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Research Article

## Optimum design of anti-buckling behavior of graphite/epoxy laminated composites by differential evolution and simulated annealing method

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### Article Info

### Abstract

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Stacking sequence design and optimization of 64 layered symmetric-balance graphite/epoxy laminated composite have been performed. The optimization problems aim to find the optimum stacking sequence maximizing the critical buckling load by single objective optimization approach. Differential Evolution (DE) and Simulated Annealing (SA) optimization algorithms are proposed to solve the problems. The effect of the aspect ratios ( $a/b$ ) and in-plane biaxial compressive loading ratios ( $N_x/N_y$ ) on critical buckling load are investigated. In order to see the effect of discrete increments of fiber orientation angle on critical buckling load, 1°, 5°, 15°, 30° and 45° fiber angle increments are also considered. The results show that (i) the proposed algorithms DE and SA exhibit comparable performance in terms of critical buckling load when compared Genetic algorithm (GA) and Generalized pattern search algorithm (GPSA), (ii) DE and SA find distinct stacking sequence configurations in terms of buckling load for the same laminated structure design problems.

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## 1. Introduction

In recent years, laminated composites have been very popular due to their high specific modulus and high specific strength values in manufacturing industry both in high-class industries like aerospace applications and middle-class industries such as marine, automotive and military applications [1, 2]. In addition to these characteristics of the laminated structures, fiber reinforced composites have a distinctive feature that allows the structural properties of composite materials such as fiber orientation and stacking sequence to be adjusted. These distinguishing features provide great possibilities for designers as an alternative to isotropic materials. Despite all of these superior properties of these materials, there are critical problems in some specific working conditions. These special conditions can be classified as overstress, over deflection, resonant vibration, and buckling. It can be said that the determination of the buckling load capacity of the laminated composite plate under the in-plane compressive loads is very important for the design of composite structures [1, 3].

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Laminated composite plates are frequently exposed to uniaxial or biaxial pressures depending on the site of use. Thin and wide composite plates are exposed to a load in the compression plane, buckling in the plate is considered a critical failure mode. Determination of the buckling load capacity under in plane biaxial compression during the design of composite plates is crucial for understanding the post-buckling behavior especially in the engineering applications such as aircraft, automotive and ships design. Many studies have been carried out in the literature to solve this problem [1, 3]. For this purpose, Chao et al. [4] studied the optimization of buckling load under uniaxial compression conditions. Erdal and Sonmez [5] determined optimum stacking sequence design of composite plate to maximize critical buckling load. Aymerich and Serra [6] maximized buckling load capacity of laminated composite plate under strength, ply-contiguity and symmetric-balance constraints. Deveci *et al.* [7] obtained optimum integer and discrete stacking sequences of laminated composite plates for maximum buckling load capacity considering Puck fiber and inter-fiber failure (IFF) criteria as nonlinear constraints. Many stochastic optimization methods have been used to optimize buckling behavior. Kim and Lee [8] reported that Genetic Algorithms (GA), Generalized Pattern Search Algorithm (GPSA), Differential Evolution (DE) and Simulated Annealing (SA) algorithms are suitable for the solution of buckling problems. Karakaya and Soykasap [9] used the GA method to maximize the critical buckling load factor of composite plates and designed optimum ones with conventional fiber angles.

In the literature, studies on optimizing the stacking sequence of laminate composites have been carried out with GPSA [9], SA [10], Scatter Search [5], Tabu Search [11], Ant Colony Optimization (ACO) [6, 12] and GA, SA [13] algorithms in order to maximize the buckling load factor.

In the present study, in order to maximize critical buckling load factor (objective function), single objective optimization of 64 layered symmetric-balance graphite/epoxy laminated composite plate is considered by utilizing modified version of two stochastic optimization methods: Differential Evolution and Simulated Annealing. Fiber orientations are selected as design variables. The effect of aspect ratio and loading ratios on critical buckling load are also investigated.

## 2. Mechanical Analysis

The used laminated composite plate is simply supported on four edge and specially orthotropic. The geometric dimension and fiber configuration of plate are length  $a$ , width  $b$ , total thickness  $h$  and fiber orientation angle  $\theta$  in the  $x, y, z$  and  $l$  directions, respectively (Fig 1). The composite plate is subjected to biaxial in-plane loads per unit length  $N_x$  and  $N_y$ . The material of the composite laminated plate is assumed to be homogeneous and the layers have equal thickness. The governing equation of the buckling process based on the classical laminated plate theory for the described symmetric laminate is given as follow [14]

$$D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} = \lambda \left( N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + N_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (1)$$

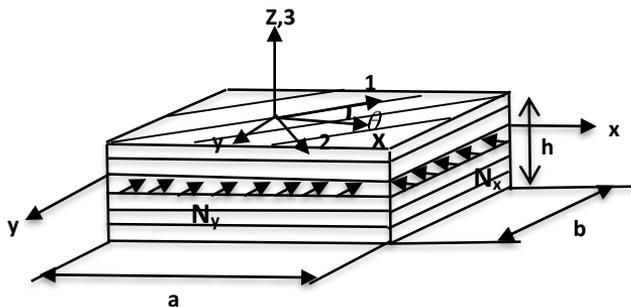


Fig. 1 Thin laminated composite plate subject to in-plane loading

where  $w$  is the deflection in the  $z$  direction and  $D_{ij}$  is the bending stiffness as

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \tag{2}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N \int_{z_k}^{z_{k+1}} \bar{Q}_{ij}^{(k)} z^2 dz \quad i, j = 1, 2, 6 \tag{3}$$

where  $N$  is the total number of plies,  $k$  is the ply number and  $\bar{Q}_{ij}^{(k)}$  is the transformed reduced stiffness of the  $k$ th layer

The boundary conditions for the simply supported plate are given as

$$\begin{aligned} w = 0 \text{ at } x = 0, \quad w = 0 \text{ at } y = 0 \\ M_x = 0 \text{ at } x = 0, \quad M_y = 0 \text{ at } y = 0 \end{aligned} \tag{4}$$

where, the moment resultants are defined as:

$$(M_x, M_y) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y) dz \tag{5}$$

where,  $\sigma_x, \sigma_y$  are the normal stress resultants in  $x$  and  $y$  directions.

For specially orthotropic laminate, the fiber configurations consist of only  $0^\circ$  and  $90^\circ$  and in this case, the element of stiffness matrix:  $A_{16}=A_{26}=B_{16}=B_{26}=D_{16}=D_{26}=0$ . Nemeth [15] has given the detail explanation about the usage of specially orthotropic case in composite laminates for buckling problems. If the laminated composite is not specially orthotropic, the bending-twisting terms  $D_{16}$  and  $D_{26}$  will be neglected only when the non-dimensional parameters fulfill the conditions.

$$\gamma \leq 0.2, \quad \delta \leq 0.2 \tag{6}$$

where,

$$\begin{aligned} \gamma &= D_{16} (D_{11}^3 D_{22})^{-1/4} \\ \delta &= D_{26} (D_{11} D_{22}^3)^{-1/4} \end{aligned} \tag{7}$$

With the substitution of Eq. (2) into Eq. (1) under Eq. (4) boundary condition, and solving eigen function problem, buckling load factor expression can be obtained as [14]

$$\lambda_b = \frac{\pi^2 \left[ D_{11} \left( \frac{m}{a} \right)^4 + 2(D_{12} + 2D_{66}) \left( \frac{m}{a} \right)^2 \left( \frac{n}{b} \right)^2 + D_{22} \left( \frac{n}{b} \right)^4 \right]}{N_x \left( \frac{m}{a} \right)^2 + N_y \left( \frac{n}{b} \right)^2 + N_{xy} \left( \frac{m}{a} \right) \left( \frac{n}{b} \right)} \quad (8)$$

where,  $m$  and  $n$  are integer numbers corresponding to different modes shapes;  $\lambda_b$  is buckling load factor;  $N_x$  and  $N_y$  are applied loads. Buckling loads are identified as  $N_{xb}=N_x \lambda_b$  and  $N_{yb}=N_y \lambda_b$ . Critical load factor  $\lambda_{cb}$  is the lowest buckling load factor, and it can be found by using appropriate combinations of  $m$  and  $n$ . For the present problem,  $m$  and  $n$  are taken to be 1 or 2, and hence the smallest of  $\lambda_b$  (1,1),  $\lambda_b$  (1,2),  $\lambda_b$  (2,1),  $\lambda_b$  (2,2) yields  $\lambda_b$ . Also, the critical load equals to  $\lambda_{cb}$  when unit loads are applied.

### 3. Optimization Algorithms

Optimization techniques can be divided into two main categories as traditional and non-traditional. One of the traditional optimization techniques Lagrange Multipliers is used to find the optimum solution of only continuous and differentiable functions. Due to the fact that the design problems of the composites have discrete search spaces, the nontraditional optimization techniques can be used. In these cases, the stochastic optimization methods such as Genetic Algorithms (GA), Generalized Pattern Search Algorithm (GPSA), Ant Colony Optimization (ACO), Differential Evolution (DE) and Simulated Annealing (SA) can be utilized. A detail explanation about stochastic optimization methods can be found in Rao [16] and in Gurdal *et al.* [17] for composite design problems and various applications. In this study, DE and SA methods are utilized to solve the defined laminated composite optimization problems. Related parameters of the algorithms used in adjusting the options correctly are listed in Tables 1.

#### 3.1 Differential Evolution Method

Differential Evolution is one of the stochastic optimization methods and a preferable to use in complex structured composite design problems such as a finding of critical buckling load, estimation of the natural frequency of the system and acquire the lightweight design. Differential Evolution algorithm contains the four main stages: initialization, mutation, crossover and selection as shown in Fig. 2. To find the optimum result, the effective parameters of the algorithm: scaling factor, crossover and population size. For more information about DE, can be referred to Storn and Price [18]. A population of solutions is handled instead of a single solution at each iteration in DE algorithm and also this algorithm is computationally expensive. Even if DE Algorithm is not guaranteed to find the global optima for all types of optimization problems, in some studies it is shown that it is relatively robust and efficient in finding global optimum compared to the other search techniques. In the Mathematica implementation of DE algorithm, it considers a population of  $r$  points,  $\{\theta_1, \theta_2, \dots, \theta_j, \dots, \theta_r\}$ . It is important that  $r$  should be much greater than the number of design variables. At the iteration process, firstly, the algorithm generates a new population that is produced by selecting random points. By introducing the real scaling factor as "rsf" and defining  $\theta_{rsf} = \theta_w + rsf(\theta_u - \theta_v)$ ,  $i^{\text{th}}$  iteration points can be obtained from the previous population. Secondly, a new point  $\theta_{new}$  is established by selecting  $j^{\text{th}}$  coordinate from  $\theta_{rsf}$  with probability  $P$ . In Mathematica

software,  $P$  can be adjusted by the option "CrossProbability". In that step, if the constraint  $f(\theta_{\text{new}}) < f(\theta_i)$  is valid then  $\theta_i$  is taken instead of  $\theta_{\text{new}}$  in the population. Stopping criteria for this process is that

- (i) if the difference between the optimum output values at the new and old populations,
- (ii) the difference between two (the new and old) points the new best points are less than the tolerances provided by the parameters which specifies how many effective digits of accuracy and precision should be sought in the final result.

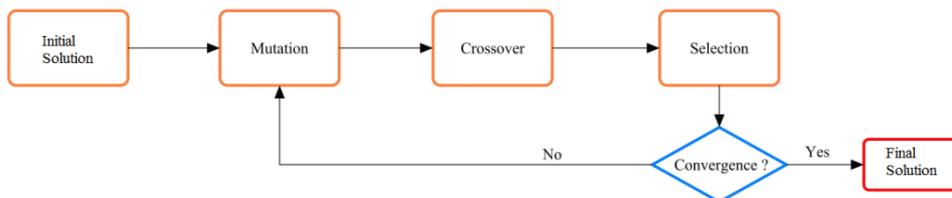


Fig. 2 Flowchart of the DE algorithm [19]

### 3.2 Simulated annealing

SA is the most popular search method based on the physical process of the annealing related to a metal object which is heated to a high temperature and permit to cool slowly. During the melting process, the material becomes a tougher material by means of the atomic structure settle to a lower energy state. In the optimization problems, SA algorithm can arrive at a better global optimum thanks to the annealing process allows the structure to get away from a local minimum. It is possible to solve mixed-integer, discrete, or continuous optimization problems by using SA. The main advantage related to SA is to be a talented algorithm for these problems [20].

Mathematica implementation of SA can be briefly explained as follows:

Firstly, an initial guess is introduced as  $\theta_{in}$ , Secondly, a new point,  $\theta_{\text{new}}$ , is produced in the neighborhood of the current point,  $\theta$  at each iteration so far,  $\theta_{\text{best}}$ , is also tracked. The main idea is here that the radius of the neighborhood decreases with the iteration. If

$f(\theta_{\text{new}}) < f(\theta_{\text{best}})$ ,  $\theta_{\text{new}}$  replaces  $\theta_{\text{best}}$  and  $\theta$ . Otherwise,  $\theta_{\text{new}}$  replaces  $\theta$  with a probability.

The distance of the new point from the current point is based on Boltzmann's probability distribution  $e^{B(i, \Delta f, f_0)}$ .

In this distribution "B" is the function defined by Boltzmann Exponent,  $i$  is the current iteration,  $\Delta f$  is the change in the fitness function value, and  $f_0$  represents the value of the objective function from the  $(i-1)$ <sup>th</sup> iteration. B is  $\frac{-\Delta f \log(i+1)}{10}$  if it is not introduced by the user. Instead of only one initial guess the Mathematica command "Simulated Annealing" uses two or more starting points. The number of initial points is given by the option Search Points, and is calculated as  $\min(2r, 50)$ , where  $r$  is the number of variables.

For each starting point, this is repeated until the maximum number of iterations is reached, the method converges to a point, or the method stays at the same point consecutively for the number of iterations given by Level Iterations.

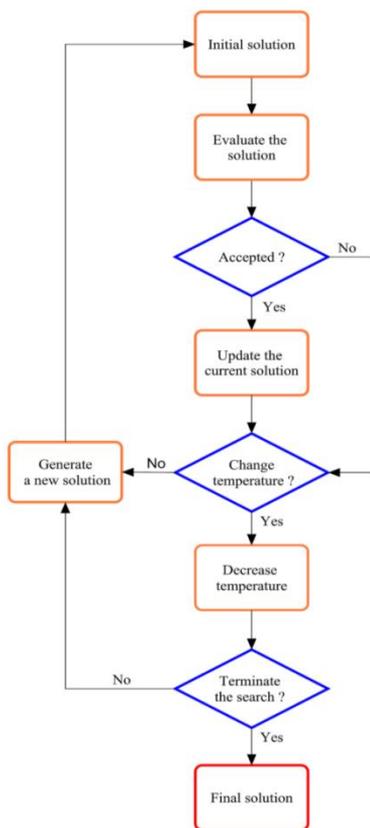


Fig. 3 Flowchart of the SA algorithm [21]

Table 1 Two optimization methods options

Options Name	DE	SA
Cross Over fractions	0.5	-
Random Seed	0	0
Scaling Factor	0.6	-
Tolerance	0.001	0.001
Mutation Fraction	0.1	-
Level Iterations	-	50
Perturbation Scale	-	1.0
Search Point	-	3000

#### 4. Problem Definition

In this study, the optimum stacking sequence designs of 64 layered symmetric-balance graphite/epoxy laminated composite plates have been investigated. Single-objective

optimization formulation has been used for mathematical verification of model problems. The critical buckling load factor ( $\lambda_{cb}$ ) is considered as the objective function. Fiber orientation angles of laminated composite plate are taken as discrete design variables. The optimization has been conducted for various aspect ratios ( $a/b$ ) and in plane biaxial compressive loading ratios ( $N_x/N_y$ ) using Differential Evolution (DE) and Simulated Annealing (SA) stochastic algorithms. In the design process, thickness of each layer is 0.127 mm and  $N_x$  has been taken as 1 N/m.  $N_y$  has been calculated from the load ratio ( $N_x/N_y$ ). In Tables 2 and 3, detailed descriptions of material properties and load cases are introduced.

Table 2 The elastic properties of graphite/epoxy layers [9]

Parameters	Graphite/Epoxy
$E_1$ , Longitudinal Modulus (MPa)	127600
$E_2$ , Transverse Modulus (MPa)	13000
$G_{12}$ , In-plane Shear Modulus (MPa)	6400
$\nu_{12}$ , Poisson's ratio	0.3
$t$ , Ply thickness (m)	$0.127 \times 10^{-3}$

Table 3 Composite plate load cases

Load case	a (m)	b (m)	$N_x$ (N/m)	$N_y$ (N/m)
LC1	0.508	0.254	1	1
LC2	0.508	0.508	1	1
LC3	0.508	1.016	1	1
LC4	0.508	0.254	1	0.5
LC5	0.508	0.508	1	0.5
LC6	0.508	1.016	1	0.5
LC7	0.508	0.254	1	2
LC8	0.508	0.508	1	2
LC9	0.508	1.016	1	2

In the following part, descriptions of the model problems (problems 1 and 3) are introduced.

### Problem 1

The problem given in Karakaya and Soykasap [9] and Deveci [22] is selected as benchmark. Stacking sequence design and optimization of 64 layered symmetric-balance graphite/epoxy composites are considered so as to maximize critical buckling load. The considered laminated composite plate is subjected to  $N_x=1$  N/m,  $N_y=1$  N/m (LC1) and  $N_x=1$  N/m,  $N_y=2$  N/m (LC9) in-plane biaxial compressive loading. Fiber orientation angles of the layers are considered in the range of 0 and 90° with 45° increments for the aspect ratio  $a/b=2$  (LC1) and  $a/b=0.5$  (LC9). The stochastic optimization methods DE, SA are

utilized and the performances of these methods are compared with those of Generalized Pattern Search Algorithm (GPSA) by Karakaya and Soykasap [9] and Genetic Algorithm (GA) by Karakaya and Soykasap [9] and Deveci [22] in terms of critical buckling load.

**Problem 2**

Organization of the problem is similar to problem 1 such that single objective optimization in order to maximize critical buckling load comprising aspect ratios  $a/b=2$  (LC1) and  $a/b=0.5$  (LC9) of the laminated composite plates for biaxial compressive loading  $N_x= 1$  N/m,  $N_y=1$  N/m (LC1) and  $N_x= 1$  N/m,  $N_y=2$  N/m (LC9). However, in order to see the effect of discrete increments of fiber orientation angle on the critical buckling load,  $1^\circ$ ,  $5^\circ$ ,  $15^\circ$ ,  $30^\circ$  and  $45^\circ$  fiber angle increments are considered. The stochastic optimization methods DE and SA are utilized.

**Problem 3**

The aim of the optimization problem is to determine the effect of the aspect ratios  $a/b$  and in-plane biaxial compressive loading ratios  $N_x/N_y$  for all the cases (LC1-LC9) on critical buckling load. Fiber orientation angles of the layers are considered in the range of  $0$  and  $90^\circ$  with  $5^\circ$  increments. The stochastic optimization method DE is used for the optimization process.

**5. Results and Discussions**

In this section, the results of buckling problems (1-3) are given based on DE and SA methods. Table 4 shows the result of the Problem 1. This problem is solved to validate the proposed Differential Evolution and Simulated Annealing optimization algorithms. It is shown that DE and SA exhibit comparable performance in terms of critical buckling load when compared Generalized Pattern Search Algorithm (GPSA) by Karakaya and Soykasap [9] and Genetic Algorithm (GA) by Karakaya and Soykasap [9] and Deveci [22]. As it is seen from the results given in the Table 4 that stochastic algorithms (DE and SA) work correctly thus they have a potential to obtain reliable results for the Problems 2 and 3.

Table 4 Optimum stacking sequence designs for load cases 1 and 9 under  $45^\circ$  increment using DE and SA

Loading cases	Stacking sequence (DE)	Stacking sequence (SA)
LC1	$[90_8/\pm 45_2/90_4/\pm 45_3/\pm 45_5]_s$	$[90_4/\pm 45/90_6/\pm 45/90_4/\pm 45_2/90_4/\pm 45/90_4]_s$
LC9	$[\pm 45_4/0_8/0_4/\pm 45_3/0_2/\pm 45_2]_s$	$[0_6/90_6/90_2/\pm 45/0_6/\pm 45_3/90_2/0_2]_s$

Table 4 cont. Optimum stacking sequence designs for load cases 1 and 9 under  $45^\circ$  increment using DE and SA

Loading cases	$\lambda_{cb}$ [9]	$\lambda_{cb}$ [22]	$\lambda_{cb}$ ( Present DE)	$\lambda_{cb}$ ( Present SA)
LC1	695,781.3	695,663.1	695,822.2	695,822.2
LC9	132,243.5	132,237.8	132,244.6	132,232.9

Table 5 shows optimum stacking sequence designs of 64-layered symmetric-balance graphite/epoxy laminated composites for maximum critical buckling load utilizing the proposed DE algorithm. The increments of fiber orientation angles of the plies are selected as 1°, 5°, 15°, 30° and 45°. For LC1, the buckling load values of graphite/epoxy composite vary in the range of 695,822.2 and 722,978.4 N/m. The lowest buckling load value (695,822.2) of graphite/epoxy is gained for the configuration  $[90_8/\pm 45_2/90_4/\pm 45_8]_s$  while the highest value of that (722,978.4) is obtained for the configuration  $[(\pm 72/\pm 73)_2/\pm 72_2/\pm 74/\pm 73/\pm 72/\pm 71/\pm 72/\pm 71/\pm 74/\pm 81/\pm 66/90_2]_s$ . For the case LC9, the buckling load values of graphite/epoxy composite vary in the range of 132,244.6 and 140,664.3 N/m. The lowest buckling load value (132,244.6) of graphite/epoxy is obtained for  $[(\pm 45/0_2)_4/0_4/\pm 45_3/0_2/\pm 45_2]_s$  while the highest value of that (140,664.3) is obtained for the configuration  $[\pm 27_5/\pm 28/\pm 27/\pm 26/\pm 27/\pm 28/\pm 27/\pm 26/\pm 30/\pm 29/\pm 23/\pm 27]_s$ .

The same problem (Problem 2) have also been solved by SA and the maximum critical buckling load values corresponding to stacking sequences are given in Table 6. For the case LC1, the buckling load values of graphite/epoxy composite vary between 695,822.2 and 721,500.6 N/m. Unlike DE method, the maximum critical buckling load value of graphite/epoxy composite is obtained for 5° fiber angle increment by SA. Even though the stacking sequences design of laminated composite under symmetric balance constraint based on DE and SA are different for 30° and 45° fiber angle increments, the maximum critical buckling load values are the same. When it is compared for 1°, 5° and 15° fiber angle increments, both optimum stacking sequence and critical buckling load have been found as distinct and DE gives higher critical buckling load values than that of SA.

Table 5 Optimum stacking sequence designs for load cases 1 and 9 under different fiber orientation increments using DE (D1, D2, D3, D4 and D5, correspond to 1°, 5°, 15°, 30° and 45° increments, respectively).

Loading Cases	LC1	LC9
Stacking Sequence (D1)	$[(\pm 72/\pm 73)_2/\pm 72_2/\pm 74/\pm 73/\pm 72/\pm 71/\pm 72/\pm 71/\pm 74/\pm 81/\pm 66/90_2]_s$	$[\pm 27_5/\pm 28/\pm 27/\pm 26/\pm 27/\pm 28/\pm 27/\pm 26/\pm 30/\pm 29/\pm 23/\pm 27]_s$
Stacking Sequence (D2)	$[\pm 70/\pm 75_2/\pm 70/\pm 75/\pm 70_2/\pm 75_2/\pm 70_2/\pm 75_3/\pm 80/\pm 50]_s$	$[\pm 25_2/\pm 30_2/\pm 25/(\pm 25/\pm 30)_3/\pm 30/\pm 25_2/\pm 30/0_2]_s$
Stacking Sequence (D3)	$[\pm 75_6/\pm 60/\pm 75/\pm 60_3/\pm 75/90_8]_s$	$[\pm 30_4/\pm 15/\pm 30/(\pm 30/\pm 15)_2/\pm 15/\pm 30_2/\pm 15_2/\pm 30]_s$
Stacking Sequence (D4)	$[90_2/(90_2/\pm 60)_4/\pm 60_3/(90_2/\pm 60)_2]_s$	$[\pm 30_5/0_2/\pm 30_2/0_2/\pm 30_2/0_4/\pm 30_3]_s$
Stacking Sequence (D5)	$[90_8/\pm 45_2/90_4/\pm 45_8]_s$	$[(\pm 45/0_2)_4/0_4/\pm 45_3/0_2/\pm 45_2]_s$

Table 5 cont. Optimum stacking sequence designs for load cases 1 and 9 under different fiber orientation increments using DE (D1, D 2, D3, D4 and D5, correspond to 1°, 5°, 15°, 30° and 45° increments, respectively).

Loading Cases	$\lambda_{cb}$ (D1)	$\lambda_{cb}$ (D2)	$\lambda_{cb}$ (D3)	$\lambda_{cb}$ (D4)	$\lambda_{cb}$ (D5)
LC1	722,978.4	722,659.5	720,153.0	714,584.3	695,822.2
LC9	140,664.3	140,510.2	139,954.5	139,660.0	132,244.6

Table 6 Optimum stacking sequence designs for load cases 1 and 9 under different fiber orientation increments using SA. (D1, D 2, D3, D4, D5, corresponds to 1°, 5°, 15°, 30°, 45° increments, respectively)

Loading Cases	LC1	LC9
Stacking Sequence(D1)	$[\pm 69/\pm 73_2/90_2/\pm 72/\pm 78/\pm 65/\pm 82/\pm 62/\pm 65/\pm 67/\pm 71/\pm 76/90_2/\pm 62/\pm 63]_s$	$[\pm 29/\pm 27/\pm 29_2/\pm 28/\pm 27_2/\pm 20/\pm 5/\pm 33/\pm 10/\pm 30/\pm 44/\pm 46/\pm 63/\pm 12]_s$
Stacking Sequence(D2)	$[\pm 70_2/\pm 75/\pm 80/\pm 75/\pm 70/\pm 80/\pm 65/\pm 80/\pm 60/\pm 65/\pm 75/\pm 85/\pm 80/\pm 60/\pm 50]$	$[\pm 25/\pm 35/\pm 20/\pm 35/\pm 30/\pm 25_3/\pm 20/\pm 25/\pm 10/\pm 30/\pm 35/\pm 20/\pm 15/\pm 25]_s$
Stacking Sequence(D3)	$[\pm 75_4/90_2/\pm 75/\pm 60_3/90_2/\pm 60_3/\pm 75_2/\pm 60]_s$	$[\pm 30_5/\pm 15/\pm 30/\pm 15_2/\pm 30_2/\pm 15_2/0_2/\pm 75/0_2]_s$
Stacking Sequence(D4)	$[90_4/\pm 60_2/90_6/\pm 60_4/90_2/\pm 60_3/90_2]_s$	$[\pm 30_6/0_4/\pm 30_2/0_2/\pm 30_2/0_2/\pm 30/\pm 6/0]_s$
Stacking Sequence(D5)	$[90_4/\pm 45/90_6/\pm 45/90_4/\pm 45_2/90_4/\pm 45/90_4]_s$	$[0_6/90_6/90_2/\pm 45/0_6/\pm 45_3/90_2/0_2]_s$

Table 6 cont. Optimum stacking sequence designs for load cases 1 and 9 under different fiber orientation increments using SA. (D1, D 2, D3, D4, D5, corresponds to 1°, 5°, 15°, 30°, 45° increments, respectively)

Loading Cases	$\lambda_{cb}$ (D1)	$\lambda_{cb}$ (D2)	$\lambda_{cb}$ (D3)	$\lambda_{cb}$ (D4)	$\lambda_{cb}$ (D5)
LC1	721,044.0	721,500.6	719,280.4	714,584.3	695,822.2
LC9	140,013.0	139,691.0	139,734.4	139,649.0	132,232.9

In order to ensure convergence performances of the proposed algorithms, two convergence graphs have been performed and presented in Fig. 4. It is seen that both of the algorithms show good performance to reach optimum fitness values.

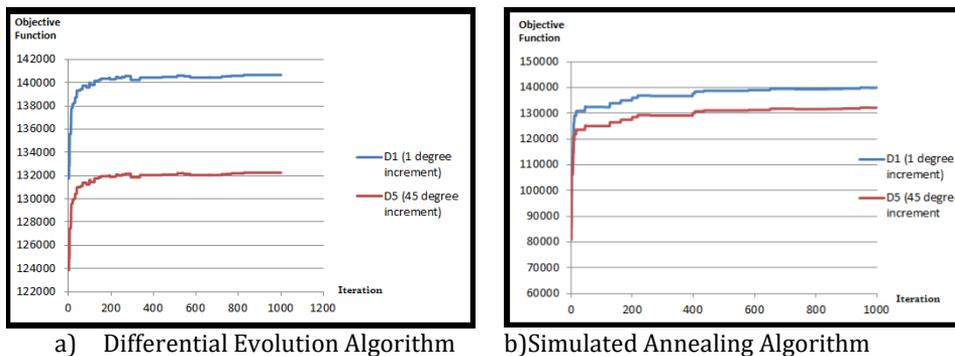


Fig. 4 The convergence graphs based on DE and SA for LC9

Figure 5 shows the effect of loading cases including different aspect ratios  $a/b = 0.5, 1, 2$  and loading ratios  $N_x/N_y = 0.5, 1$  and  $2$  on critical buckling load (Problem 3). According to results, it can be stated that (i) Load case 4 ( $a/b = 2; N_x/N_y = 2$ ) gives the highest critical buckling load, (ii) Load case 9 ( $a/b = 0.5; N_x/N_y = 0.5$ ) gives the lowest critical buckling load, (iii) Load case 3, 6 and 8 give the approximately the same result.

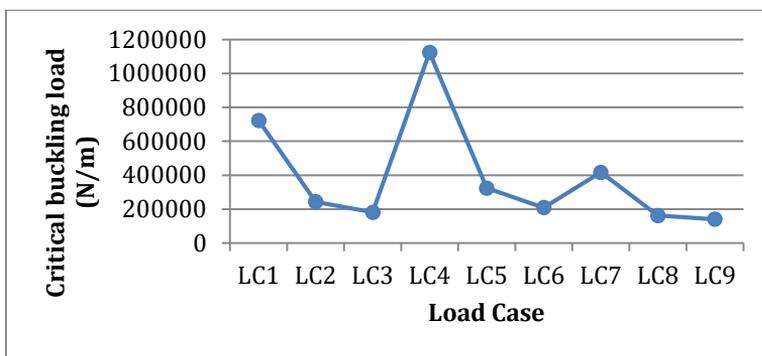


Fig. 5 The effect of load case on critical buckling load (Problem 3)

Table 7 shows the optimum stacking sequence designs of 64-layered symmetric-balance graphite/epoxy laminated composites for maximum critical buckling load utilizing the proposed DE algorithm (Problem 3). The increments of fiber orientation angles of the plies are selected as  $5^\circ$ . According to results, the stacking sequence configurations of laminated composite plates are obtained as  $[\pm 45_{16}]_s$  for LC2, LC5 and LC8 cases. It is mean that Although biaxial compressive loading ratios ( $N_x/N_y$ ) are distinct for Square composite plates ( $a/b=1$ , shown in Table 3 for LC2, LC5 and LC8), stacking sequence configurations do not change.

Table 7 Optimum stacking sequence designs for load cases 1 - 9 under 5-degree fiber orientation increments using DE.

Loading Cases	Stacking Sequence	$\lambda_{cb}$
LC1	$[\pm 70/\pm 75_2/\pm 70/\pm 75/\pm 70_2/\pm 75_2/\pm 70_2/\pm 75_3/\pm 80/\pm 50]_s$	722,659.5
LC2	$[\pm 45_{16}]_s$	242,844.4
LC3	$[\pm 20_2/\pm 15_5/\pm 20_4/\pm 15/\pm 20/\pm 15/\pm 25/0]_s$	180,667.2
LC4	$[\pm 60/\pm 65/\pm 60/\pm 65_4/\pm 60_2/\pm 65_3/\pm 60/\pm 65_2/\pm 60]_s$	1,124,072.3
LC5	$[\pm 45_{16}]_s$	323,792.5
LC6	$[\pm 5_2/\pm 10_2/\pm 5/\pm 10_2/\pm 5/\pm 10/0_4/\pm 5_2/0_6]_s$	208,273.4
LC7	$[90_2/\pm 85/\pm 80/\pm 85_2/\pm 80/\pm 85_3/\pm 80/90_2/\pm 85_3/\pm 80/90_2]_s$	416,550.6
LC8	$[\pm 45_{16}]_s$	161,896.2
LC9	$[\pm 25_2/\pm 30_2/\pm 25_4/\pm 30_4/\pm 25_2/\pm 30/0_2]_s$	140,510.2

## 6. Conclusion

In this study, the optimum stacking sequence designs of 64 layered symmetric-balance graphite/epoxy laminated composite plates have been investigated using Differential Evolution (DE) and Simulated Annealing (SA) algorithms. For this reason, three optimization problems have been introduced to see the effect of different load cases (LC1-LC9) correspond to aspect ratios  $a/b$  and loading ratios  $N_x/N_y$  on critical buckling load. Also the effect of fiber orientation angle increments ( $1^\circ$ ,  $5^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ) on buckling load are investigated. It can be concluded from the results that

- The stochastic optimization algorithms DE and SA have been performed for the same laminated composite design problems (Problem1 and Problem 2), successfully. Thus, this attempt has improved reliability and robustness of the process and also provided to avoid inherent scattering of the proposed algorithms. According to given convergence graphs, it is seen that both of the algorithms show good performance to reach optimum fitness values.
- The results based on stochastic optimization algorithms DE and SA have been compared to results given in Karakaya and Soykasap [9] by GA and GPSA methods and Deveci [22] by GA method for the same laminated composite structure design and optimization problems. Regarding the results, DE and SA show comparable performance to obtain the maximum critical buckling load.
- The critical buckling load performance of graphite/epoxy laminated composite obtained using conventional design variable ( $45^\circ$  increment) is approximately 3.75% and 5.5% lower than that obtained utilizing  $1^\circ$  increment design variable for LC1 and LC9, respectively. These results are also valid for both Table 5 and Table 6.
- LC 4 ( $a/b=2$ ;  $N_x/N_y=2$ ) gives the highest critical buckling load and LC 9 ( $a/b=0.5$ ;  $N_x/N_y=0.5$ ) gives the lowest critical buckling load for 64 layered graphite/epoxy composite.

## References

- [1] Halpin JC. *Primer on Composite Materials Analysis (Revised)*, CRC Press, Florida, NW, USA, 2017.
- [2] Polat H. Türkiye Ekonomisinde İmalat Sanayi. Dicle Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi, 2011; 1(2): 24-39.

- [3] Vasiliev VV, Morozov EV. *Advanced Mechanics of Composite Materials and Structural Elements*, Elsevier, Massachusetts, MA, USA, 2013.
- [4] Chao C, Koh S, Sun C. Optimization of buckling and yield strengths of laminated composites. *AIAA Journal*, 1975; 13(9): 1131-1132. <https://doi.org/10.2514/3.60515>
- [5] Erdal O, Sonmez FO. Optimum design of composite laminates for maximum buckling load capacity using simulated annealing. *Composite Structures*, 2005; 71(1): 45-52. <https://doi.org/10.1016/j.compstruct.2004.09.008>
- [6] Aymerich F, Serra M. Optimization of laminate stacking sequence for maximum buckling load using the ant colony optimization (ACO) metaheuristic. *Composites Part A: Applied Science and Manufacturing*, 2008; 39(2): 262-272. <https://doi.org/10.1016/j.compositesa.2007.10.011>
- [7] Deveci HA, Aydin L, Artem HS. Buckling optimization of composite laminates using a hybrid algorithm under Puck failure criterion constraint. *Journal of Reinforced Plastics and Composites*, 2016; 35(16): 1233-1247. <https://doi.org/10.1177/0731684416646860>
- [8] Kim C, Lee J. Optimal design of laminated composite plates for maximum buckling load using genetic algorithm. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 2005; 219(9): 869-878. <https://doi.org/10.1243/095440605X31751>
- [9] Karakaya S, Soykasap O. Buckling optimization of laminated composite plates using genetic algorithm and generalized pattern search algorithm. *Structural and Multidisciplinary Optimization*, 2009; 39(5): 477-486. <https://doi.org/10.1007/s00158-008-0344-2>
- [10] Pai N, Kaw A, Weng M. Optimization of laminate stacking sequence for failure load maximization using Tabu search. *Composites Part B: Engineering*, 2003; 34(4): 405-413. [https://doi.org/10.1016/S1359-8368\(02\)00135-X](https://doi.org/10.1016/S1359-8368(02)00135-X)
- [11] Rao ARM, Arvind N. A scatter search algorithm for stacking sequence optimization of laminate composites. *Composite Structures*, 2005; 70(4): 383-402. <https://doi.org/10.1016/j.compstruct.2004.09.031>
- [12] Sebaey TA, Lopes CS, Blanco N, Costa J. Ant colony optimization for dispersed laminated composite panels under biaxial loading. *Composite Structures*, 2011; 94(1): 31-36. <https://doi.org/10.1016/j.compstruct.2011.07.021>
- [13] Di Sciuva M, Gherlone M, Lomario D. Multiconstrained optimization of laminated and sandwich plates using evolutionary algorithms and higher-order plate theories. *Composite Structures*, 2003; 59(1): 149-154. [https://doi.org/10.1016/S0263-8223\(02\)00132-0](https://doi.org/10.1016/S0263-8223(02)00132-0)
- [14] Reddy JN. *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, CRC Press, Florida, NW, USA, 2004. <https://doi.org/10.1201/b12409>
- [15] Nemeth MP. Importance of anisotropy on buckling of compression-loaded symmetric composite plates. *AIAA Journal*, 1986; 24(11): 1831-1835. <https://doi.org/10.2514/3.9531>
- [16] Rao SS. *Engineering Optimization: Theory and Practice*, John Wiley and Sons, New York, NY, USA, 2009.
- [17] Gurdal Z, Haftka RT, Hajela P. *Design and Optimization of Laminated Composite Materials*, John Wiley and Sons, New York, NY, USA, 1999.
- [18] Storn R and Price K. Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 1997; 11(4): 341-359. <https://doi.org/10.1023/A:1008202821328>
- [19] Savran M, Aydin L. Stochastic optimization of graphite-flax/epoxy hybrid laminated composite for maximum fundamental frequency and minimum cost. *Engineering Structures*, 2018. 174: 675-687. <https://doi.org/10.1016/j.engstruct.2018.07.043>

- [20] Ozturk S, Aydin L, Kucukdogan N, Celik E. Optimization of lapping processes of silicon wafer for photovoltaic applications. *Solar Energy*, 2018; 164: 1-11. <https://doi.org/10.1016/j.solener.2018.02.039>
- [21] Ozturk S, Aydin L, Celik E. A comprehensive study on slicing processes optimization of silicon ingot for photovoltaic applications. *Solar Energy*, 2018; 161: 109-124. <https://doi.org/10.1016/j.solener.2017.12.040>
- [22] Deveci HA. Optimum design of anti-buckling behavior of the laminated composites considering puck failure criterion by genetic algorithm. MSc Thesis, İzmir Institute of Technology, İzmir, Turkey, 34-68, 2011.