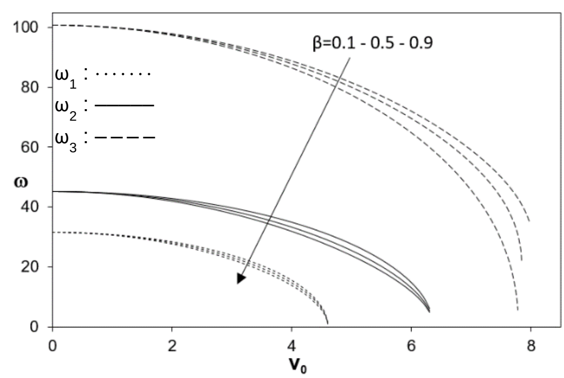

Fig. 7 Variation of the first three modes with fluid velocity for η =0.5, v_f=0.5 parameters and for different β values.

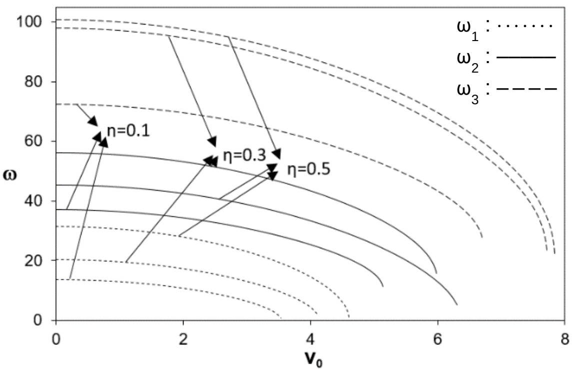

Fig. 8 Variation of the first three modes with fluid velocity for β=0.5, v_f=0.5 parameters and for different η locations.

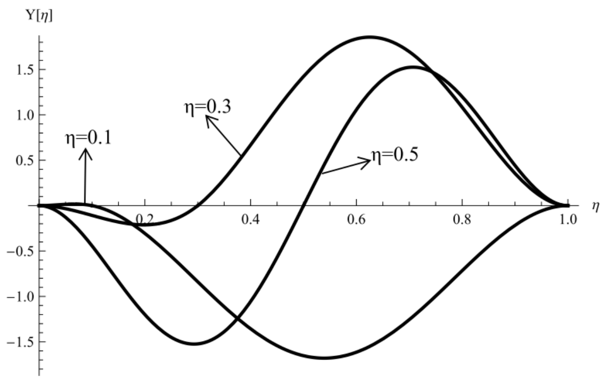

Fig. 9 First mode shapes for v_f=0.5, β=0.5, v_0=2 and different η locations.

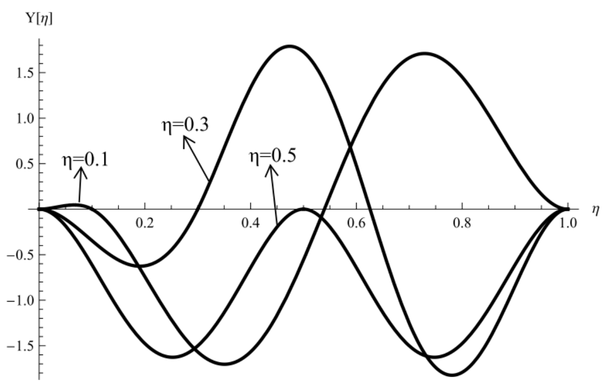

Fig. 10 Second mode shapes for v_f=0.5, β=0.5, v_0=2 and different η locations

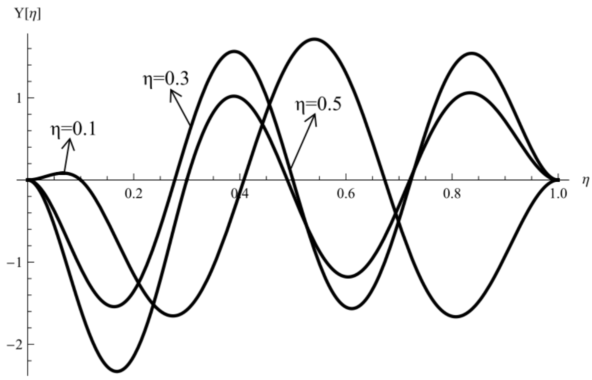

Fig. 11 Third mode shapes for v_f=0.5, β=0.5, v_0=2 and different η locations

Table 1 Variation of the first three modes with critical fluid velocity for β=0.5, v_f=0.5 and different η locations.

η	ω_1	ω_2	ω_3
0.1	3.53	5.13	6.71
0.3	4.13	5.99	7.77
0.5	4.6	6.36	7.84

4. Conclusions

This study discusses the fluid-carrying pipe with multiple supports and axial motion. A specific situation is examined where the pipe is attached to the ground with embedded supports on its ends and there is a simple support on the middle section. It is assumed that fluid velocity changes harmonically around an average velocity ($v(t)=v_0+\varepsilon v_1\sin\Omega t$). Taking into consideration the nonlinear effects caused by extension of pipes, equations of motion and limit conditions were found by Hamilton's principle. Such equations of motion were non-dimensionalized, hence the dependence on material and geometric structure was eliminated. Approximate solutions were found using the multiple time-scaled method. The first term in the perturbation series constitutes the linear problem. Solution of the linear problem enabled precise calculation of natural frequencies for different positions of the support at the middle part, different pipe coefficients, different rates of fullness and values of fluid velocity. First, second and third natural frequencies were found for different parameters from the solution of the linear problem and these results were shown as graphs. According to these results, an increase in pipe coefficient for all locations of support also increases the values of natural frequency. On the other hand, when the velocity of fluid increases, natural frequencies tend to decrease. Values of natural frequency cannot be obtained after a certain value of velocity, which is considered critical velocity. For all positions of support, when the rate of fullness of the fluid-carrying pipe increases, the values of natural frequencies start from the same point. As the rate of fullness increases, frequencies decrease for the same value of fluid velocity. Whereas, when the location of support is close to the midpoint, natural frequency values increase. The highest frequency value occurs at the position η=0.3 and the lowest frequency value occurs at the position η=0.1 only for the second natural frequency values. When the fluid velocity increases, natural frequencies tend to decrease.

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