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Research Article

Boundary layer flow of Gold –thorium water based nanofluids over a moving semi-infinite plat

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Abstract

The 2-dimensional steady state boundary layer flow of nanofluids over an impermeable semi-infinite moving flat plate is studied. It is assumed that the flat plate moves with a constant velocity. Utilizing similarity transformation, the nonlinear governing equations are transformed to ordinary differential equations and then the resulting ODE is solved using the homotopy perturbation method. The strength of HPM solutions were verified by comparing with numerical results obtained using Runge-Kutta Gill method with shooting technique. Two types of nanoparticles gold and thorium in the water based fluid are considered. The Dimensionless velocity profiles are addressed for various nanoparticles and for different values of the nanoparticle volume fraction. The outcome of the nanoparticle volume fraction on the flow characteristics and mainly on the velocity gradient $f''(0)$ is investigated. It is finding that thorium nanoparticles have the highest velocity compared to Gold nanoparticles, that is the Thorium nanoparticle density is low compared to Gold nanoparticle density. The enhancing values nanoparticle volume fraction slowdown the fluid velocity and velocity gradientis high for Gold compared to Thorium.

1. Introduction

Gold nanoparticles have advantages over other metal nanoparticles due to their biocompatibility and non-cytotoxicity. Gold is utilized internally in human from last fifty years due to their chemical inertness. The size of Gold nanoparticles can be minimized during their synthesis and function alization with various groups. Gold nanoparticles accumulate in the tumor cells and illustrate optical scattering. So these can act as the probe for the microscopic study of cancer cells. These are also used in chemotherapy and diagnosis of cancer cell. Gold-water nanofluids using molecular dynamics nanofluids belong to a new class of fluids with enhanced thermo physical properties and heat transfer performance.

Nanofluid is a considerable factor affecting the next major industrial revolution of the recent century. Many researchers have focused on modeling the thermal conductivity and obtained different viscosities of nanofluid. Ultra high- performance cooling is one of the most vital needs of many industrial technologies. Choi et al. [1] further a little quantity of nanoparticles to conventional heat transfer fluids and scrutinized the increase of thermal conductivity. Das [2] studied the effect of nanofluid flow past a permeable stretching sheet

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with slip, thermal buoyancy and heat Source/sink by numerically. Makinde and Aziz [3] offered a numerical study on the boundary layer flow induced in a nanofluid due to a linearly stretching sheet with a convective boundary condition. Thiagarajan and Selvaraj [4] investigated nanofluid MHD stagnation point flow over a flat plate with heat transfer. Sobamowo et al. [5-8] presented the various types of nanofluid boundary layer flow problems. Anwar et al. [9] investigated the effect of free convection boundary layer nanofluid flow through a non-linear stretching surface. Bachok et al. [10] presented boundary-layer flow nanofluid past a moving semi-infinite flat plate in unvarying free stream, and establish that dual solutions exist when the plate and the free stream shift in the opposite directions. Bachok et al. [11] presented the problem of the identical free stream of nanofluid parallel to a fixed or moving flat plate by numerically. Presently, number of researchers studied numerical investigation of nanofluid flow over various types of plate problems [Hayate et al. [12, 13], Sheikholeslami [14, 15], Ahmad et al. [16]]. He [17-20] expanded the homotopy perturbation method for solving linear, nonlinear and initial and boundary value problems by combining the standard homotopy and the perturbation methods. Recently, Sobamowo et al. [21-24] studied the homotopy perturbation method for different type of fluids with boundary value problems. Oguntala et al. [25] investigated the homotopy perturbation method for heat transfer process on inclination with porous fin heat sink. By making use of above research work, the plan of the current investigation to study the effect of Gold – Thorium water based nanofluid through a semi-infinite moving plate. The similarity transformations are used and solved by both HPM method and Runge-Kutta Gill method; the solutions are compared with the help of graphs.

2.Mathematical formulation

Consider the 2-dimensional laminar flow through a continuously moving flat horizontal plate embedded in Gold – Thorium water-based nanofluid. The nanofluid can contain each of six types of nanoparticles including gold and thorium. It is considered that the plate has a constant velocity. A uniform spherical size and shape is assumed for the nanoparticles. It is also assumed that the base fluid and the nanoparticles are in the thermal equilibrium, and no velocity slip occurs between the base fluid and the nanoparticles Raftari et al. [26]. Considering these assumptions the laminar boundary layer equations are as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

The principal boundary conditions are as follows:

$$\begin{aligned} &u=U_w, v=0, \text{ at } y=0 \\ &u\to 0 \text{ as } y\to\infty \end{aligned} \tag{3}$$

In which U_w is the plate velocity which is constant, and x and y directions with corresponding velocity components are u and v respectively. Where the viscosity of the nanofluid μ_{nf}, the density of the nanofluid ρ_{nf}.

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \ \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{4}$$

Where μ_f is the viscosity of the fluid, ρ_f and ρ_s are the reference density of the fluid fraction and solid fraction respectively, and ϕ is the nanoparticle volume fraction.

The similarity variable and stream functions are defined as follows,

$$\eta = \frac{y}{x}\mathrm{Re}_x^{\frac{1}{2}}, \ f(\eta) = \frac{\psi(x,y)}{(U_w v_f x)^{\frac{1}{2}}} \tag{5}$$

Where the local Reynolds number is $Re_x = U_w x / v_f$, in whichthe kinematic viscosity of the base fluid (water) is v_f. The stream function is $\psi(x,y)$ which identically satisfies Eq.(1) and is defined as $u = \frac{\partial \Psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$.

The dimensionless momentum and boundary conditions are as follows;

$$f''' + \frac{1}{2}(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)ff'' = 0 \tag{6}$$

with the boundary conditions

$$f(0)=0, f'(0)=1, \ f'(\infty)=0 \tag{7}$$

The significant quantity is the local skin friction coefficient $C_{f,x}$defined as $c_{x,f} = \tau_w / \rho_f U_w^2$ in which the plate surface shear stress is given as $\tau_w = \mu_{nf}(du / dy)_{y=0}$ Use of the similarity parameters (5) gives [27].

$$C_{f,x}\mathrm{Re}_x^{0.5} = \frac{f''(0)}{(1-\phi)^{2.5}} \tag{8}$$

3. Solution by homotopy perturbation method (HPM)

Using HPM [17, 18, 19 and 20], the original nonlinear ODE (which cannot be solved easily) is divided into some linear ODEs.

At first, the governing ODE (6) and the boundary conditions (7) are written as:

$$u''' + \frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)uu'' = 0; \tag{9}$$

$$u(0)=0, \ u'(0)=1, \ u'(\infty)=0 \tag{10}$$

Then, a homotopy is constructed in the following form:

$$u''' - \alpha^2 u' + p\left(\frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)uu'' + \alpha^2 u'\right) = 0. \tag{11}$$

According to HPM, the following serious in terms of powers of $\rho$is substituted in Eq. (11):

$$u = u_0 + pu_1 + p^2 u_2 + \ldots \tag{12}$$

After some algebraic manipulation, equating the identical powers of$\rho$to zero gives:

$$p^0 : u_0''' - \alpha^2 u_0' = 0; u_0(0) = 0,\ u_0'(0) = 1, u_0'(\infty) = 0; \tag{13}$$

$$\left.\begin{aligned} &p^1 : u_1''' - \alpha^2 u_1' + \frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)u_0 u_0'' + \alpha^2 u_0' = 0; \\ &u_1(0) = 0,\ u_1'(0) = 0,\ u_1'(\infty) = 0; \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} &p^2 : u_2''' - \alpha^2 u_2' + \frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)\times\left(u_0 u_1'' + u_1 u_0''\right) + \alpha^2 u_1' = 0; \\ &u_2(0) = 0,\ u_2'(0) = 0,\ u_2'(\infty) = 0 \end{aligned}\right\} \tag{15}$$

Eq. (13) forρ^0 has the following solution:

$$u_0(\eta) = \frac{1}{\alpha}(1 - \exp(-\alpha\eta)). \tag{16}$$

Here$a$is a constant which is further to be determined. If solution (16) for u_0 is substituted in the equation forρ^1, Eq. (14), it will become as:

$$u_1''' - \alpha^2 u_1' = \left[\frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right) - \alpha^2\right]\times\exp(-\alpha\eta) - \frac{1}{2}(1-\varphi)^{2.5}\left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)\times\exp(-2\alpha\eta) \tag{17}$$

Eq. (17) for U_1 can be solved in an unbounded domain under the boundary conditions $u_1(0) = 0, u_1'(0) = 0, u_1'(\infty) = 0$ (as it is shown in the Appendix) [17],

which givesu_1 as:

$$u_1(\eta) = \left(\frac{1}{2\alpha} - \frac{\Omega}{6\alpha^3}\right) + \frac{\Omega}{12\alpha^3}\exp(-2\alpha\eta) + \left(-\frac{1}{2\alpha} + \frac{\Omega}{12\alpha^3}\right)\exp(-\alpha\eta). \tag{18}$$

in which$a = \left({}^{\Omega}/_{2}\right)^{0,5}$and $\Omega = (1-\varphi)^{2,5}\left[1-\varphi+\varphi\left({}^{\rho_S}/_{\rho_f}\right)\right]$ it should be noted that α can be $\alpha = \pm\left(\frac{\Omega}{2}\right)^{2.5}$, but here as α is demanded to be positive $(a > 0)$,therefore$a = \left({}^{\Omega}/_{2}\right)^{2,5}$. Thus the first order approximate semi analytical solution $f(\eta) = u(\eta) + u_1(\eta)$becomes as:

$$f(\eta)=\left(\frac{3}{2\alpha}-\frac{\Omega}{6\alpha^{3}}\right)+\frac{\Omega}{12\alpha^{3}}\exp(-2\alpha\eta)+\left(-\frac{3}{2\alpha}+\frac{\Omega}{12\alpha^{3}}\right)\exp(-\alpha\eta). \tag{19}$$

According to Eq. (19), the dimensionless plate surface shear stress is as:

$$f''(0)=-\frac{3}{2}\alpha+\frac{5\Omega}{12\alpha} \tag{20}$$

4. Numerical Analysis

The equations (6) with boundary conditions are solved numerically using the Runge-Kutta Gill method algorithm with a systematic governing of $f''(0)$ by the shooting technique until the boundary conditions are satisfied. The step size is taken as $\Delta\eta = 0.01$. The process is repeated until the results are correct up to the desired accuracy of 10^{-5}level. Numerical results are found for several values of the nanoparticle volume fraction ϕ on velocity$f'(\eta)$ and velocity gradient$f''(0)$. Table 1 compares the HPM solution and numerical solution values of the dimensionless fluid velocity gradient at the surface $f''(0)$ for gold and thorium water nanofluids for various values of the nanoparticle volume fractionϕ.

Table 1. Values of velocity gradient $f''(0)$ for some values ϕ for thorium and gold water nanofluids.

φ	Thorium (HPM)	Thorium (Numerical)	Gold (HPM)	Gold (Numerical)
0	-0.471404	-0.44411	-0.471404	-0.44411
0.1	-0.594801	-0.55999	-0.695050	-0.65412
0.2	-0.632370	-0.59599	-0.769766	-0.72461

Table 2. Thermophysical properties of base fluid and the nanoparticles at 288K

	Water	Thorium	Gold
$\rho(kg/m^3)$	1000.5	11724	19300
$C_p(j/kgK)$	4181.8	118	126
$k(W/mK)$	0.59	54	318

5. Results and Discussion

In this study, boundary layer flow of nanofluids over a semi-infinite moving flat plate embedded in the water-based nanofluid is investigated analytically utilizing homotopy perturbation method. Also, comparison between the numerical results and HPM solution of velocity including different values of active parameters is shown in this figure. In table. 2 the density of water and nanopaticles used in the present study are given.

Figure 1 and 2 display the effect of nanoparticle volume fraction on the velocity profiles of the thorium and gold water nanofluids. It is clear that an increase in the nanoparticle volume fraction decreases the velocity profiles. This phenomenon occurs because presence of the nanoparticles leads to further thinning of boundary layer thickness. The physical meaning is the increasing value of nanoparticle volume fraction means the fluid density is increased, so fluid velocity is reduced [6].

From Figure 3 it is observed that the thorium nanoparticles have the highest value of velocity profile compared to gold nanoparticles. The velocity profile of a nanofluid is based on the density of the nanofluids. The reason is thorium nanoparticle have low density compared to gold nanoparticle.

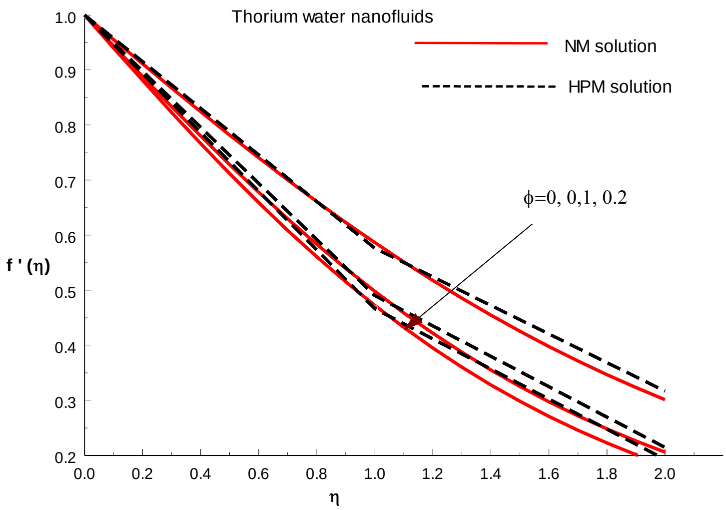

Fig. 1 Velocity profiles $f'(\eta)$ for different nanoparticle volume fractions for thorium water nanofluid.

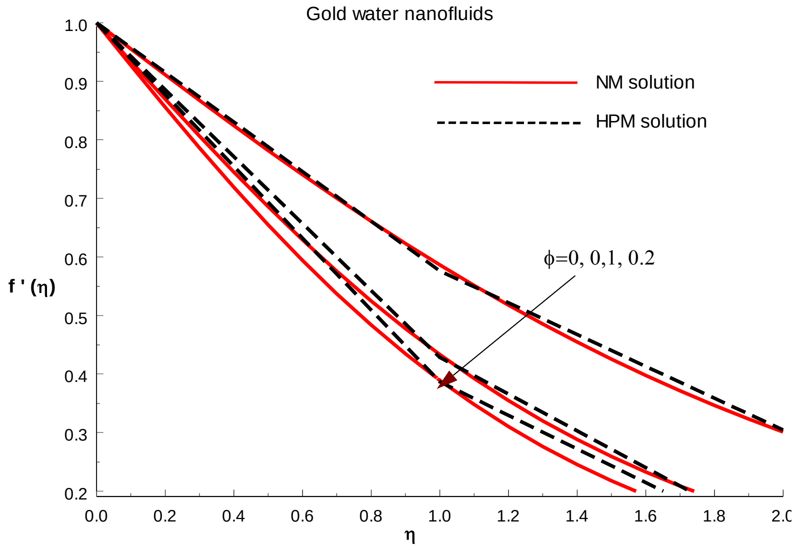

Fig. 2 Velocity profiles $f'(\eta)$ for different nanoparticle volume fractions for gold water nanofluid.

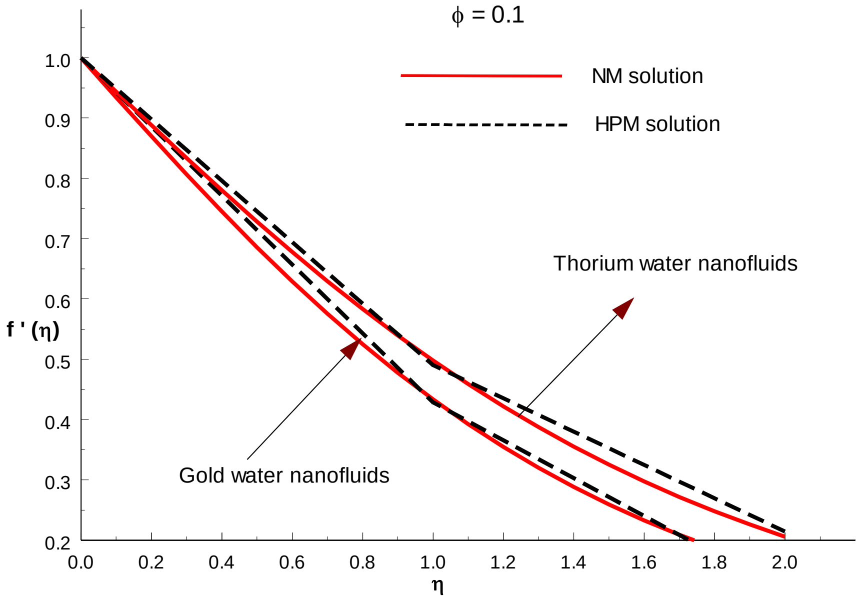

Fig. 3 Velocity profiles $f'(\eta)$ for different nanoparticles when $\phi = 0.1$.

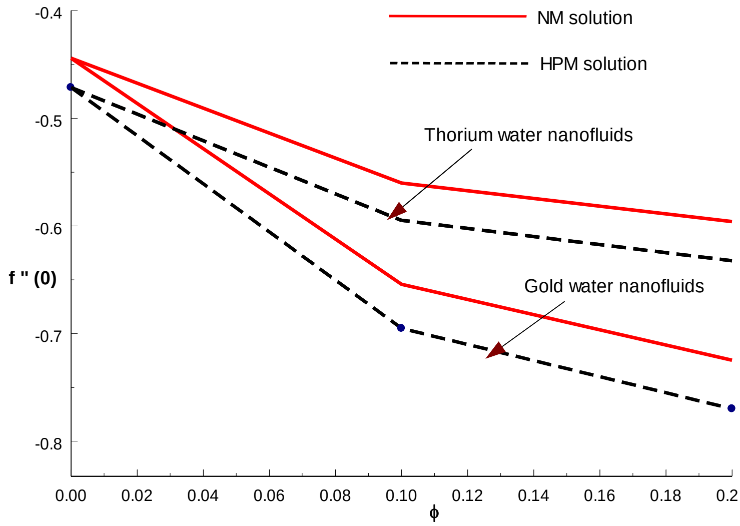

Fig. 4 Effect of the nanoparticle volume fraction ϕ on the fluid velocity gradient for different types of nanofluids.

Figure 4 presents the variations of $f''(0)$with $\phi$for various nanoparticles (Gold and Thorium) using HPM solution from Table 1. It is seen that with the increase of ϕ the magnitude of $f''(0)$increases for thorium-water and gold-water working fluids. Comparison of Figure 4 with the nanoparticles densities in table 2 makes it clear that the nanoparticles with higher density result in higher magnitudes of $f''(0)$.

6. Conclusion

In this work was to examined the effect of convective boundary layer flow of a Gold – Thorium water based nanofluid through a moving flat plate by both analytically and numerically. Analytically by HPM method and numerically by Runge-Kutta Gill method. The effects of different nanofluids on Skin friction and velocity profiles are discussed with the help graph. The main concluding observations can be summarized as follows:

- The two dimensional boundary layer flows of nanofluids over an impermeable consciously moving horizontal plate is studied. The results show that the present HPM solution with only two terms agrees within 3% error with numerical solutions for the velocity gradient at the plate surface.
- Thorium nanoparticles have the highest value of velocity profile compared to gold nanoparticles.
- Velocity gradient decreases with increasing values of nanoparticle volume fraction for both of thorium water nanofluid and gold water nanofluid
- Increasing values nanoparticle volume fraction decreases the velocity profile.

Nomenclature

(u, v)	velocity of the fluid in the x, y directions respectively(m/s)
Cf_x	skin friction coefficient
Re_x	local Reynolds number
x	distance along the surface (m)
y	distance normal to the surface (m)
f	dimensionless stream functions

Greek Symbols

η	similarity variable
ρ	density of the fluid (kg/m3)
ν_f	kinematic viscosity of the base fluid (water)
ρ_f	density of the fluid fraction
ρ_s	density of solid fraction
μ_f	the viscosity of the fluid
μ_{nf}	viscosity of the nanofluid
ρ_{nf}	the density of the nanofluid
ϕ	the nanoparticle volume fraction

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