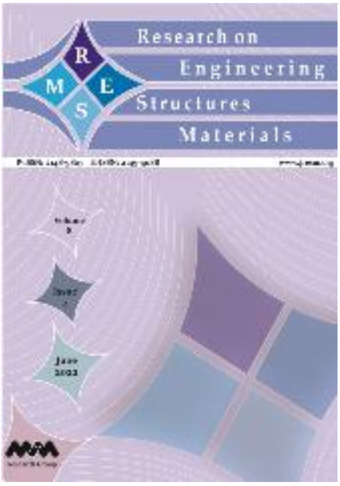
Research on Engineering Structures & Materials

journal homepage: http://www.jresm.org

Research Group



Identification of distributed impact force using the finite element model based on regularization method

Abdelali El-Bakari, Abdellatif Khamlichi, Eric Jacquelin

Online Publication Date: 20 Jan 2023

URL: http://www.jresm.org/archive/resm2022.586me1116.html

DOI: http://dx.doi.org/10.17515/resm2022.586me1116

Journal Abbreviation: *Res. Eng. Struct. Mater.*

To cite this article

El-Bakari A, Khamlichi A, Jacquelin E. Identification of distributed impact force using the finite element model based on regularization method. *Res. Eng. Struct. Mater*., 2023; 9(2): 331-349.

Research on Engineering Structures & Materials

journal homepage: http://www.jresm.org

MIM Research Group

Research Article

Identification of distributed impact force using the finite element model based on regularization method

Abdelali El-Bakari[*1,2,3,a], Abdellatif Khamlichi [1,b], Eric Jacquelin[4,5,c]

[1]*Department STIC, National School of Applied Sciences, Abdelmalek Essaadi University, Tetouan, Morocco*
[2]*Communication Systems and Detections Laboratory, FS, Abdelmalek Essaadi University, Tetouan, Morocco*
[3]*MASGC, National School of Applied Sciences, Abdelmalek Essaadi University, Tetouan, Morocco*
[4]*Université Claude Bernard Lyon, Villeurbanne, France*
[5]*IFSTTAR, UMR_T9406, LMBC Laboratoire de Biomécanique et Mécanique des chocs, Bron, France*

Article Info

Article history:

Received 16 Nov 2022
Revised 04 Jan 2023
Accepted 18 Jan 2023

Keywords:

Regularization problem; Deconvolution; Impact characteristics; Identification; Modal truncation

Abstract

The aim of this paper is to invent a technique based on the finite element method which makes it possible to identify the impact load occurring on the elastic bodies. Identifying the periodical signal like a distributed pressure caused by an impact that occurs on the elastic plate can contribute to structural health monitoring. The inverse formulation of the objective function based on the finite element method has been established. The first part of this work will be devoted to modal analysis, the objective of which is to formulate the transfer function between the impact zone and the sensors implemented at known positions in the frequency domain. To switch from the frequency domain to the time domain, the inverse fast Fourier Transformation is used. Here, the identification of the impact force acting on linear elastic structures such as plates was considered. In the first place, it is convenient to look for the parameters that define the impact area using a minimization technique based on the Maxwell-Betti theorem. Once, the localization is determined, the reconstruction of the impact force-signal characteristics is done by the regularization methods. The regularization based on the truncation of the generalized singular values decomposition (TGSVD) has been adapted in this work. The TGSVD shows a relative error equal to 2.10%. This value is lower than that of literature value where Tikhonov regularization with the L-curve criterion was used & results in a relative error of about 40%". Further, the current work for a more complicated case that generates a non-punctual impact for the same type of force shows the efficiency of the TGSVD method to reconstruct the impact signal. The influence of sensor locations, modal truncation, noise level, and discrete transfer functions on the reconstructed impact characteristics are discussed.

1. Introduction

The direct measurement of the force developed during an impact is not possible, especially because the projectile is any kind and the impact zone is not known a priori. We can also note that the measure may be intrusive and difficult to perform in the case of a distributed force. On the other hand, the measurement of a vibration response at a point in the structure such as acceleration, displacement, or deformation can easily be performed. This is why indirect methods based on inverse problem-solving are the only viable methods for identifying the characteristics of an impact. The reconstruction of the characteristics of dynamic impact forces calls into question the ill-posed problem. The problem is generally solved by the regularization method [1]. Over the past decade years, we have seen the extensive application of the regularization-based inverse problem to identify the impact signal applied to engineering structures in order to monitor structural health [2-4].

*Corresponding author: aelbakari@uae.ac.ma
[a] orcid.org/0000-0002-4196-6640; [b] orcid.org/0000-0001-6954-774X; [c] orcid.org/0000-0001-8415-6712
DOI: http://dx.doi.org/10.17515/resm2022.586me1116

Assessing random impact forces is very important because it helps to prevent system failures. In many practical situations, such as a high-speed impact of an object onto a structure, which can be assimilated to a non-punctual impact: a bird impacting the airplane fuselage, direct measurement of impact signal is complicated, prohibitive, and cumbersome, especially for large structures. While the main arguments concern the difficulty of installing the sensors and the changes in dynamic properties, the prior identification of impact zone characteristics can make investigations more efficient. The identification of the characteristics of the force generated by the impact can be used to better evaluate the monitoring of the structure, thus minimizing the experimental effort required [5- 7].

The identification of the impact force includes two relevant aspects, namely the reconstruction of the impact signal and the estimation of the impact force position. For the reconstruction of impact forces, the technique most used by many researchers in the elastic field is deconvolution based on the hypothesis that the response of a body subjected to impact force depends linearly on an impact force. This technique can be used both in the time domain and in the frequency domain. Based on the deconvolution problem Jacquelin et al. [8] used generalized singular value decomposition (GSVD) to reconstruct the impact force acting on a rectangular plate. Rezayat et al. [9] proposed an approach based on sparse regularization to identify dynamic forces in the frequency domain by grouping fast iterative shrinkage-thresholding algorithms. Qiao et al. [10] proposed a sparse regularization method to solve an arbitrary identification force by developing a dictionary composed of a set of basic functions such as sine, cosine, spline functions, and wavelets.

For its time domain applications, the dynamic responses provided by a remote sensor have been considered in He et al. [11]. They proposed a method based on empirical decomposition with intermittent criteria and finite element modeling for reconstruction impact force characteristics. For a linear elastic structure, the impact identification problem is usually solved in the time domain by using the deconvolution procedure method from the deformation response [12-13]. Doyle [14] proposed a wavelet deconvolution method for impact force identification, which produced a significantly more accurate solution than frequency-domain methods. However, the resolution of the above problems by the direct method for example the elimination of gauss gives unstable results; on the other hand, the method of least squares is widely used to improve the precision of the solutions [15-16]. In some cases, it is preferable to use a residual error minimization technique between the measured and numerically calculated responses in the time domain, instead of obtaining the history of the impact forces by the process of direct deconvolution of a convolution integral [17]. The finite element method was used in the approach proposed by Liu et al. [18] to characterize the impact force based on stress wave, they highlighted four sensors symmetrically implemented to measure responses in terms of time-varying strain. Employing fast Fourier transformation (FFT), we can associate a form of integral deconvolution problem in the frequency domain to predict the impact force on beams from the measured strain data [19-20]. He et al. [21] employed the regularization based on the TSVD method to identify the impact force in the frequency domain and remarked that the position of the respondent sensors strongly influenced the condition number of the FRF matrix. Liu et al. [22] proposed a nonconvex overlapping group sparsity to solve the impact force identification problem, and also compared the nonconvex regularization method and the standard L1-norm method.

Furthermore, for the location of the impact zone, several efficient methods have been proposed by many researchers based on the beam theory. In the case of a punctual impact, the location of the impact area can be easily achieved by minimizing the error between the measured and calculated responses. Inoue et al. [20] proposed a technique that uses the arrival time of each frequency component of a pulse detected through wavelet transformation to determine the location of the force on the beam. Liu [23] proposed an algorithm for solving a force identification problem based on TSVD and the Tikhonov filter-LS schemes. Yen and Wu [24] used Green's functions to identify the characteristics of the impact area with a significant number of sensors implemented on the isotropic plate to measure deformation responses.

In most practical cases, the impact is distributed on a structural patch, the problem is more complex because it is to identify pressure distribution and not just a single force. Assuming uniform pressure, a new parameter appears in the problem representing the extent of the affected area. El-Bakari et al. [25] proposed an improvement approach based on heuristic optimization like a particle swarm optimization algorithm to minimize the fitness function that doesn't involve pressure. The technique is based on weighting coefficients to stabilize the objective function, four cases were proposed in their work. Liu et al. [26] employed the modified second-order Tikhonov regularization method (TRM) and the non-negativity of the identified loads, the non-negative least squares (NNLS) to identify the distributed loads acting on irregular structures.

In this work, we propose an efficient method in order to identify the history and location of impact forces occurring on a linear elastic plate. Localization has been resolved as a minimization problem. Modal analysis based on the finite element method has made it possible to derive FRF (frequency response functions) between the excitation points and the measured response by implementing sensors. Using the inverse fast Fourier transform (IFFT), the associated temporal response functions were computed. Then a Toeplitz matrix was created. It gives the displacement associated with the applied pressure acting on a rectangular patch of the plate. The robustness of this method will be discussed as a function of the noise level.

2. Theoretical Problem Formulation

We consider a homogeneous and isotropic elastic rectangular plate of dimensions $\mathrm{a} \times b \times h$, with $\mathrm{a} = 0.6$ m, $\mathrm{b} = 0.4$ m, and $h = 5 \times 10^{-3}$m as shown in Fig. 1. The force modeling impact is selected to result from periodic dynamic load identification assuming a uniform pressure, $p(t)$ where t is time. it is applied to a rectangular patch in the middle of the plate which is centered on a point S_0 having a coordinate (a/2, b/2) and having length δa and width δb.

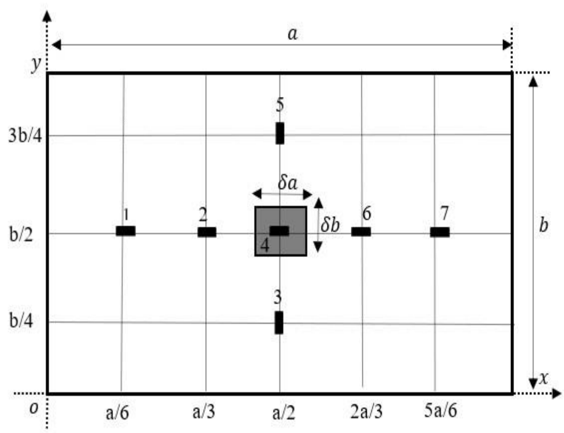


Fig. 1 Dimension of rectangular plate and sensors position

A finite element model of a rectangular plate is formulated in this part. As a result of the applied pressure, the displacement y is measured at certain points belonging to all points marked with symbol ■ which are shown in Fig. 1. The coordinates and node values of the sensors implemented on the structure are given in Table 1.

Table 1. Positions of sensors implemented on a rectangular plate

Sensors	#1	#2	#3	#4	#5	#6	#7
Coordinates	(a/6, b/2)	(a/3, b/2)	(a/2, b/4)	(a/2, b/2)	(a/2, 3b/4)	(2a/3, b/2)	(5a/6, b/2)
Nodes	121	231	368	363	358	473	583

Taking into account a sampled time interval of duration T_c and of length N the solution of the dynamical system is obtained by a linear convolution having the following form

$$y(k) = \sum_i^k h(j)p(k-j) \qquad k = 1,2,\dots,N \tag{1}$$

where $y(k)$ is the discrete dynamic response as observed by the implemented sensor and $p(k)$ the discrete input pressure, h is a discrete system's impulsion response function of the linear elastic system. The sampling rate is thus given by $\Delta t = T_c/(N-1)$.

In the frequency domain, there is also only a relationship with input/output defined by the assembling of the Eq. (1) for $k = 1,2,\dots,N$,

$$Y = HP \tag{2}$$

where H is the Toeplitz-like transfer matrix, P is the pressure signal vector, and Y is the measured signal vector.

The H-matrix is obtained by employing a FEM of the plate. Designating the index i and j respectively the measurement DOF displacement, and any displacement DOF that is normal to the zone of applied impact, the FRF is collected from the calculated modes by FEM as

$$F_{ip}(\omega) = \sum_{j=1}^{N_{impact}} F_{ij}(\omega) \tag{3}$$

with

$$F_{ij}(\omega) = \sum_{k=1}^{N_{modes}} \frac{\Phi_{ik}\Phi_{jk}}{\omega^2-\omega_k^2+2i\beta_k\omega_k\omega} \tag{4}$$

where N_{impact} is the number of total degrees of freedom of the normal displacements with respect to the affected area, N_{modes} the number total of modes that are selected in modal expansion, Φ_{ik} is the ith component of the kth mode normalized concerning mass, ω_k the kth mode angular frequency, β_k the damping coefficient for the kth mode, and the ω angular frequency.

Using the IFFT of the FRF given by Eq. (3)., the impulse function response h in the time domain can be obtained.

Generally, the matrix H can be written in the following form

$$H = \begin{bmatrix} h(1) & \cdots & h(N) \\ \vdots & \ddots & \vdots \\ h(N) & \dots & h(1) \end{bmatrix} \tag{5}$$

In this study, the $N = 1024$ is selected in order to calculate easily the IFFT.

3. Regularization of The Inverse Problem

The direct inversion of the problem defined by Eq. (2) gives an unstable solution because the matrix is ill-conditioned. This means that its direct inversion of an ill-conditioned matrix can significantly amplify the error in the dynamic response. And as a result, lead to an unstable numerical solution that doesn't make physical sense. For the above reason, the deconvolution problem given in Eqs (2) and (5) needs to be regularized. The most popular ones are Tikhonov regularization [27] and Singular Value Decomposition (SVD) based methods [28], including truncated SVD (TSVD). In this work, the regularization technique based on the truncated generalized singular value decomposition (TGSVD) is considered. It should be mentioned that the simplest SVD method has not managed to regulate the identification problem considered in this work. The TGSVD-regularized solution of the problem given in Eqs (2) and (5) can be written like this

$$P = X\Phi\Delta^{-1}\mathrm{U}^{t}W = H^{*}W \tag{6}$$

where X, Δ, U are the singular factors of H , Φ is the filter factor and $H^{*} = X\Phi\Delta^{-1}\mathrm{U}^{t}$ is the regularized pseudo-inverse of H .

The presence of generalized singular values in the matrix H requires the introduction of the filter factors' goal in order to minimize the influence of the low amplitude of the latter. The Φ filter specified by the truncation method states:

$$\Phi_{ij} = f_i\delta_{ij} \quad i, j = 1, \dots, n \tag{7}$$

where $f_i = 0$ if $i \leq k$ and $f_i = 1$ otherwise.

To build the filter Φ within the framework of the truncation method, the rank k should be determined. Various criteria can be used for that. Here, when working with sinusoidal pressure signals, numerical experimentation has shown that $k \in [N-10, N]$ with $N = 1024$.

4. Case Study

4.1. Description Problem

The rectangular plate shown in Fig. 1 is selected to have the following dimensions: length $a = 0.6$ m, width $b = 0.4$ m, and height $h = 5 \times 10^{-3}$m. It is supposedly embedded on these four edges. The material Young modulus is $E = 210$ GPa and the Poisson coefficient is $\mu = 0.3$. The density is $\rho = 7800$ kg. m^{-3} . The flexural rigidity modulus of the plate is given by $D = Eh^3/(12(1-\mu^2))$.

A FEM simulation was carried out using the Abaqus software package. Since the type and size of mesh elements do not affect the displacement results, the simulation is performed using Abaqus eight-node linear brick, reduced integration with hourglass control (C3D8R). The C3D8R elements have only one point of integration which can pose a problem of convergence in terms of displacement. This is why in first-order, reduced-integration elements in Abaqus include hourglass control. The C3D8R elements and intermediate-size mesh for a linear 3D model of the rectangular plate do not affect the convergence in terms of the displacement node element. The mesh intermediate is described by the following properties:

- number of elements: 651;
- number of nodes: 1408;

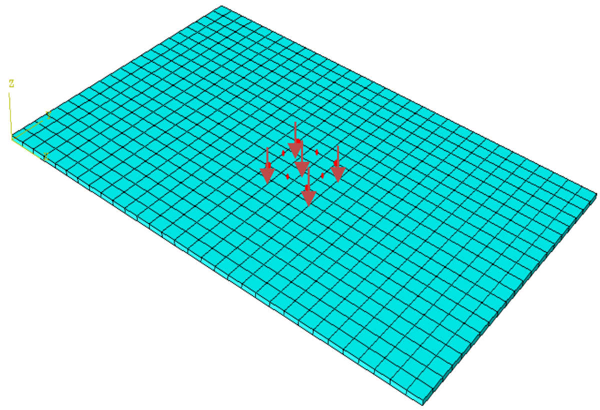


Fig. 2 Rectangular plate meshed; the impact zone (red color) subjected to the uniform pressure $p(t)$

Fig. 2 shows the intermediate mesh using the Abaqus-FEM model where the impact area is represented in red color. The latter extends over a surface (patch) in the middle of the plate simply supported. S_0 , the coordinate of the impact center is given $S_0 = (0.3\text{m}, 0.2\text{ m})$, and the extents of the patch are defined by $\delta a = \delta b = 0.06$ m. The displacement measured by the sensors located on the x-axis and y-axis symmetry of the upper face of the plate having a normal parallel to the z-axis. Only the z-displacement is assumed to be measured.

The characteristics of a periodical-like sinus profile which represents the impact pressure are given in figure 3. The maximal peak value of the pressure is 3×10^4Pa, and the pulse width of each peak is $T = 0.02$s. The total duration of the signal considered in this study is $T_c = 6T$.

Fig. 4 shows the displacements in the z-direction measured by sensors implemented at different locations on the top face of the plate using the Abaqus implicit dynamic procedure: the nodes considered are located on the plane of symmetry of the top face, as indicated in Table 1.

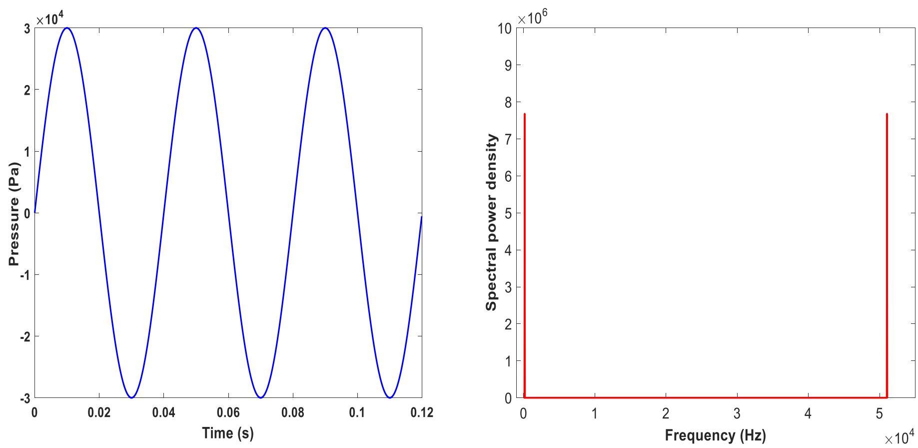


Fig. 3 Impact pressure signal characteristics; Pressure transient and Spectral content

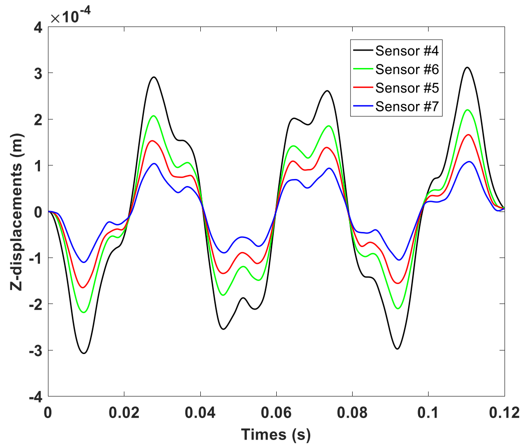


Fig. 4 Z-displacement as computed by using Abaqus dynamic implicit procedure

According to the study of convergence as a function of the size of the elements, an intermediate mesh was obtained without weighing down the machine by a refined mesh. Since the structure has no geometric defect at the time of the discretization the results remain unchanged vis-à-vis relation to the size of the element. The choice of the position of the sensors is not arbitrary it is an objective to show that the symmetry presents similar results. For this, it is enough to take the displacements given by sensors #3, #4, #6 et #7.

4.2. Modal Analysis

It is necessary to understand the vibration characteristics of a structure before proceeding with the identifying problem. This section focuses on the study of natural frequencies and structural modes shapes using the finite element method. This will allow us to determine the modes that have significant participation, as well as the corresponding frequencies, including the interest to develop the transfer function. The first 20 frequencies of the rectangular plate are given in Fig. 5. Fig. 6 illustrates some representative modes in the z-direction. The purpose of this modal study is to determine the maximum number of modes that contribute to the transfer function in order to complete the inverse formulation used to identify the impact force that occurs on the plate.

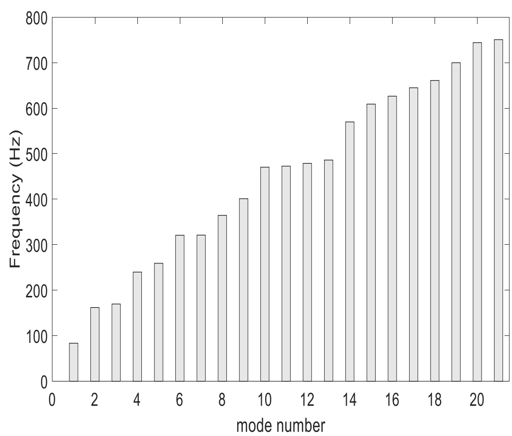


Fig. 5 Frequencies of the first 20 modes of the rectangular plate

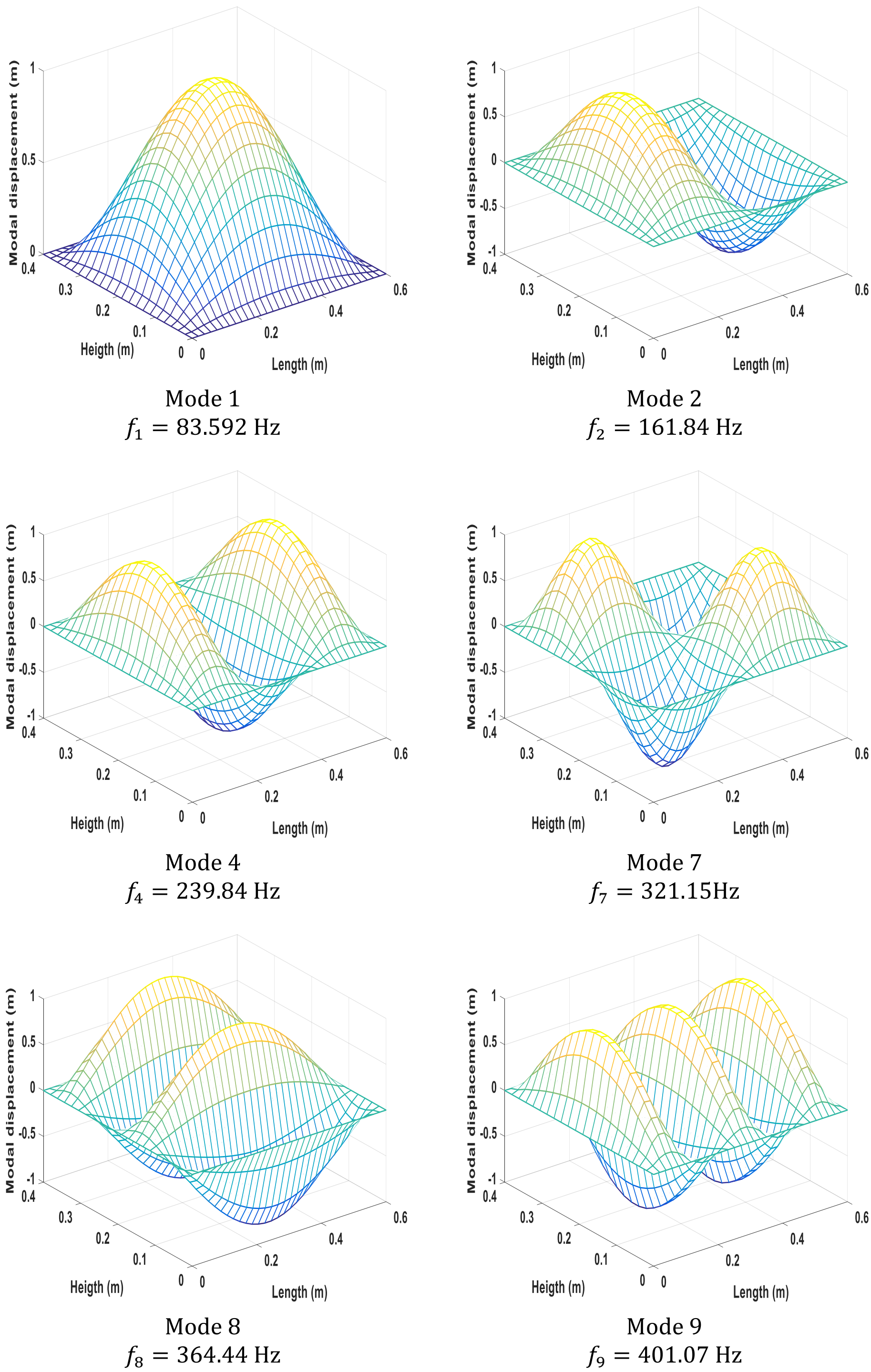
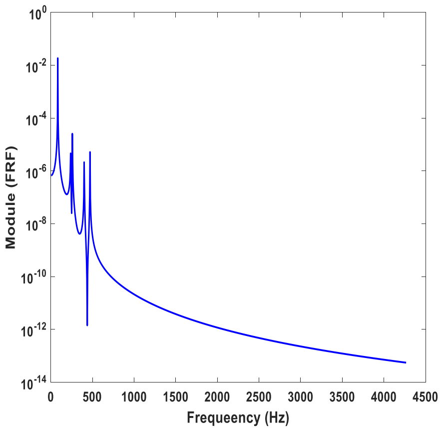


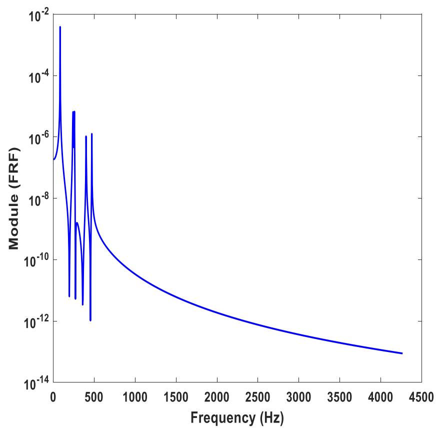
Fig. 6 Modes shapes of the plate, the z-displacement degrees of freedom are depicted (a/h=120)

The frequency response function is calculated between the measurement sensor position and the excitation points using Eqs. (3)-(4). The $\beta_k = 5 \times 10^{-4}$ damping coefficient was used to avoid the singularities that may be present in the transfer matrix. Figure 7 gives the FRF for nodes 363 and 583 (according to the finite element mesh labeling).

Modal truncation convergence is reached here by considering only the first plate mode, but for the solution of the localization problem, it is necessary to keep three modes. The two signals compare well but a phase distortion can be noticed while computing the response using the discrete response function. The IFFT was calculated here by using N= 1024 points. The time domain used was [0,0.12s]. $\Delta f = 8{,}33$ Hz and $\Delta t = 1.1718 \times 10^{-4}$ s. With sufficient accuracy, one can assume now that the direct problem is well described by Eqs. (2)-(5) resulting from modal and fast Fourier analyses.



Node 363 is located at coordinate (a/2, b/2)



Node 583 is located at coordinate (5a/6, b/2)

Fig. 7 FRF between sensor locations nodes and the impacted zone

4.3. Identification Problem

In this section, the formulation of inverse problem analysis was considered in two steps; the first one consists in finding the impact zone characteristics from measured displacement responses by the sensors placed on the top face of the rectangular plate. And in the second step, the reconstruction of the time history of the signal will be discussed through the regularization method. As regards the location, this means locating the impact zone and reconstructing the pressure signal. In the first step, searching the impact center position of the impacted zone, denoted here S_0, then as a second step the impact zone extensions δa and δb are determined.

To carry out this identification work, a flow chart summarizing the identification procedure is proposed in Fig.8.

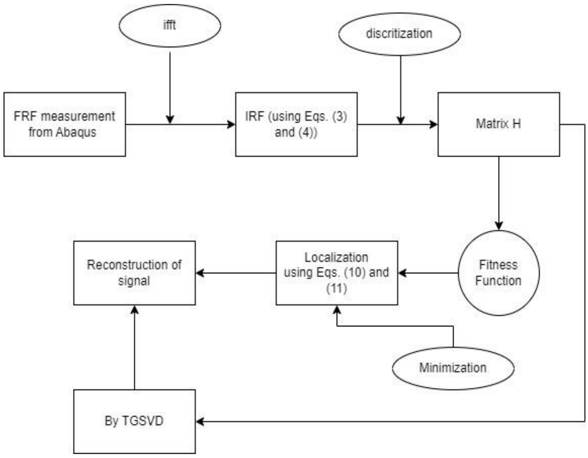


Fig. 8 Flow chart of the proposed identification method

4.3.1 Localization of impact force

Based on parametric modeling of dynamic responses associated, we consider a pair of points located on the plane of symmetry of the top face of the plate and having respectively a coordinate (x_i, y_i) and (x_j, y_j). The responses write

$$\begin{aligned} Y_i &= H_i(S_0, \delta a, \delta b)\ Q \\ Y_j &= H_j(S_0, \delta a, \delta b)\ Q \end{aligned} \tag{8}$$

where Q is the resultant pressure force located at the unknown point (x, y), H_i and H_j are the Toeplitz matrices connecting this unknown point with the known senor locations i and j. Using commutative propriety$H_i(S_0, \delta a, \delta b)H_j(S_0, \delta a, \delta b) = H_j(S_0, \delta a, \delta b)H_i(S_0, \delta a, \delta b)$, one can prove the following Maxwell-Betti equation [18].

$$H_i(S_0, \delta a, \delta b)Y_j = H_j(S_0, \delta a, \delta b)Y_i \tag{9}$$

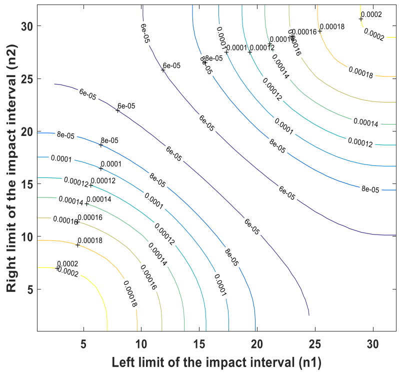
Parameter S_0 should then minimize the objective function

$$S_0 = \underset{x,y}{\operatorname{argmin}}\{\frac{\|H_i(S_0,\delta a,\delta b)Y_j - H_j(S_0,\delta a,\delta b)Y_i\|}{\|H_i(S_0,\delta a,\delta b)\| + \|H_j(S_0,\delta a,\delta b)\|}\} \tag{10}$$

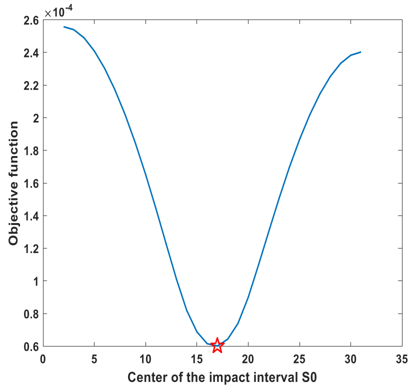
where (x, y) are not located on the fixed boundary of the plate and$\|\quad\|$ is the matrix norm 1. Having had determined S_0 a second minimization is performed in order to determine the limits of the impact interval in the x-axis (u_{10}, u_{20}) with parameter $\delta a = |u_{20} - u_{10}|$ and in the y-axis(v_{10}, v_{20}) with parameter $\delta b = |v_{20} - v_{10}|$ by solution of

$$(\delta a, \delta b) = \underset{x,y}{\operatorname{argmin}}\{\frac{\|H_i(u_1,u_2,v_1,v_2)Y_j - H_j(u_1,u_2,v_1,v_2)Y_i\|}{\|H_i(u_1,u_2,v_1,v_2)\| + \|H_j(u_1,u_2,v_1,v_2)\|}\} \tag{11}$$

It should be noted that, in order to avoid trivial solutions corresponding to the fixed boundaries and to points that are closer to them (because the modal displacements are small), the objective function is divided by the sum of norms of the Toeplitz matrices. One should also consider more than one mode in evaluating the FTF, otherwise, the numerator of the loss function is constant and minimization has a trivial multi-solution.



Isovalues of the objective function defined by Eq. (11) as a function of the nodes limiting the impact interval



Variation of the objective function defined by Eq. (10) as a function of the axial node number, the star shows the obtained minimum

Fig. 9 Solutions of the minimization problems defined by Eqs (10) and (11)

Fig. 9 illustrates the objective function defined in Eq. (10) as a function of the local axial node number of the plate. This function is shown to have a minimum which is the point located at the coordinate distance $S_0(0.3\text{m}, 0.2\text{m})$ corresponding to the center of the plate. The minimization of the fitness function gives the minimum at abscissa n=17 (the value of the objective function is $S_0 = 6.05 * 10^{-5}$) which corresponds to node 363 in the finite element mesh and has an index equal to 364.

Scanning all nodes (coordinate patch), we find the minimum of Eq. (10) obtained at n=17, this means that the minimum is obtained for all rectangular patch nodes. This technique makes it possible to find all areas of impact;

Minimization of the objective function defined by Eq. (11) yields the interval in:

- the x-axis $[0.2614285\text{ m}, 0.3385715\text{ m}]$ $(\delta a = 0.077143\text{ m})$ which is not far from the exact solution$[0.27\text{ m}, 0.33\text{ m}]$
- y-axis $[0.161425\text{ m}, 0.238575\text{m}](\delta b = 0.07715\text{ m})$ which is not far from the exact solution $[0.17\text{ m}, 0.23\text{ m}]$.

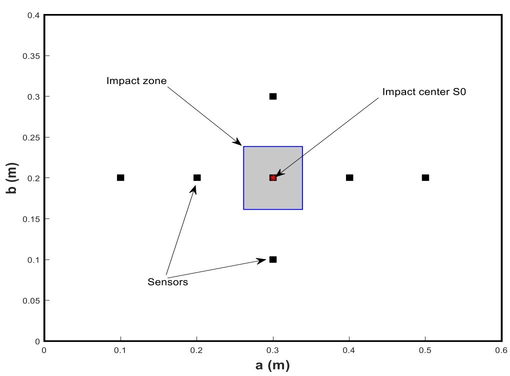


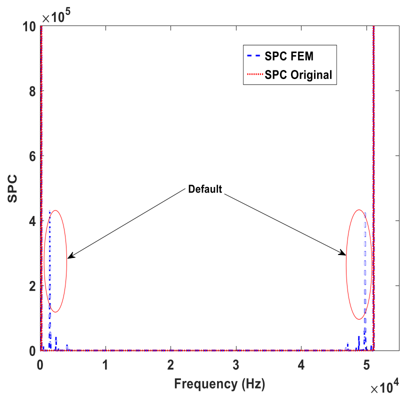
Fig. 10 Localization of impact force

Fig. 10 summarizes the problem of localization which has been challenged by Eqs. (9) and (10). It is clear that localization has been perfectly ensured by minimizing the objective functions defined by the Eqs. (9) and (10).

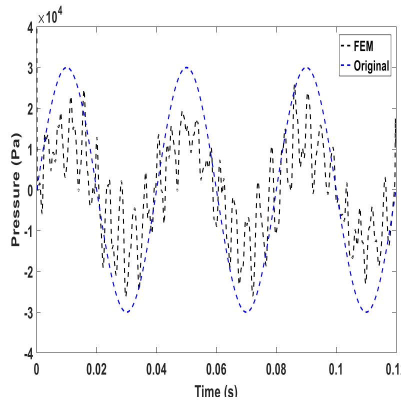
Now that S_0, δa and δb are calculated, the Toeplitz matrix $H_i(S_0, \delta a, \delta b)$ is completely known. The following section will be devoted to solving the inverse problem using the coupling FEM-TGSVD to reconstruct the impact signal.

4.3.2 Reconstruction of Signal Force

This part of the work analyzes the possibility of reconstruction force by using only the finite element method without the regularization technique. In the first order, the mains SPC is used to compare the different signals reconstructed. Figure 11 shows the SPC comparison with three positions of sensors implemented at points (j_1, j_2, and j_3) to measure the displacement response of the respective nodes (583, 358, 473), for each position we can only reconstruct the impact force. In Figures 11a, b, and c the first sensor is placed at i_1 corresponding to node 363 to give the excitation signal, and sensors measuring the displacement response are placed at j_1, j_2 and j_3 successively.

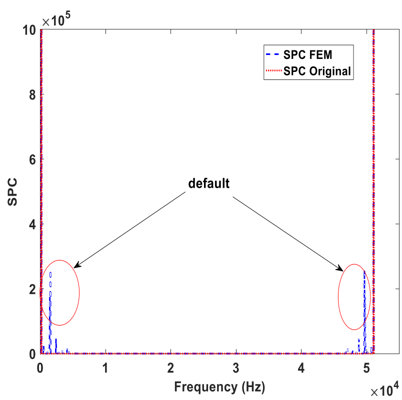


Spectral content

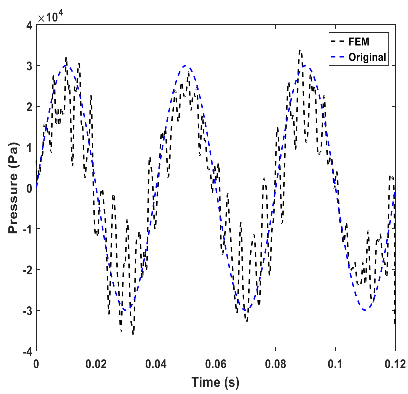


Impact force reconstruction

(a) Reconstruction of Impact pressure signal characteristics: Configuration 1 (#4, #7)

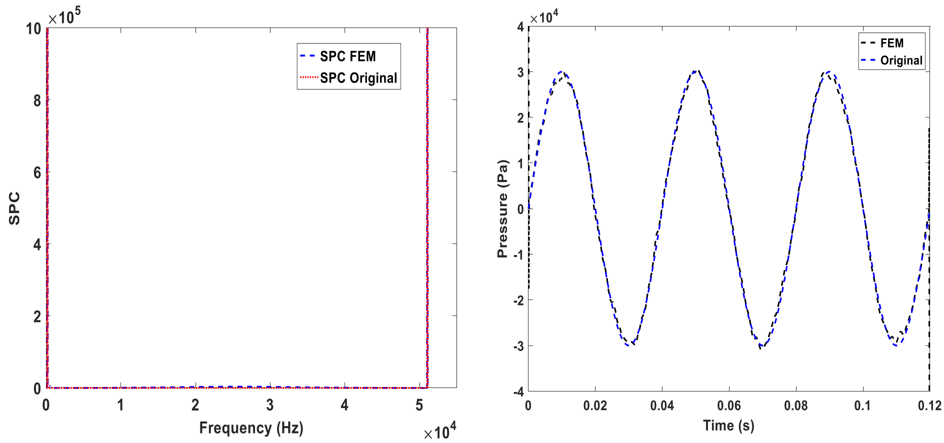


Spectral content



Impact force reconstruction

(b) Reconstruction of Impact pressure signal characteristics: Configuration 2 (#4, #5)



Spectral content   Impact force reconstruction

(c) Reconstruction of Impact pressure signal characteristics: Configuration 3 (#4, #6)

Fig. 11 Identification of impact force and a spectral of power by using three configurations of position sensors

Fig. 11 shows the impact force identification by using only the finite element method as stated by Eq. (8). The first point to make from Fig. 10 is that from the power spectrum, you can predict the quality of the reconstructed signal. The reconstruction is poor in Figs. 10a and 11b correspond to the first configuration when using the sensor excitation #4 (node corresponding 363) and the sensor response #7 (node corresponding 358) and for the second configuration (#4, #5). Configuration three (#4, #6) corresponding to the nodes (363, 374) gives a very good reconstruction, which Can be explained by the following:

- The position of the measured response sensor is crucial.
- Do not be near or far from the impact area.
- The best result is for the displacement Y_{j1} measured by sensor #6 which is in node 474.

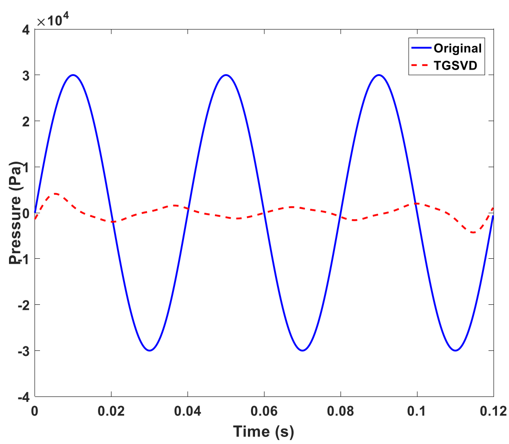


Fig. 12 Reconstruction of impact force when using 3 modes and filter truncation order k=10

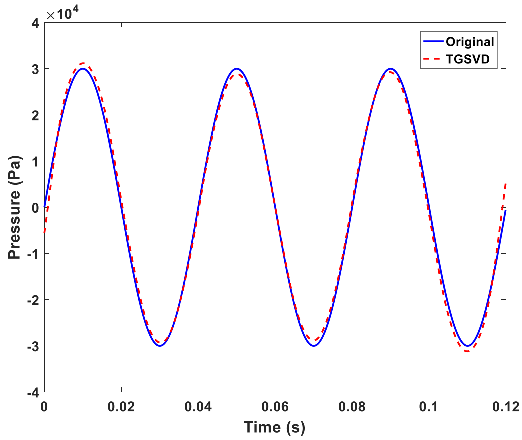


Fig. 13 Reconstruction of impact force when using 3 modes and filter truncation order k=20

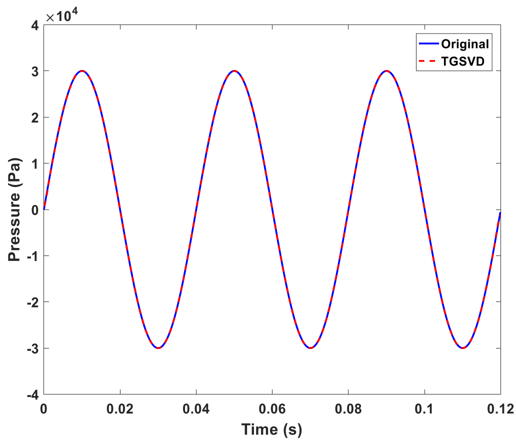


Fig. 14 Reconstruction of impact force when using 3 modes and filter truncation order k=80,

This method requires a laborious calculation and several tests in order to find the best place to implement the measurement sensor. This part was the object of a scanning calculation on the whole plate which is not easy in the case of any plate or structure that has rather large dimensions. To make the reconstruction method more reliable, we use the technique of generalized regularization in order of truncation.

Figs. 12, 13, and 14 illustrate the various results found by using the finite element method and TGSVD regularization method; it is important to remember that the position of the sensors in this method does not influence enough the good reconstruction of the impact signal. The only parameter that influenced the quality of the reconstructed signal is the truncation mode.

From figs. 12, 13, and 14 we can observe the effect of regularization in establishing the transfer function conducted to the reconstruction impact force. One can see that the reconstruction is bad without or with a wrong truncation filtering order. Those figures give a comparison with the signal reconstructed by using the TGSVD technique regularization and real impact force. For the filter, the truncation order equals 10 the reconstruction is bad (Fig.12). For a truncation number equal to 20, the reconstructed signal takes a form that is very close to the real signal (Fig.13).

The signal profile obtained by the solution of the inverse problem coincides perfectly with the actual input force (Fig. 14). It is shown in this work that the mode number does not have much influence on the quality of the reconstructed signal, for this reason, the number of modes has been reduced to three. It is convenient to report that the best results obtained for the truncation filter order were equal to 80.

Compared to the work of Qiao et al. [10] who discussed the determination of a harmonic impact force for three types of sinusoidal forces at certain frequencies 60Hz, 80 Hz, and 150 Hz. In this identification procedure, they used SpaRSA identification and Tikhonov regularization with the L-curve criterion. they showed a clear advantage of SpaRSA over the Tikhonov method. This work for a more complicated case that generates a non-punctual impact for the same type of force shows the efficiency of the TGSVD method to reconstruct the impact signal. Moreover, the calculation of the relative error between the exact and identified forces is envisaged in order to test the performance of the TGSVD-based identification method and compare it with the work of Qiao et al. [10]. It is defined as

$$Relative\ error = \frac{\left\|F_{exact} - F_{identified}\right\|_2}{\|F_{exact}\|_2} \times 100\% \tag{12}$$

All calculations are completed and show a clear advantage of the TGSVD which shows a relative error equal to 2.10%. This value is lower than the one displayed in the work of Qiao et al. In addition to this, the Tikhonov method can generate a relative error of about 40% [10].

4.3.3 Effect of Noise Measurement

To test the robustness of the TGSVD regularization method, we consider four levels of measurement noise that affect the amplitude of the responses recorded by the sensors implement on the structure. The noise level considered is given by the following formula: $r = \alpha(-1 + ran\,\mathrm{d}(\mu))$, with α representing the noise level considered in this study and μ varies randomly between [0,1].

Figs. 15, 16, and 17 show the noise effect on the quality of the reconstructed signal, in fact, to achieve a good quality impact signal reconstruction, the noise level must not exceed 1%.

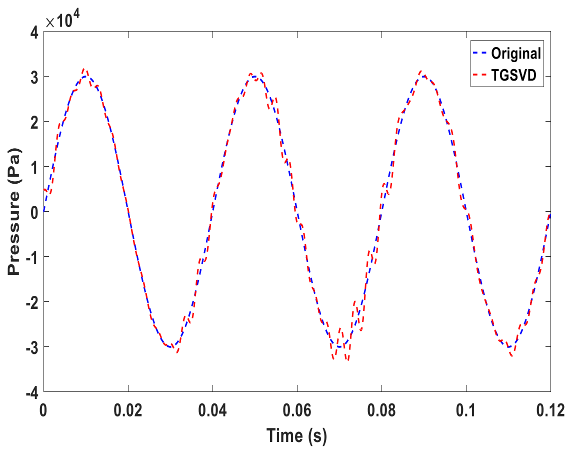


Fig. 15 Reconstruction of impact force when using 0.5% of measured noise (k=80),

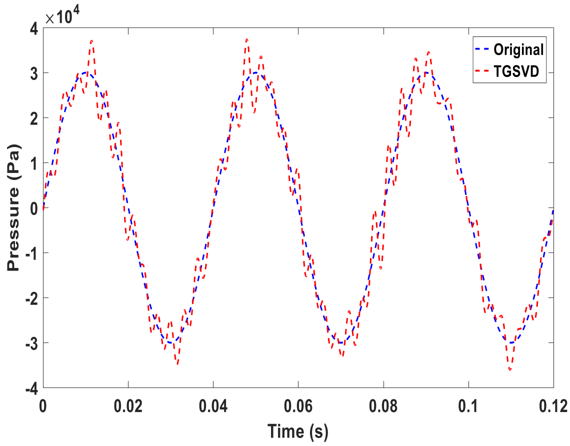


Fig. 16 Reconstruction of impact force when using 1% of measured noise (k=80),

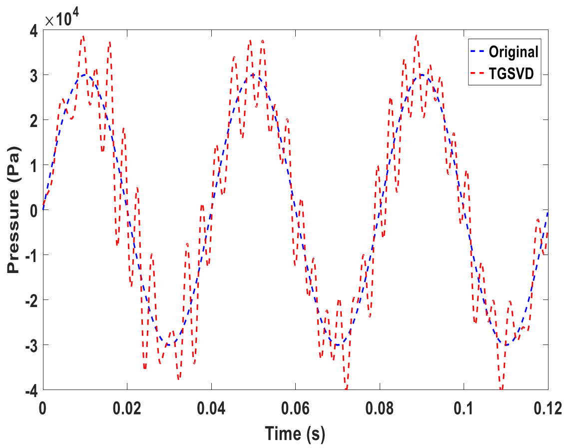


Fig. 17 Reconstruction of impact force when using 2% of measured noise (k=80),

It is clear from this part of the study which dealt with the influence of the noise level on the reconstructed signal quality, that a poor-quality sensor generates a significant noise that impacts the quality of the results and will not provide enough information about the impact signal. It should be noted that other case studies have been carried out in this work for a higher noise level and also for reconstruction based on the finite element method and the transient dynamic response. The latter gives a bad reconstruction from the first noise level. This makes the use of the TGSVD method essential in the reconstruction of the impact signal.

5. Conclusions

In this current study, the identification of a distributed impact acting on a patch of an elastic rectangular plate was performed. The identification procedure followed in this work is to decouple the reconstruction of impact force history from localization. The first step is to do optimization work ensured by a program under Matlab that allows localizing the impact area. whereas in the second step, the reconstruction of impact force history was performed using the finite element method (FEM) and then the truncated generalized singular value decomposition (TGSVD).

The proposed approach was applied to a homogeneous isotropic linear structure. The finite element method makes it possible to construct a modal or dynamic transient model. The generalized Toeplitz matrix was obtained by the IFFT of the FRF. After the reconstruction of the matrix elements, the minimization technique of the objective function was used in order de define the impact characteristics that define the impact zone. Localization has been solved for a linear structure. Once the impact zone is determined, the history of impact load is identified using the truncated generalized singular value decomposition (TGSVD) and finite element method. The impact force reconstruction is performed by two separate methods, trying to identify the history of the impact load by using just the finite element method without any means of regularization. It turned out that the reconstruction is possible for some sensors. This reconstruction is not resistant to the effects of low noise, it simply comes down to the sensitivity of the transfer matrix inversion. This makes the use of the regularization method mandatory for these kinds of identification problems.

The TGSVD shows a relative error equal to 2.10%. This value is lower than that of literature value where Tikhonov regularization with the L-curve criterion was used & results in a relative error of about 40%". Further, the current work for a more complicated case that generates a non-punctual impact for the same type of force shows the efficiency of the TGSVD method to reconstruct the impact signal.

The TGSVD was used to regularize the reconstruction problem. The efficiency of the approach used in this work was discussed in terms of the sensor's position when using only dynamical transitive responses without and with the TGSVD regularization technique, the modal truncation order and sensitivity of the measurement noise affected the responses. The modal convergence was achieved using only three modes. One should recall that the studied case is a symmetric problem and the impact frequency content is low. The proposed method can however be generalized to a more complicated situation without major problems.

Nomenclature

FEM- Finite Element Method

FRF- Frequency Response Function

TGSVD- Truncated Generalized Singular Value Decomposition

IFFT- Inverse Fast Fourier Transformation

P- the pressure signal vector

Y - the displacement vector

H - the Toeplitz matrix

j_i - positions sensors

T_c - the total duration of the signal

S_0 - impact center

$\delta a, \delta b$ - length and width of impact zone characteristics

E - elastic modulus

D - the flexural rigidity modulus of the plate

μ - Poisson ration

h -thickness of the plate

Φ_{ik}- the ith component of the kth mode normalized concerning mass

ω – angular frequency

β - damping coefficient

Competing Interests

The authors declare no competing financial interests.

References

[1] Calvetti D, Morigi S, Reichel L, et al. Tikhonov regularization and the L-curve for large discrete ill-posed problems. Journal of Computational and Applied Mathematics, 2000; 123(1-2): 423-446. https://doi.org/10.1016/S0377-0427(00)00414-3

[2] Nordström LJ. A dynamic programming algorithm for input estimation on linear time-variant systems. Computer Methods in Applied Mechanics and Engineering, 2006; 195: 6407-6427. https://doi.org/10.1016/j.cma.2006.01.002

[3] Qiao B, Liu J, Liu J, et al. An enhanced sparse regularization method for impact force identification. Mechanical Systems and Signal Processing, 2019; 126: 341-367. https://doi.org/10.1016/j.ymssp.2019.02.039

[4] Jia Y, Yang Z, Song Q. Experimental study of random dynamic loads identification based on weighted regularization method. Journal of Sound and Vibration, 2015; 342 :113-123. https://doi.org/10.1016/j.jsv.2014.12.010

[5] Chen Z, Chan THT, Yu L. Comparison of regularization methods for moving force identification with ill-posed problems. Journal of Sound and Vibration, 2020; 478:115349. https://doi.org/10.1016/j.jsv.2020.115349

[6] Hu N, Fukunaga H, Matsumoto S, at al. An efficient approach for identifying impact force using embedded piezoelectric sensors. International Journal of Impact Engineering, 2007; 34:1258-1271. https://doi.org/10.1016/j.ijimpeng.2006.05.004

[7] Wang L, Liu Y, Liu D, et al. A novel dynamic reliability-based topology optimization (DRBTO) framework for continuum structures via interval-process collocation and the first-passage theories. Computer Methods in Applied Mechanics and Engineering, 2021; 386: 114107. https://doi.org/10.1016/j.cma.2021.114107

[8] Jacquelin E, Bennani A, Hamelin P. Force reconstruction: analysis and regularization of a deconvolution problem. Journal of Sound and Vibration, 2003; 265 (1): 81-107. https://doi.org/10.1016/S0022-460X(02)01441-4

[9] Rezayat A, Nassiri V, Pauw BD, Ertveldt J, Vanlanduit S, Guillaume P. Identification of dynamic forces using group-sparsity in frequency domain. Mechanical Systems and Signal Processing, 2016; 70: 756-768. https://doi.org/10.1016/j.ymssp.2015.09.015

[10] Qiao B, Zhang X, Wang C, Zhang H, Chen X. Sparse regularization for force identification using dictionaries, Journal of Sound and Vibration, 2016; 368: 71-86. https://doi.org/10.1016/j.jsv.2016.01.030

[11] He J, Guan X, Liu Y. Structural response reconstruction based on empirical mode decomposition in time domain. Mechanical Systems and Signal Processing, 2012; 28:348-366. https://doi.org/10.1016/j.ymssp.2011.12.010

[12] W Feng, Q Li, Q Lu. Force localization and reconstruction based on a novel sparse Kalman filter. Mechanical Systems and Signal Processing, 2020, 144: 106890. https://doi.org/10.1016/j.ymssp.2020.106890

[13] Qiao B, Mao Z, Liu J, et al. Group sparse regularization for impact force identification in time domain. Journal of Sound of Vibration, 2019; 445: 44-63. https://doi.org/10.1016/j.jsv.2019.01.004

[14] Doyle JF. A wavelet deconvolution method for impact force identification. Experimental Mechanics, 1997; 37 (4): 403-408. https://doi.org/10.1007/BF02317305
[15] Chang C, Sun CT. Determining transverse impact force on a composite laminate by signal deconvolution. Experimental Mechanics, 1989; 29:414-419. https://doi.org/10.1007/BF02323860
[16] Xu B, He J, Rovekamp R, et al. Structural parameters and dynamic loading identification from incomplete measurements: approach and validation. Mechanical Systems and Signal Processing, 2012; 28: 244-257. https://doi.org/10.1016/j.ymssp.2011.07.008
[17] Yu Z, Mahdavi SH, Xu C. Time-domain spectral element method for impact identification of frame structures using enhanced GAs. KSCE Journal of Civil Engineering, 2019; 23: 678-690. https://doi.org/10.1007/s12205-018-0478-8
[18] Liu J, Hei C, Luo M, Yang D, Sun C, Feng A. A study on impact force detection method based on piezoelectric sensing. Sensors, 2022; 22: 5167. https://doi.org/10.3390/s22145167
[19] Doyle JF. An experimental method for determining the dynamic contact law. Experimental Mechanics ,1984; 24:10-16. https://doi.org/10.1007/BF02323199
[20] Inoue H, Harrigan J, Reid S. Review of inverse analysis for indirect measurement of impact force. Applied Mechanics Reviews, 2001; 54 (6): 503-524. https://doi.org/10.1115/1.1420194
[21] He Z, Lin X, Li E. A novel method for load bounds identification for uncertain structures in frequency domain. International Journal for Computational Methods, 2018; 15:1850051 ü. https://doi.org/10.1142/S0219876218500512
[22] Liu J, Qiao B, Chen Y, et al. Impact force reconstruction and localization using nonconvex overlapping group sparsity. Mechanical Systems and Signal Processing, 2022; 162: 107983. https://doi.org/10.1016/j.ymssp.2021.107983
[23] Liu Y, Shepard WS Jr. Dynamic force identification based on enhanced least squares and total least-squares schemes in the frequency domain. Journal of Sound of Vibration, 2005, 282: 37-60. https://doi.org/10.1016/j.jsv.2004.02.041
[24] Yen CS, Wu E. On the inverse problem of rectangular plates subjected to elastic impact, Part 1: method development and numerical verification. Journal of Applied Mechanics, 1995; 62: 692-698. https://doi.org/10.1115/1.2896002
[25] El-Bakari A, Khamlichi A, Dkiouak R, et al. Assessing impact force localization by using a particle swarm algorithm. Journal of sound of vibration. 2014; 333(6): 1554-1561. https://doi.org/10.1016/j.jsv.2013.11.032
[26] Liu H, Liu Q, Liu B, et al. An efficient and robust method for structural distributed load identification based on mesh superposition approach. Mechanical Systems and Signal Processing, 2021;151:107383. https://doi.org/10.1016/j.ymssp.2020.107383
[27] Tikhonov AN, Arsenin VY. Solutions of ill-posed problems. Wiley, New York, USA,1977.
[28] Hansen PC. Rank-deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion, vol. 4, Siam, USA, 2005.