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Research Article

Accuracy evaluation of the output of the spindle assembly of the NT-250I lathe machine

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Abstract

The article is devoted to the analysis of the dynamic properties of the spindle assembly. We study elastic properties of supports and mathematical modeling of it. We calculated contact and elastic deformations in roller bearings. We also consider to simulation of contact-elastic displacements of a high-precision roller bearing. Based on a mathematical model, it is shown that the amplitude of oscillations at the spindle speed significantly affects to the quality of work. Due to the non-linearity of the bending coefficients and periodic changes of parameters we show non-linear damping of the rolling bearing.

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1. Introduction

The spindle assembly consists of a spindle, supports, and a driving element, that they are usually enclosed in a separate housing. In fact, the spindle unit is a rotary system that has own design features. The design of spindle unit is determined primarily by the scope or manufacturing areas.

The design of the spindle is determined by the following features: a) the size of the spindle, the distance between the supports, and the presence of hole in it for passing materials or other purposes; b) driving parts (gears, pulleys) and their location on the spindle; c) design of supports and type of bearings; d) the method of fixture to the workpiece or tool, that affects the design of the front end of the spindle. Dissipative characteristics of the spindle assembly and frequency response (AFC) of the spindle assembly is determined by following parameters: spindle diameter/length, the distance between the supports, damping parameters and inertial and intrinsic rigidity

Most papers [1,2,3,4,5,6] analyzed influence of the designing parameters of control system for the dynamic quality. The designs of spindle assemblies and their typical calculations are given in these papers [7,8,9,10]. Scientific works on spindle rolling bearings is done by following authors such as: V.S. Balasanyan, V.B. Balmont, V.V. Bushuev. A. Jones, T. Harris, Z.M. Levina, A.M. Figatner, V.E. Pusha A.V. and many other scientists [11, 12, 13,14,15]

A number of works are devoted to the study of the rigidity of bearings: Atstupinas R.V. [16], Baranova I.A. [17], Zhuravleva V.F. [18], Kovaleva M.P. [19] and others. The evaluation of bearing stiffness without technological errors are presented in works [17], in other papers the condition of contact of all balls with rings are carried out [13,15,16], the possibility of incomplete ball contact with rings is taken into account. rings. The elastic properties of ball

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bearings, as a factor that affects on accuracy of machines, were the subject of research in the works of Novikov L.Z. [18], Kelzon A.S. [19].

According to the majority of researchers, the main sources of noise and vibration of the control system on rolling bearings are the cyclic change in the compliance (stiffness) of the bearing under load and vibration that caused by geometric errors in bearing parts [36]. The elastic properties of bearing assemblies have a significant impact on the spectrum of natural vibration frequencies of mechanical systems.

The accuracy of the spindle assembly is one of the most important indicators of machine accuracy, that has a direct impact on the parameters of the workpiece. Methods for controlling the accuracy of spindle assemblies are in accordance with GOST 22267-76, as well as standards of accuracy and rigidity for individual types of machine tools are provided for a set of checks, the obtained results make it possible to evaluate only the geometric parameters of the machine. Meanwhile, it is known that such checks are insufficient for a reasonable conclusion about the output accuracy of the machine. So, exaggeration of the radial runout of the spindle may not degrade the accuracy of processing.

In this regard, it is advisable to develop such a calculation system that would directly relate the errors of the elements of the spindle assembly with the errors of the machined surfaces. In this case, the spindle assembly should be considered as part of the shaping system of the machine, and all its links and their relative movements (except for the elements of the spindle assembly) should be considered as absolutely accurate.

Here, the problem is solved on the example of analyzing the influence of spindle supports accuracy on a set of machine output parameters such as: geometric accuracy, deviations in the shape, position and dimensions of the machined surface. The purpose of the analysis is to develop requirements for the accuracy of support bearings, as well as to establish the relationship between the input and output errors of the machine for typical turning schemes. A mathematical model of the accuracy of the spindle assembly has been developed, which takes into account the geometric and static errors of the support elements. The model is based on formulas that has input errors, but the spindle assembly errors are taken equal to zero. While constructing a calculation model, the spindle assembly supports are represented by a system of springs. Then six components of the spindle coordinate system position error can be obtained from six static equilibrium conditions, which, in accordance with [20], have a form;

$$C_{\Delta} = P \quad (1)$$

where C is a 6x6 symmetrical stiffness matrix; Δ is the I vector of the generalized error. $\Delta = (\delta_x, \delta_y, \delta_z, \alpha, \beta, \gamma)^T$; P - generalized force vector; $P = (P_x, P_y, P_z, M_x, M_y, M_z)^T$.

The forces on the rolling elements under the radial load of the bearing are unevenly distributed (Fig. 1). In the perception of the load, only rolling elements located on an arc not exceeding 180° (loaded zone). The most loaded is a ball or roller located in the direction of the force on the bearing.

The problem of distribution of forces between rolling bodies is statically indeterminate. Rolling bodies located symmetrically with respect to the plane of action of the force are equally loaded. Let us denote the force on the most loaded rolling body through F_0 ; on a body located in relation to the load plane at an angle γ (equal to the angular step), - through F_1 , at an angle 2γ - through F_2 , at an angle $n\gamma$ - through F_n , where n - half of the rolling elements in the loaded area.

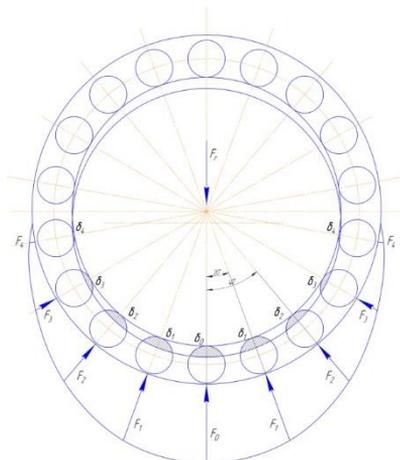


Fig. 1 Scheme of forces distribution on the bearing

2. Calculation and Analysis

We assume for simplicity that the rolling elements are located symmetrically with respect to the load plane.

Equilibrium condition of the inner ring loaded with a radial force is F_r :

$$F_r = F_0 + 2F_1 \cos \gamma + 2 F_2 \cos 2\gamma \dots + 2F_n \cos n\gamma \tag{2}$$

In addition to the static equation, we use the displacement equation. Neglecting the bending of the rings and assuming no radial clearance in the bearing, it can be assumed that the convergence of the rolling elements and the rings is equal to the corresponding projections of the total displacement of the ring δ_0 ,

$$\delta_1 = \delta_0 \cos \gamma, \delta_2 = \delta_0 \cos 2\gamma, \delta_i = \delta_0 \cos i\gamma; \tag{3}$$

where i - rolling element number.

For ball bearings, the relationship between δ balls and the compressive load F , as follows from the problem of elasticity theory about the compression of elastic bodies,

$$\delta = cF^{\frac{2}{3}} \tag{4}$$

where c - proportionality factor.

The non-linear nature of the dependence is explained by the growth of the contact area with increasing force. Expressing in the equations of displacements the approach in terms of forces, one can write:

$$F_1 = F_0 \cos^{3/2}\gamma, F_2 = F_0 \cos^{3/2}2\gamma, \dots, F_i = F_0 \cos^{3/2}i\gamma \tag{5}$$

Substituting these dependencies into the equilibrium equation, we obtain

$$F_r = F_0 \left(1 + 2 \sum_1^n \cos^{5/2}i\gamma \right) \tag{6}$$

From here we determine F_0 , and simultaneously multiply the numerator and denominator of the right-hand side by z and introduce the notation

$$k = \frac{z}{1+2\sum_1^n \cos^5/2iy} \quad (7)$$

Then

$$F_0 = \frac{k \cdot Fr}{z} \quad (8)$$

where z - total number of rolling elements and for bearings with number of balls $z = 10 \dots 20$ $k = 4.37 \pm 0.01$.

The bearings operated under normal conditions with clearance, balls with an arc is less than 1800, and the most loaded ball is compressed with a force greater than about 10%. The single-row ball bearings is taken $k = 5$ and $F_0 = 5Fr/z$.

In spherical double-row ball bearings, taking into account some uneven distribution of forces between the rows, the force on the most loaded ball is estimated $F_0 = 6Fr/z(\cos\alpha)$, where α - angle of inclination of the contact line, z - number of balls in both rows.

The problem is also solved for roller bearings, only the relationship between the convergence of rollers, rings and the compressive load assumed to be linear $\delta = c1F$ (where $c1$ -coefficient).

Similar to ball bearings for roller bearings, the highest force is $F_0 = kFr/z$. For roller bearings with number of rollers $z = 10 \dots 20$, $k = 4$. Taking into account the influence of the gap for the calculation, it was taken $k = 4,6$, and $k = 5,2$ for double row roller bearings. It is considering the uneven distribution of forces between the rows [3, 4].

In angular contact bearings under radial load, the forces on balls and rollers are greater than in radial bearings in relation to $1/\cos\alpha$, where α is the contact angle of balls or rollers and rings.

The load distribution between the rolling elements can be somewhat leveled by elastic deformations of the housings. The hole should take the form of an elliptical extended cylinder in the direction of the load. This is possible by designing axle boxes of railway rolling stock.

Axial force with precise manufacturing and the absence of mutual misalignment of the rings is distributed evenly between the rolling elements.

2.1 Calculation Without Taking Into Account the Influence of the Centrifugal Forces of the Rollers

Diameter of outdoor treadmill

$$D_H = D_B + 2d \quad (9)$$

where: D_B - diameter of the inner treadmill

d - roller diameter

Reduced curvature of internal contact

$$\sum \rho_B = \frac{z}{a} + \frac{z}{D_B} \quad (10)$$

Outdoor contact

$$\sum \rho_H = \frac{2}{d} + \frac{2}{D_H} \tag{11}$$

Rolling speed in contacts

$$(U_a + U_b) = \frac{\pi n}{60} \cdot D_B \frac{D_B + 2d}{D_B + d} \tag{12}$$

U_a - speed of the outer part

U_b is the speed of the inside

n - speed

Zero approximation for force in contacts per unit of roller contact length

$$\vec{P}_0 = \frac{\delta_\Sigma \cdot 10^8}{61 \cdot [14,832 - lg(\sum \rho_B + \sum \rho_H)]} \tag{13}$$

Thicknesses of lubricating layers in contacts

$$h_{B(H)i=0.796} \frac{[\mu_0 \cdot (U_a + U_b)]^{0.75} \cdot \alpha^{0.6}}{[\rho_{B(H)i}]^{0.15} \cdot [\sum \rho_{B(H)}]^{0.4}} = \frac{B_{B(H)i}}{[\rho_{B(H)i}]^{0.15}} \tag{14}$$

where: μ_0 - lubrication factor

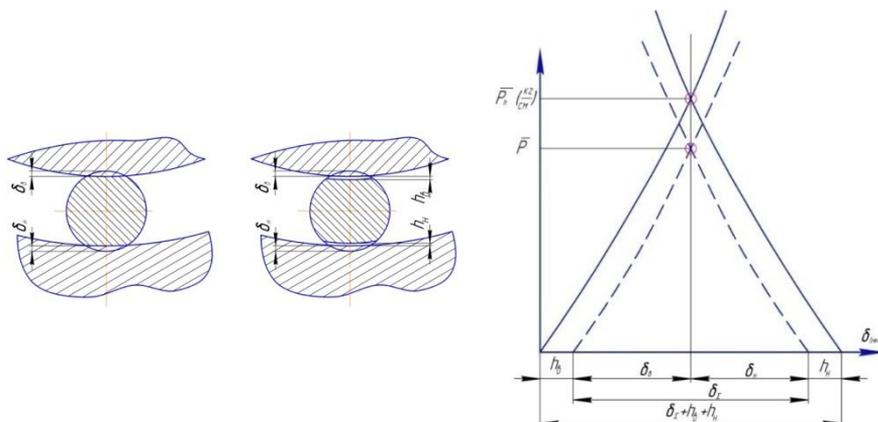


Fig. 1 Forces in contacts without taking into account the centrifugal forces of the rollers

$$\vec{P}_0 = \frac{(\delta_\Sigma + h_{B_{i-1}} + h_{H_{i-1}}) \cdot 10^8}{61 \cdot [15.7 - lg((\vec{P}_{i-1}) \sum \rho_B + \sum \rho_H)]} \tag{15}$$

Here $\delta_\Sigma + h_{B_{i-1}} + h_{H_{i-1}}$ - total tightness on the roller, it is considered that the influence of the wedging action of the lubricating layer in the contacts. The forces are calculated using the inertia method according to (15) with simultaneous refinement of the layer thicknesses (14) $i = 1.2.3.4 \dots$ to $\frac{P_i - P_{i-1}}{P_i} \leq 0,00001$

2.2. Calculation Taking Into Account the Influence of the Centrifugal Forces of the Rollers

Centrifugal force of the roller per unit contact length is

$$\bar{C} = \frac{0,855}{10^8} \cdot d^2 \cdot n^2 \frac{l}{l_p} \left(\frac{D_B^2}{D_B+d} \right) \tag{16}$$

where:

l - roller length

l_p - contact length

First, the calculation is carried out assuming that there is no contact with the inner ring.

Force on the outer ring: $\vec{P}_H = \vec{C}$

Contact deformation with outer ring:

$$\delta_{H_c} = \frac{61}{10^8} \cdot \bar{C} \cdot [7,85 - \lg(\bar{C} \cdot \Sigma \rho_H)] \tag{17}$$

Thickness of the lubricating layer in contact with the outer ring:

$$h_{H_c} = 0.796 \frac{[\mu_0 \cdot (U_a + U_b)]^{0.75} \cdot \alpha^{0.6}}{[\bar{C}]^{0.15} \cdot [\Sigma \rho_H]^{0.4}} = \frac{B_H}{[\bar{C}]^{0.15}} \tag{18}$$

If $(\delta_{H_c} + h_{H_c}) \geq \delta_\Sigma$ then there is really no contact with the inner ring and the durability of the outer ring, and therefore the bearing.

$$H_H = \frac{10^7}{60 \cdot i_H \cdot n} \cdot \left(\frac{P_{\delta_H}}{\bar{C}} \right)^{\frac{10}{3}} \tag{19}$$

It is clear that the calculation of the bearing life in this case is formal, because in the absence of contact with the inner ring, the normal operation of the bearing is impossible.

If

$$(\delta_{H_c} + h_{H_c}) < \delta_\Sigma \tag{20}$$

Then the roller is in contact with both rings and the calculation is carried out in the following sequence.

We set the force in contact of the roller with the outer ring

$$\bar{P}_H = \bar{P} + 0,35 \cdot \bar{C} \tag{21}$$

Then the zero approximation for the force in contact with the inner ring

$$\bar{P}_H = \bar{P} + 0,35 \cdot \bar{C} \tag{22}$$

Contact deformation with outer ring:

$$\delta_{H_1} = \frac{61}{10^8} \cdot \bar{P}_{H1} \cdot [7,85 - \lg(\bar{P}_{H1} \cdot \Sigma \rho_H)] \tag{23}$$

Layer thicknesses in contact

$$h_H = \frac{B_H}{[\bar{P}_{H_c}]^{0.15}} \qquad h_{B_{10}} = \frac{B_B}{[\bar{P}_{B_{10}}]^{0.15}} \tag{23}$$

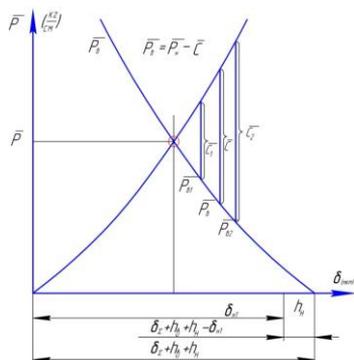


Fig. 3 Force approximation

The next approximation for the force in contact with the inner ring is determined by iteration over

$$\bar{P}_{B1i} = \frac{\delta_{\Sigma} + h_{B1i-1} + h_{H1} - \delta_{H1}}{10^8 \cdot [7,85 - \lg(\bar{P}_{B1i-1} \cdot \Sigma \rho_B)]} \delta_{H1} \tag{25}$$

Here $i = 1.2.3.4 \dots$ to $\frac{P_i - P_{i-1}}{P_i} \leq 0,00001$. The value of the centrifugal force $C1$, corresponded to the accepted force P_{H1} , in contact of the roller with the outer ring

$$\bar{C} = \bar{P}_{H1} - \bar{P}_{B1i} \tag{26}$$

We set the following value of the force in contact of the roller with the outer ring

$$\bar{P}_{H2} = \bar{P} + 0,45 \cdot \bar{C} \tag{27}$$

We repeat the calculation once again with followings: $P_{B20}, \delta_{H2}, h_{H2}, h_{H20}, P_{B2i}, C2$. The next value of the force in contact of the roller with the outer ring is determined by the formula

$$\bar{P}_{H3} = \bar{P}_{H1} + (\bar{P}_{H2} - \bar{P}_{H1}) \cdot \frac{\bar{C} - \bar{C}_1}{\bar{C}_2 - \bar{C}_1} \tag{28}$$

Further, the calculation is carried out by the iteration method zero approximation for the force in contact with the inner ring

$$\bar{P}_{B30} = \bar{P}_{H3} - \bar{C} \tag{29}$$

Contact deformation with outer ring:

$$\delta_{H3} = \frac{61}{10^8} \cdot \bar{P}_{H3} \cdot [7,85 - \lg(\bar{P}_{H3} \cdot \Sigma \rho_H)] \tag{30}$$

Layer thicknesses in contact

$$h_{H3} = \frac{B_H}{[\bar{P}_{H3}]^{0,15}} \quad h_{B30} = \frac{B_H}{[\bar{P}_{B30}]^{0,15}} \tag{31}$$

The iteration of the force in contact of the roller with the inner ring is determined by the iteration method

$$\bar{P}_{B3i} = \frac{\delta_{\Sigma} + h_{B3i-1} + h_{H3} - \delta_{H3}}{10^8 [7,85 - \lg(\bar{P}_{B3i-1} \cdot \Sigma \rho_B)]} \delta_{H1} \quad (32)$$

Here $i = 1.2.3.4 \dots$ to $\frac{P_i - P_{i-1}}{P_i} \leq 0,00001$

The value of the centrifugal force $Cv3$, corresponding to the accepted force $Pnv3$, in contact of the roller with the outer ring

$$\bar{C}_3 = \bar{P}_{H3} - \bar{P}_{B3i} \quad (33)$$

The iteration of the force in contact of the roller with the outer ring is determined by the formula

$$\bar{P}_{Hi} = \bar{P}_{H1} + (\bar{P}_{Hi-1} - \bar{P}_{H1}) \cdot \frac{\bar{C} - \bar{C}_1}{\bar{C}_{i-1} - \bar{C}_1} \quad (34)$$

After determining the forces in the contacts, the chipping durability is determined taking into account centrifugal forces.

Contact stresses

$$\sigma_{B(H)} = 610 \cdot \sqrt{\bar{P}_{B(H)} \cdot \Sigma \rho_{B(H)}} \quad (35)$$

Contact area half-width

$$b_{B(H)} = \frac{104}{10^5} \cdot \sqrt{\frac{\bar{P}_{B(H)}}{\Sigma \rho_{B(H)}}} \quad (36)$$

Calculation of contact deformations of the roller.

Fig. 4 shows the diagram of roller bearing position relative to the outer and inner races. On Fig. 4 δ_H and δ_B are the total contact deformations of the roller, respectively, between the outer and inner races.

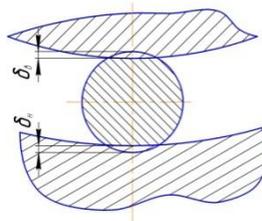


Fig. 2 The total contact strain between the roller and the outer and inner races

Diameter of outdoor treadmill:

$$D_H = D_B + 2d \quad (37)$$

Reduced curvature of internal contact:

$$\Sigma \rho_B = \frac{2}{d} + \frac{2}{D_B} \quad (38)$$

Outdoor contact:

$$\sum \rho_H = \frac{2}{d} + \frac{2}{D_H} \tag{39}$$

Total contact strain between roller and outer and inner race.

$$\delta_\Sigma = \frac{F_0 \cdot 61[14,832 - \lg(\sum \rho_B + \sum \rho_H)]}{10^8} \tag{40}$$

3. Results of Calculation

The calculation results of a high-precision roller bearing brand 2-697920L2 installed in the spindle unit of a lathe model NT-250I that is shown in table 1, with the following data $d = 14$ mm, $D_B = 110$ mm, $z = 20$. Since the bearing is double, then z is taken equal to 40.

Table 1. Calculation results

Bearing load P, kg	Force acting on the most loaded roller, $F_0 H$	Contact deformation δ_Σ , mkm
400	1019,2	4,3
300	764,4	3,2
200	509,6	2,1
100	254,8	1

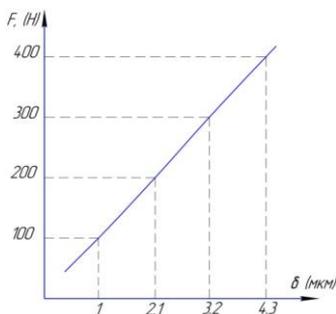


Fig. 3 Graph of the dependence of the contact deformations of the roller depending on its loading

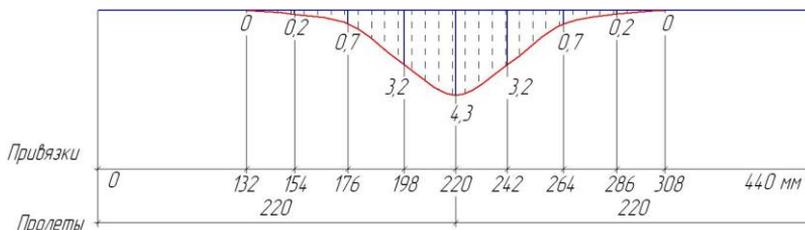


Fig. 4 Graph of the distribution of contact deformations on the outer race of the bearing

We make a static calculation of a beam with one intermediate support under the action of concentrated forces acting on the outer race of the bearing, located in a vertical plane.

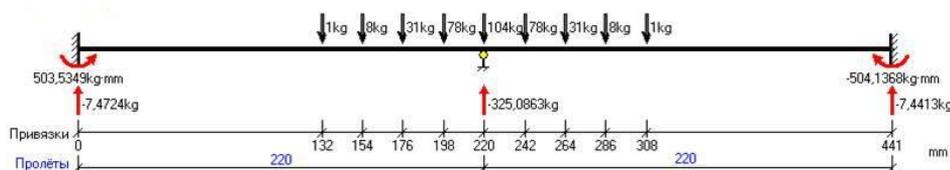


Fig. 5 Geometric diagram of a beam with fixed loads

Border conditions

The boundary conditions at each edge can be:

- hard termination;
- hinge;
- free edge.

Relationships between Deflection, Section Rotation, and M and Q plots. Very interesting relationships are known from the course "Strength of Materials", namely [21,22]:

- the angle of rotation of the section U is the derivative of the vertical displacement w
- the bending moment in section M is the derivative of the angle of rotation U multiplied by EJx;
- shear force in section Q is the derivative of the bending moment M;
- distributed load q is the derivative of the shear force Q

Thus, there are equalities:

$$U(z) = w'(z); \tag{41}$$

$$M(z) = EJ_x \cdot U'(z); \tag{42}$$

$$Q(z) = M'(z); \tag{43}$$

$$q(z) = Q'(z) \tag{44}$$

When constructing diagrams, we will be guided by formulas (40-43).

The beam is statically indeterminate: the unknown reactions of the intermediate supports and force factors at the ends of the beam cannot be found from the equations of statics. To solve the problem, the method of initial parameters was applied. The differential equation of the bent axis of the beam in this case has the form:

$$EJ_x w(z) = EJ_x w_0 + EJ_x \theta_0 z + M_0 \frac{z^2}{2} + Q_0 \frac{z^3}{6} + \sum_{x_k < z} M_k \frac{(z-x_k)^2}{2} + \sum_{x_k < z} F_k \frac{(z-x_k)^3}{6} + \sum_{x_k^n < z} q_k \frac{(z-x_k^n)^4}{24} - \sum_{x_k^k < z} q_k \frac{(z-x_k^k)^4}{24} \tag{45}$$

where:

$EJ_x w_0$ - deflection in the left section (up to a factor EJ_x);

$EJ_x Q_0$ - angle of rotation of the left section (also up to a factor EJ_x);

M_0 и Q_0 - bending moment and shear force in the left section.

All these parameters (they are called initial) are unknown. In each of the sums, the summation is carried out over all force factors located to the left of the current section. The second sum (concentrated forces F_k) takes into account the unknown reactions of the supports R_1, R_2, \dots . Thus, in equation (45) there are $n+4$ unknowns, where n is the number of intermediate supports. If all these unknowns are found, then it will be possible to construct a displacement diagram using formula (45) and other diagrams using derivatives of (45), which, by (41-43), given rotation angles:

$$EJ_x \theta(z) = EJ_x \theta_0 + M_0 z + Q_0 \frac{z^2}{2} + \sum_{x_k < z} M_k (z - x_k) + \sum_{x_k < z} F_k \frac{(z - x_k)^2}{2} + \sum_{x_k^n < z} q_k \frac{(z - x_k^n)^3}{6} - \sum_{x_k^k < z} q_k \frac{(z - x_k^k)^3}{6} \tag{46}$$

bending moments:

$$M(z) = M_0 + Q_0 z + \sum_{x_k < z} M_k + \sum_{x_k < z} F_k (z - x_k) + \sum_{x_k^n < z} q_k \frac{(z - x_k^n)^2}{2} - \sum_{x_k^k < z} q_k \frac{(z - x_k^k)^2}{2} \tag{47}$$

cutting forces:

$$Q(z) = Q_0 + \sum_{x_k < z} F_k + \sum_{x_k^n < z} q_k (z - x_k^n) - \sum_{x_k^k < z} q_k (z - x_k^k) \tag{48}$$

To find these $n + 4$ unknowns, there are the same number of equations:

- under each support, the displacement is zero - only n equations of the form (45) at those points where the supports are located;
- 2 of any parameters on each edge of the beam are equal to zero - 4 equations in total.
- Depending on the type of boundary conditions will be equal to zero:
- in rigid embedment - movement and angle of rotation;
- when hinged - displacement and bending moment;
- at the free edge - bending moment and shear force.

Here the equation $n+4 = 3+4 = 7$ are compiled with 7 unknowns. We compiled and solved this system of equations using the Gaussian method with the choice of the leading element for each column.

After compiling and solving the system of equations described in the previous section and calculating all the necessary data for formulas (45-48), we plot displacements, rotation angles, bending moments and shear forces, while the EJ_x multiplier is calculated depending on the current element assortment. Depend on this multiplier, as can be seen from equations (45-48), only displacements, that is, deflections and angles of rotation.

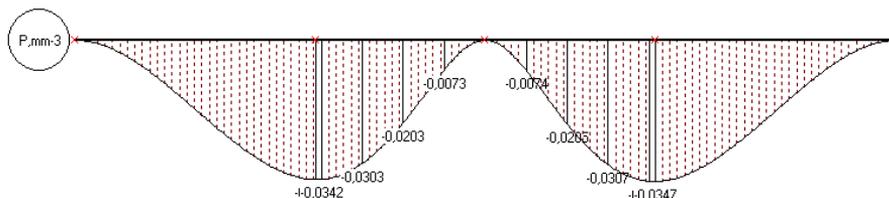


Fig. 8 Diagram of rotation angles [deg-2]

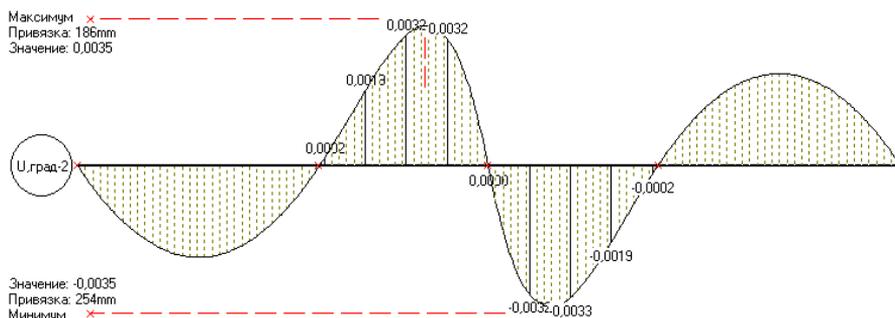


Fig. 9 Plot of bending moments [kgmm]

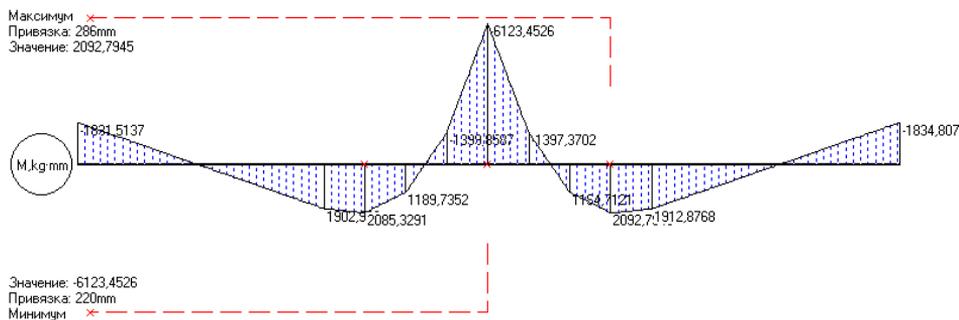


Fig. 10 Diagram of shear forces [kg]

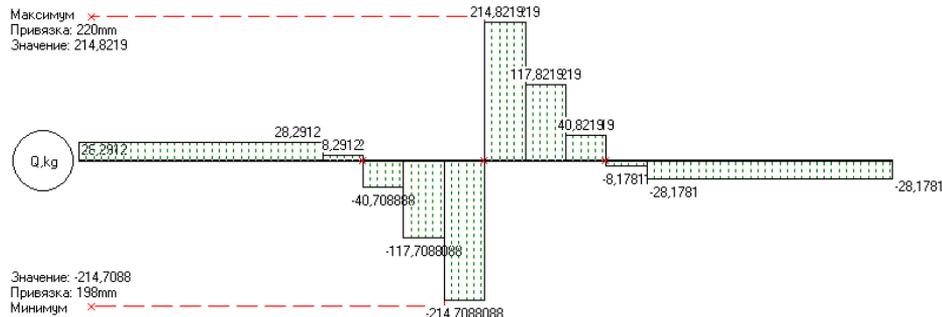


Fig. 11 Obtained results in different values

3.1 Calculation Results

Item characteristics:

Sort: Stripe 7x77

Mass 1 m.p. = 4,21 kg

Moment of inertia, $J_x = 712,0000 \text{ sm}^4$

Moment of resistance, $W_x = 102,0000 \text{ sm}^3$

Static half-section moment, $S_x = 58,4000 \text{ sm}^3$

Steel brand – SHX15 GOST 801-78

Design steel resistance, $R_y = 230 \text{ MPa}$

Design shear strength of steel, $R_s = 0,58 \cdot R_y = 133,40 \text{ MPa}$

Relative deflection - $1/250 \text{ span}$

Elastic modulus, $E = 211000 \text{ MPa}$

Verification of strength and stiffness conditions:

Stresses in the beam: normal = $M_{max} / W_x = 0,5887 < R_y = 230 \text{ MPa}$

tangent = $Q_{max} \cdot S_x / (J_x \cdot t_{cr}) = 3,1417 < R_s = 0,58 \cdot R_y = 133,4 \text{ MPa}$

Maximum deflection (with safety factor) = 0.0347 mm^{-3} which is $1/6373316$ of the maximum span of 221 mm .

Thus, the total displacement of a roller bearing is the sum of the contact deformations of the rollers and the elastic deformations of the outer race of the bearing. The addition is made graphically, by superimposing on each other diagrams of contact and elastic deformations occurring in the bearing. At the same time, at the point of the smallest elastic deformation of the outer cage, the largest contact deformations of the rollers are observed.

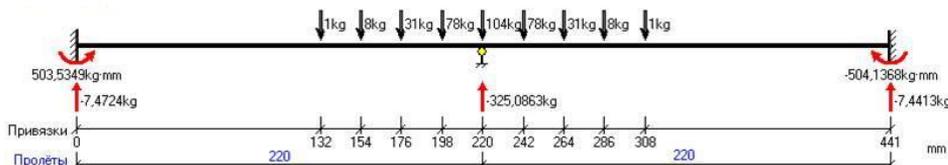


Fig. 12 Reaming the outer race of a bearing with specified loads

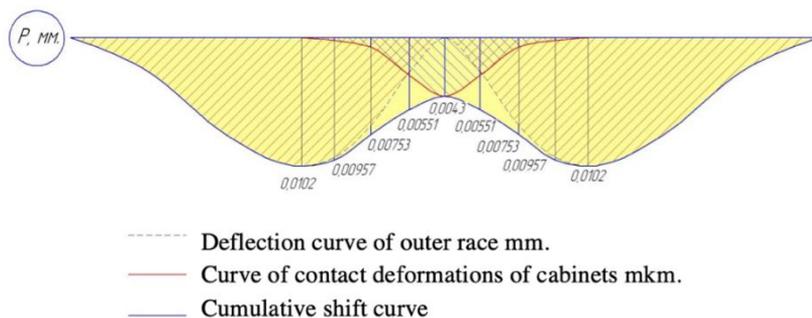


Fig. 13 Diagram of total displacements, mm

4. Numerical Simulation Results and Discussions

The APM FEM system is a tool integrated into KOMPAS-3D for the preparation and subsequent finite element analysis of a three-dimensional solid model (parts or assemblies).

Preparation of a geometric 3D model and setting the material is carried out by means of the KOMPAS-3D system. With APM FEM, you can apply various types of loads, specify boundary conditions, create a finite element mesh, and perform calculations. In this case, the procedure for generating finite elements is carried out automatically.

At the bottom of the dialog is a table of coefficients used in the calculation. Each material can be given a specific set of coefficients. More detailed information about the coefficients can be found in the documentation for the system APM Structure 3D.

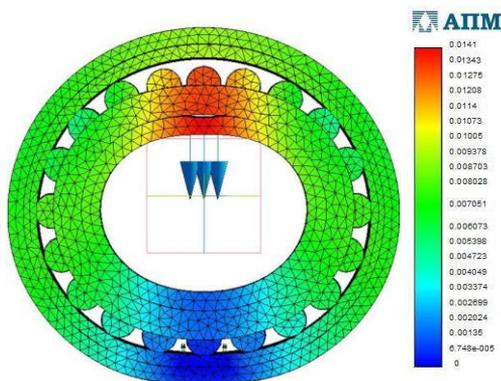


Fig. 14 Static calculation graph

Table 2. Material name: Tube 77x7 Steel SHX15 GOST 800-78

Compressive yield strength [MPa]	1670
Young's modulus [MPa]	211000
Poisson's ratio	0.3
Density [kg/m ³]	7812
Thermal expansion coefficient [1/C]	0.000151
Thermal conductivity coefficient [W/(mK)]	40
Compressive strength [MPa]	410
Fatigue limit (n) [MPa]	209
Fatigue limit (k) [MPa]	139
Name	Meaning
Model weight [kg]	6.645652
Model center of gravity [m]	(-0.002646 ; 0.000105 ; 0)
Moments of inertia of the model relative to the center of mass [kg*m ²]	(0.004798 ; 0.011922 ; 0.011956)
Reaction moment relative to the center of mass [N*m]	(0.268123 ; -0.327184 ; -3.588786)
Total reaction of supports [N]	(-0 ; 3923.000135 ; -0)
Absolute reaction value [N]	3923.000135
Absolute torque value [Nm]	3.61363

Table 3. Static calculation results

Name	Type	Minimum value	Maximum value
Total linear displacement	USUM [mm]	0	0.0141

Reshetov D.N. put forward the hypothesis that the hole should take the form of an elliptical cylinder, elongated in the direction of the load. However, no specific form was proposed. We calculated the shape of the hole. The geometry of which was designed taking into

account the deformation of the bearing. The hole must take the form of an elliptical cylinder, elongated in the direction of the load [7, 8].

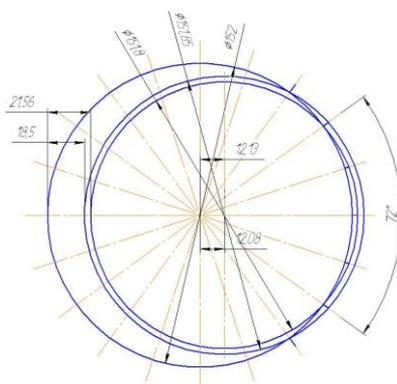
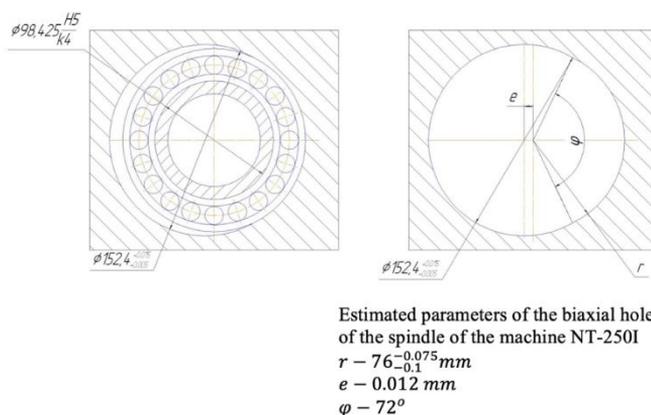


Fig. 15 Suggested hole shape



proportion of contact deformations in splined joints is overall balance, which is associated with the use of a large number of movable gears. Thus, the spindle in the process of rotation changes its orientation relative to the cutting tool;

- Analytical functional dependences of the oscillations of the support elements on the force impact from the rotating spindle are obtained, taking into account the preload and the cutting force brought to the support, the own inertial parameters of the SHU support and the stiffness characteristics of the "node-frame" contacts;
- A mathematical model of the accuracy of the spindle assembly has been developed, which takes into account the geometric and static errors of the support elements.
- Calculation of contact and elastic deformations in roller bearings has been developed, as well as contact-elastic displacements of a high-precision roller bearing brand 2-697920L2, installed in the spindle unit of a lathe model NT-250I, have been calculated;
- The shape of the hole is proposed, the geometry of which was designed taking into account the deformation of the bearing. As a result, it was possible to minimize the values of contact and elastic displacements, contact displacements by a factor of two, and elastic displacements by a factor of 3. Thus, for parts with a diameter of up to 50 mm, it becomes possible to achieve 8-9 accuracy grades.

References

- [1] Gasparov ES. Ensuring the dynamic quality of high-speed spindle assemblies based on modeling and in-place assessment of the state of supports; Doctoral dissertation, Ulyan State Technical University.
- [2] Sayfidinov O, Bognár G. Review on Relationship Between the Universality Class of the Kardar-Parisi-Zhang Equation and the Ballistic Deposition Model. *International Journal of Applied Mechanics and Engineering*, 2021: 26;4 206-216. <https://doi.org/10.2478/ijame-2021-0060>
- [3] Van V. Study of the influence of milling parameters on the accuracy of machining non-rigid parts.
- [4] Sayfidinov O, Bognár G, Kovács E. Solution of the 1D KPZ Equation by Explicit Methods. *Symmetry*, 2022: 14;4 699. <https://doi.org/10.3390/sym14040699>
- [5] Lizogub VA. Improving the accuracy and productivity of cutting based on the analysis of the design parameters of spindle units on the rolling bearings of machine tools.
- [6] Pelevin ON. Modeling of dynamic performance of spindle hydrostatic bearings of processing technological equipment: specialty 05.02. 02 - Mechanical engineering, drive systems and machine parts.
- [7] Push AV. Spindle knots. Quality and reliability.
- [8] Sayfidinov O, Bognár G. Kardar-Parisi-Zhang interface growing equation with different noise terms. In *AIP Conference Proceedings 2022*: 2425;1 290006 AIP Publishing LLC. <https://doi.org/10.1063/5.0081584>
- [9] Muminov RO, Kuziev DA, Zotov VV, Sazankova ES. Performability of electro-hydro-mechanical rotary head of drill rig in open pit mining: A case-study <https://rudmet.ru/catalog/journal/2129/>
- [10] Artemenko OR. Promotion of the practicality of milling typography with numerical programming, Master's thesis.
- [11] Balmont VB, Gorelik IG, Figatner AM. Calculations of high-speed spindle units. Moscow: VNIITEMR, 1987: 1;52.
- [12] Balmont VB, Matveev VA. Instrument rolling supports. M.: Mashinostroenie. 1984.
- [13] Muminov RO, Raykhanova GE, Kuziev DA. Improving the reliability and durability of drilling rigs by reducing dynamic loads // *Coal*. - Moscow, 2021: 5; 32-36. <https://doi.org/10.18796/0041-5790-2021-5-32-36>

- [14] Kulvets AP. Vibrations of rigid shafts rotating in precision bearings / A.P. Kulvets // Proceedings of universities of the Lithuanian SSR. Vibrotechnics, 1973: 3;20 333-339.
- [15] Levina ZM. Calculation of stiffness of modern spindle bearings. Machine tools and tools, 1982: 10; 1-3.
- [16] Sayfidinov O, Bognár GV. Numerical solutions of the Kardar-Parisi-Zhang interface growing equation with different noise terms. In Vehicle and Automotive Engineering, 2020: Nov 25; 302-311. https://doi.org/10.1007/978-981-15-9529-5_27
- [17] Figatner AM. Precision rolling bearings of modern machine tools. Moscow: NIIMash. 1981.
- [18] Jones AB. A general theory for elastically constrained ball and radial roller bearings under arbitrary load and speed conditions.
- [19] Atstupinas RV. Questions of the dynamics of a precision rigid rotor in elastic rolling bearings: Cand. dis. ... Candidate of Technical Sciences: 1970 / Atstupinas R.V. Kaunas, 1970. 24 p.
- [20] Atstupinas RV. Investigation of the radial rigidity of rolling bearings, taking into account their manufacture. / R.V. Atstupinas and others // Vibrotechnics, Vilnius, 1970, No. 4.
- [21] Yaxshiyev S, Ashurov K, Mamadiyarov AJ. Dynamics of spindle assembly of metal-cutting machine. International Journal of Engineering and Advanced Technology (IJEAT) ISSN: 2249 - 8958, Volume-9 Issue-3, February 2020 <https://doi.org/10.35940/ijeat.C6119.029320>
- [22] Atullaev AO, Yakhshiev S, Mamadiyarov A. Method for calculating the dynamic characteristics of the spindle assembly Development of science and technology BMTI Uzbekistan. ISSN 2181-8193-2020.