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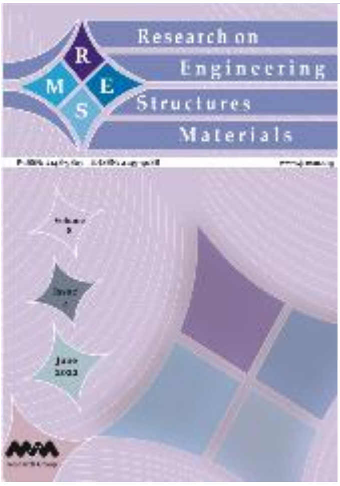

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*Research Article*

# Estimating vibration period of reinforced concrete moment resisting frame buildings

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## Abstract

One of the initial steps in the analysis and design of buildings subjected to lateral loads is estimating the fundamental vibration period of the building. Design codes and standards recommend conducting modal analysis to investigate the fundamental vibration period of the building. On the other hand, these design codes and standards specify simple empirical models that relate the fundamental period to the building height as an alternative approach to modal analysis. In this study, extensive modal analyses were conducted to investigate the fundamental period of 382 building models. Modal analyses were conducted to evaluate the effect of design parameters on the fundamental vibration period of reinforced concrete moment resisting frame buildings. The effect of each design parameter was identified using sensitivity analysis. Finally, a simple model was developed in this study based on the results of modal analysis to estimate the fundamental vibration period of the buildings. Main design parameters including building height, spans length, columns elasticity and columns size were considered in the developed model. The proposed model was validated against modal analysis in which a mean value of the proposed model to modal analysis predictions ratio equal to 1.00 ± 0.155 with coefficient of variation equal to 15.38 were obtained.

## 1. Introduction

The fundamental vibration period is a main parameter that appears in base shear and lateral forces equations used for the analysis and design of buildings subjected to either wind loads or seismic excitation. The fundamental vibration period is mainly depending on stiffness and mass of the building which is a function of several parameters including building height, structural system, material properties, members dimensions and plan area of the building. Design codes and standards recommended using either modal analysis or a specified simple time period building height relation in order to predict the fundamental period of the buildings [1–3]. Researchers used recorded vibration time during earthquakes or ambient vibration experiments for developing empirical models. Goel and Chopra [4] and Salama [5] considered vibration of reinforced concrete moment-resisting frame buildings during earthquake in different regions in the United States. Hong and Hwang [6] and Chiauzzi et al. [7] considered vibration of reinforced concrete buildings during earthquake in Taiwan and Canada, respectively. On the other hand, several models were developed based on the recorded ambient vibrations of the buildings in different regions in the world. Guler et al. [8], Inel et al. [9] and Kaplan et al. [10] considered buildings in Turkey. Velani and Ramancharla [11] and Velani and Kumar [12] considered buildings in India. Jalali and Milani [13], Gallipoli et al. [14], Al-Nimry et al. [15] and Pan et al. [16] considered ambient vibration of buildings in Iran, Europe, Jordan and Singapore, respectively. Almost all the previous developed models give direct relations of the

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fundamental vibration period to buildings height. However, it was shown that previous simple models resulted in significantly large variations in predictions [17–24].

Several models to predict fundamental vibration time were developed in the literature based on modal analysis considering numerical modelling of the buildings. Crowley and Pinho [25] and Crowley and Pinho [26] studied several existing European buildings and developed an equation that relates fundamental vibration periods to building height. Amanat and Hoque [27] modified code specified fundamental period to building height relation by introducing span length, number of spans and infill amount factors based on finite element modeling and modal analyses. Rimal and Maskey [28] considered building height, building plan and number of bays in their proposed model that developed based on finite element modelling and modal analyses results. Koçak et al. [29] proposed empirical relation of fundamental period to building height, modulus of elasticity of infill walls and thickness of the infill walls based on numerical analysis considering 270 building models. Kewate and Murudi [30] modeled 21 existing reinforced concrete moment resisting frame buildings in India and proposed an equation that relates the fundamental vibration period to building height. Multiple fundamental period to building height relations for different seismic intensities were developed by Verderame et al. [31] based on modal analyses results. Mohamed et al. [32] used applied element method in modeling reinforced concrete buildings and conducted nonlinear dynamic analysis in which the fundamental period was extracted from the time history curve. An equation relates fundamental vibration period to building height, building width to length ratio and column size was developed. Joshi et al. [33] analyzed 206 building models to generate vibration period data and used the generated data in genetic programming for developing multiple models according to different limits of building height. Also, Hadzima-Nyarko and Draganic [34] used genetic algorithms to develop a fundamental period formulas based on the modal analysis results of finite element models. Asteris et al. [35] proposed using artificial neural network to predict the fundamental vibration period of buildings considering the effect of several design parameters including building height, spans length, number of spans, infill strength and amount. The developed artificial neural network model was trained and verified against numerical modeling and modal analyses results. Al-Balhawi and Zhang [36] investigated and developed a vibration period model for reinforced concrete tall buildings having moment resisting frames with shear walls system. Noor et al. [37] modeled 21 existing reinforced concrete buildings in India and proposed simple fundamental period to building height model based on the analyses results. Sharma et al. [38] developed artificial neural network model to predict fundamental period of reinforced concrete buildings based on finite element analysis results of modal analyses considering the effect of pile soil interaction. Gravett et al. [39] developed fundamental period formula based on the analyses results of 475 building models considering the effect of soil structure interaction and foundation types including separate footings and mat foundations. Ruggieri et al. [40] modeled newly constructed 40 reinforced concrete buildings using finite element method and conducted modal analyses. The results were utilized in regression analysis to derive a model to estimate fundamental period of buildings. Mirrashid and Naderpour [41] proposed fundamental period models of infilled reinforced concrete buildings using artificial neural network and neuro fuzzy methods considering the influence of number of stories, number of spans, spans length, infill stiffness and ratio of openings to infill.

In this study, recorded vibration periods of buildings located in different regions in the world were collected and compared with predictions of different models. Also, extensive modal analyses were conducted considering 382 reinforced concrete moment resisting frame (RC MRF) building models developed in this study to investigate the effect of different design parameters on the fundamental vibration period of the buildings. Models were built and modal analyses were conducted using commercially available structural analysis and design software SAP 2000 [42]. Sensitivity analyses were conducted in which the effects of main design parameters including building height, spans length, columns stiffness was considered. The main objective of this study was to develop a simple but more precise model to estimate the fundamental vibration period of reinforced concrete moment resisting frame buildings based on the results of the modal analyses and considering the main influencing design parameters. The developed model is recommended for practicing engineers as an alternative to modal analysis.

The following section includes collecting some of actual records of fundamental periods as well as exploring several available models to predict fundamental period in which the results are compared. The third section includes presenting numerical modeling and modal analyses to investigate the fundamental period of 382 building models developed in this study. A proposed model in this study to estimate the fundamental vibration period is presented in fourth section followed by section five that includes the sensitive analysis to investigate the influence of each design parameter of the proposed model based on the results of modal analysis presented in this study. Then, the proposed model has verified in the sixth section. Finally, study considerations are highlighted and main conclusions points are drawn.

## 2. Available Period of Vibration Models

The design codes and standards specify different fundamental period to building height relations [1–3]. Also, several models were presented in the literature. Most of the available models consider the fundamental period of the building as a function of to its height or a number of stories. The available models were derived by regression analysis considering actual vibration records during earthquakes or ambient vibration of buildings located in different regions in the world. Table 1 presents some of these available models. The terms $T_f$ and H represent the time of the fundamental vibration period and the building height, respectively.

Table 1. Available vibration time model

| Researcher | Model | Location |
|---|---|---|
| ASCE/SEI 7-16 [1] | $T_f = 0.0466\, H^{0.9}$ | USA |
| Eurocode 8 [2] | $T_f = 0.075\, H^{3/4}$ | Europe |
| FEMA 450 [3] | $T_f = 0.0524\, H^{0.9}$ | USA |
| Hong and Hwang [6] | $T_f = 0.0294\, H^{0.804}$ | Taiwan |
| Michel et al. [43] | $T_f = 0.013\, H$ | France |
| Chiauzzi et al. [7] | $T_f = 0.037\, H^{0.76}$ | Canada |
| Guler et al. [8] | $T_f = 0.026\, H^{0.9}$ | Turkey |
| Velani and Ramancharla [11] | $T_f = 0.009\, H^{1.1}$ | India |
| Inel et al. [9] | $T_f = 0.0343\, H^{0.762}$ | Turkey |

To evaluate the validity of the available models, predictions using these models are compared with actual records gathered from published works. Data of vibration period of 255 RC MRF buildings located in different regions in the world were collected and presented in Table 2 with measured time periods illustrated in Fig. 1. A comparison between predictions using available models and actual records of fundamental vibration time is presented in Fig. 2. Fig. 2 shows that almost all the existing formulas resulted in substantially different predictions. Also, a significant discrepancy between the predictions and measured values have been demonstrated. Consequently, adopting simple relation of building height to fundamental period resulted in inadequate predictions.

Table 2. Data of measured fundamental period of RC MRF buildings published in the literature

| Reference | Location | No. of data | Building height (m) |
|---|---|---|---|
| Goel and Chopra [4] | USA | 34 | 9 – 56 |
| Hong and Hwang [6] | Taiwan | 19 | 27 – 77 |
| Jalali and Milani [13] | Iran | 11 | 17 – 53 |
| Guler et al. [8] | Turkey | 6 | 12 – 30 |
| Chiauzzi et al. [7] | Canada | 12 | 12 – 70 |
| Gallipoli et al. [14] | Europe | 113 | 3 – 51 |
| Ditommaso et al. [44] | Italy | 14 | 5.2 – 24.7 |
| Al-Nimry et al. [15] | Jordan | 25 | 4.4 – 20.5 |
| Velani and Ramancharla [11] | India | 21 | 72 – 147 |
|  | Total | 255 | 3 – 147 |

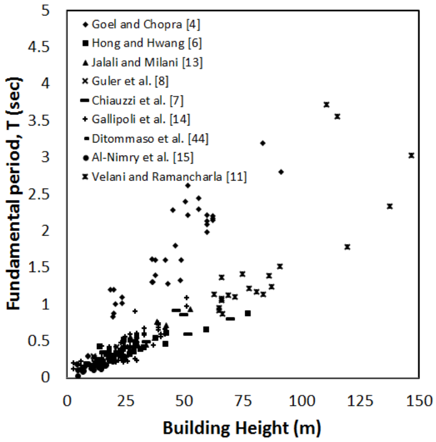


Fig. 1 Recorded fundamental vibration time of RC MRF buildings located in different regions in the world

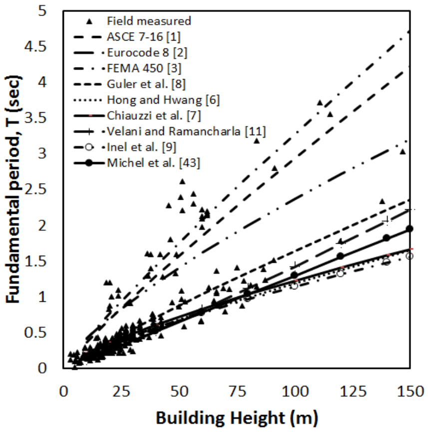


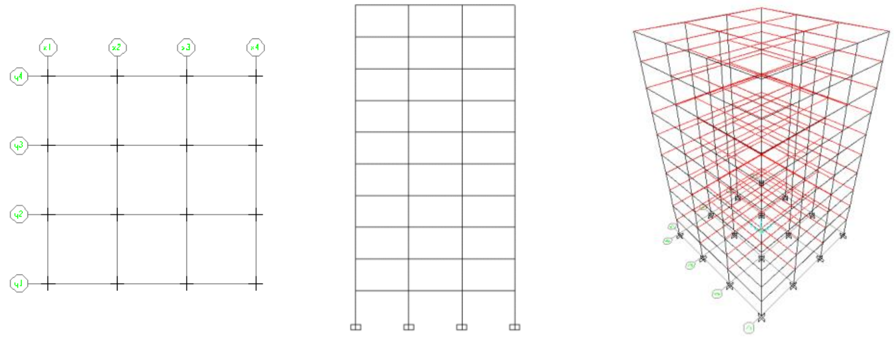
Fig. 2 Comparison between the recorded and predicted fundamental vibration time of RC MRF buildings

## 3. Numerical Investigations

In this study, modal analysis was conducted considering 382 models of reinforced concrete moment resisting frame buildings to determine the fundamental time period. Buildings were modelled using three-dimensional finite elements using the structural analysis and design software SAP2000 [42]. Beams and columns were simulated using three-dimensional frame elements and slabs were modelled using three-dimensional plate rectangular elements. The slabs thickness was assigned equal to 180 mm for all building models and the beams web width and total depth were set equal to 0.3 m and 0.50 m, respectively. The parametric investigations include the effect of building height, column size, span length and material properties on the time period of the buildings (T). The considered height of each storey of was 3 m. Table 3 gives the limits of the design parameters of the building models that considered in the investigations. Figure 3 illustrates a model structure of the buildings using the structural analysis and design program SAP2000 [42]. Table 4 illustrates the details of the parameters of the considered models in this study including height of the building (H), the span length (L), the compressive strength of concrete (f'c), the side dimension of the column (D) and the number of spans in orthogonal directions (Bx and By). Results of modal analyses for the considered models are illustrated in Table 4 and Fig. 4. Fig. 4 shows very wide range of predicted vibration periods corresponding to building height. The variation in the fundamental period can be seen to stem from the effect of different design parameters other than the building height. Also, it is shown that the numerical results presented in Fig. 4 cover the range of almost all measured values that shown in Figs. 1 and 2.

Table 3. Limits of the design parameters of the considered building models in the numerical analysis

| Parameter | Values | Range |
|---|---|---|
| Building height (m) | 3, 15, 30, 45, 60, 90 | 3 – 90 |
| Columns side dimension (m) | 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.2 | 0.3 – 1.2 |
| Compressive strength of concrete $f_c'$ (MPa) | 30, 40, 50, 60, 70, 80, 90 | 30 – 90 |
| Span length (m) | 4, 5, 6, 7 | 4 – 7 |



(a) (b) (c)

Fig. 3 Building Model considered in this study using SAP2000 [42], a) plan view, b) side view and c) three dimensional view

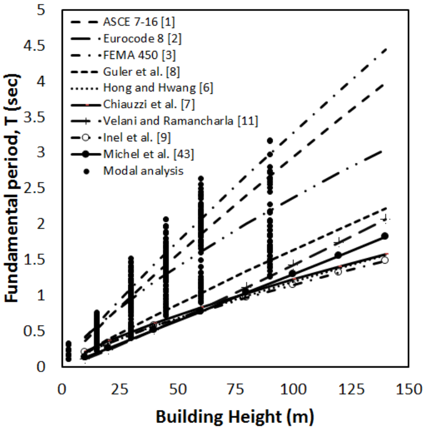


Fig. 4 Comparison of numerical fundamental period of buildings with different specified models

Table 4. Details of the design parameters of the considered models

| Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) | Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 4 | 70 | 0.3 | 3 | 3 | 0.1556 | 59 | 30 | 4 | 70 | 0.6 | 3 | 3 | 0.7301 |
| 2 | 15 | 4 | 70 | 0.3 | 3 | 3 | 0.6178 | 60 | 45 | 4 | 70 | 0.6 | 3 | 3 | 1.1468 |
| 3 | 30 | 4 | 70 | 0.3 | 3 | 3 | 1.2317 | 61 | 60 | 4 | 70 | 0.6 | 3 | 3 | 1.5995 |
| 4 | 45 | 4 | 70 | 0.3 | 3 | 3 | 1.8965 | 62 | 15 | 4 | 60 | 0.6 | 3 | 3 | 0.3532 |
| 5 | 15 | 4 | 60 | 0.3 | 3 | 3 | 0.6421 | 63 | 30 | 4 | 60 | 0.6 | 3 | 3 | 0.7588 |
| 6 | 30 | 4 | 60 | 0.3 | 3 | 3 | 1.2800 | 64 | 45 | 4 | 60 | 0.6 | 3 | 3 | 1.1919 |
| 7 | 45 | 4 | 60 | 0.3 | 3 | 3 | 1.9710 | 65 | 60 | 4 | 60 | 0.6 | 3 | 3 | 1.6623 |
| 8 | 15 | 4 | 50 | 0.3 | 3 | 3 | 0.6720 | 66 | 15 | 4 | 50 | 0.6 | 3 | 3 | 0.3697 |
| 9 | 30 | 4 | 50 | 0.3 | 3 | 3 | 1.3397 | 67 | 30 | 4 | 50 | 0.6 | 3 | 3 | 0.7942 |
| 10 | 45 | 4 | 50 | 0.3 | 3 | 3 | 2.0630 | 68 | 45 | 4 | 50 | 0.6 | 3 | 3 | 1.2474 |
| 11 | 3 | 4 | 40 | 0.3 | 3 | 3 | 0.1789 | 69 | 60 | 4 | 50 | 0.6 | 3 | 3 | 1.7399 |
| 12 | 15 | 4 | 40 | 0.3 | 3 | 3 | 0.7106 | 70 | 15 | 4 | 40 | 0.6 | 3 | 3 | 0.3909 |
| 13 | 30 | 4 | 40 | 0.3 | 3 | 3 | 1.4166 | 71 | 30 | 4 | 40 | 0.6 | 3 | 3 | 0.8397 |
| 14 | 15 | 4 | 30 | 0.3 | 3 | 3 | 0.7635 | 72 | 45 | 4 | 40 | 0.6 | 3 | 3 | 1.3190 |
| 15 | 30 | 4 | 30 | 0.3 | 3 | 3 | 1.5222 | 73 | 60 | 4 | 40 | 0.6 | 3 | 3 | 1.8397 |
| 16 | 15 | 4 | 70 | 0.4 | 3 | 3 | 0.4393 | 74 | 15 | 4 | 30 | 0.6 | 3 | 3 | 0.4200 |
| 17 | 30 | 4 | 70 | 0.4 | 3 | 3 | 0.8972 | 75 | 30 | 4 | 30 | 0.6 | 3 | 3 | 0.9023 |
| 18 | 45 | 4 | 70 | 0.4 | 3 | 3 | 1.3963 | 76 | 45 | 4 | 30 | 0.6 | 3 | 3 | 1.4174 |
| 19 | 15 | 4 | 60 | 0.4 | 3 | 3 | 0.4566 | 77 | 60 | 4 | 30 | 0.6 | 3 | 3 | 1.9769 |
| 20 | 30 | 4 | 60 | 0.4 | 3 | 3 | 0.9325 | 78 | 15 | 5 | 70 | 0.4 | 3 | 3 | 0.5088 |
| 21 | 45 | 4 | 60 | 0.4 | 3 | 3 | 1.4511 | 79 | 30 | 5 | 70 | 0.4 | 3 | 3 | 1.0393 |
| 22 | 60 | 4 | 60 | 0.4 | 3 | 3 | 2.0288 | 80 | 45 | 5 | 70 | 0.4 | 3 | 3 | 1.5576 |
| 23 | 3 | 4 | 50 | 0.4 | 3 | 3 | 0.1063 | 81 | 60 | 5 | 70 | 0.4 | 3 | 3 | 2.1995 |
| 24 | 15 | 4 | 50 | 0.4 | 3 | 3 | 0.4779 | 82 | 15 | 5 | 60 | 0.4 | 3 | 3 | 0.5288 |
| 25 | 30 | 4 | 50 | 0.4 | 3 | 3 | 0.9760 | 83 | 30 | 5 | 60 | 0.4 | 3 | 3 | 1.0801 |
| 26 | 45 | 4 | 50 | 0.4 | 3 | 3 | 1.5188 | 84 | 45 | 5 | 60 | 0.4 | 3 | 3 | 1.6188 |
| 27 | 60 | 4 | 50 | 0.4 | 3 | 3 | 2.1234 | 85 | 60 | 5 | 60 | 0.4 | 3 | 3 | 2.2859 |
| 28 | 3 | 4 | 40 | 0.4 | 3 | 3 | 0.1124 | 86 | 15 | 5 | 50 | 0.4 | 3 | 3 | 0.5534 |
| 29 | 15 | 4 | 40 | 0.4 | 3 | 3 | 0.5053 | 87 | 30 | 5 | 50 | 0.4 | 3 | 3 | 1.1305 |
| 30 | 30 | 4 | 40 | 0.4 | 3 | 3 | 1.0319 | 88 | 45 | 5 | 50 | 0.4 | 3 | 3 | 1.6943 |
| 31 | 45 | 4 | 40 | 0.4 | 3 | 3 | 1.6059 | 89 | 15 | 5 | 40 | 0.4 | 3 | 3 | 0.5852 |
| 32 | 60 | 4 | 40 | 0.4 | 3 | 3 | 2.2452 | 90 | 30 | 5 | 40 | 0.4 | 3 | 3 | 1.1953 |
| 33 | 15 | 4 | 30 | 0.4 | 3 | 3 | 0.5430 | 91 | 15 | 5 | 30 | 0.4 | 3 | 3 | 0.6288 |
| 34 | 30 | 4 | 30 | 0.4 | 3 | 3 | 1.1089 | 92 | 30 | 5 | 30 | 0.4 | 3 | 3 | 1.2845 |
| 35 | 45 | 4 | 30 | 0.4 | 3 | 3 | 1.7257 | 93 | 15 | 5 | 70 | 0.5 | 3 | 3 | 0.4235 |
| 36 | 60 | 4 | 30 | 0.4 | 3 | 3 | 2.4127 | 94 | 30 | 5 | 70 | 0.5 | 3 | 3 | 0.8851 |
| 37 | 15 | 4 | 70 | 0.5 | 3 | 3 | 0.3711 | 95 | 45 | 5 | 70 | 0.5 | 3 | 3 | 1.3357 |
| 38 | 30 | 4 | 70 | 0.5 | 3 | 3 | 0.7762 | 96 | 60 | 5 | 70 | 0.5 | 3 | 3 | 1.8852 |
| 39 | 45 | 4 | 70 | 0.5 | 3 | 3 | 1.2138 | 97 | 15 | 5 | 60 | 0.5 | 3 | 3 | 0.4401 |
| 40 | 60 | 4 | 70 | 0.5 | 3 | 3 | 1.6964 | 98 | 30 | 5 | 60 | 0.5 | 3 | 3 | 0.9198 |
| 41 | 15 | 4 | 60 | 0.5 | 3 | 3 | 0.3857 | 99 | 45 | 5 | 60 | 0.5 | 3 | 3 | 1.3882 |
| 42 | 30 | 4 | 60 | 0.5 | 3 | 3 | 0.8067 | 100 | 60 | 5 | 60 | 0.5 | 3 | 3 | 1.9593 |
| 43 | 45 | 4 | 60 | 0.5 | 3 | 3 | 1.2615 | 101 | 15 | 5 | 50 | 0.5 | 3 | 3 | 0.4607 |
| 44 | 60 | 4 | 60 | 0.5 | 3 | 3 | 1.7631 | 102 | 60 | 5 | 80 | 0.7 | 3 | 3 | 1.1800 |
| 45 | 15 | 4 | 50 | 0.5 | 3 | 3 | 0.4037 | 103 | 30 | 5 | 50 | 0.5 | 3 | 3 | 0.9627 |
| 46 | 30 | 4 | 50 | 0.5 | 3 | 3 | 0.8444 | 104 | 45 | 5 | 50 | 0.5 | 3 | 3 | 1.4529 |
| 47 | 45 | 4 | 50 | 0.5 | 3 | 3 | 1.3203 | 105 | 60 | 5 | 50 | 0.5 | 3 | 3 | 2.0507 |
| 48 | 60 | 4 | 50 | 0.5 | 3 | 3 | 1.8453 | 106 | 15 | 5 | 40 | 0.5 | 3 | 3 | 0.4871 |
| 49 | 15 | 4 | 40 | 0.5 | 3 | 3 | 0.4269 | 107 | 30 | 5 | 40 | 0.5 | 3 | 3 | 1.0180 |
| 50 | 30 | 4 | 40 | 0.5 | 3 | 3 | 0.8928 | 108 | 45 | 5 | 40 | 0.5 | 3 | 3 | 1.5363 |
| 51 | 45 | 5 | 60 | 0.7 | 6 | 3 | 0.9160 | 109 | 60 | 5 | 40 | 0.5 | 3 | 3 | 2.1683 |
| 52 | 45 | 4 | 40 | 0.5 | 3 | 3 | 1.3961 | 110 | 15 | 5 | 30 | 0.5 | 3 | 3 | 0.5234 |
| 53 | 60 | 4 | 40 | 0.5 | 3 | 3 | 1.9512 | 111 | 30 | 5 | 30 | 0.5 | 3 | 3 | 1.0939 |
| 54 | 15 | 4 | 30 | 0.5 | 3 | 3 | 0.4587 | 112 | 15 | 5 | 70 | 0.6 | 3 | 3 | 0.3831 |
| 55 | 30 | 4 | 30 | 0.5 | 3 | 3 | 0.9594 | 113 | 30 | 5 | 70 | 0.6 | 3 | 3 | 0.8215 |
| 56 | 45 | 4 | 30 | 0.5 | 3 | 3 | 1.5002 | 114 | 45 | 5 | 70 | 0.6 | 3 | 3 | 1.2487 |
| 57 | 60 | 4 | 30 | 0.5 | 3 | 3 | 2.0967 | 115 | 60 | 5 | 70 | 0.6 | 3 | 3 | 1.7591 |
| 58 | 15 | 4 | 70 | 0.6 | 3 | 3 | 0.3399 | 116 | 15 | 5 | 60 | 0.6 | 3 | 3 | 0.3981 |

Table 4. Continued

| Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) | Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 117 | 30 | 5 | 60 | 0.6 | 3 | 3 | 0.8538 | 172 | 15 | 6 | 30 | 0.5 | 3 | 3 | 0.5991 |
| 118 | 45 | 5 | 60 | 0.6 | 3 | 3 | 1.2977 | 173 | 30 | 6 | 30 | 0.5 | 3 | 3 | 1.2262 |
| 119 | 60 | 5 | 60 | 0.6 | 3 | 3 | 1.8283 | 174 | 15 | 6 | 70 | 0.6 | 3 | 3 | 0.4339 |
| 120 | 15 | 5 | 50 | 0.6 | 3 | 3 | 0.4167 | 175 | 30 | 6 | 70 | 0.6 | 3 | 3 | 0.9108 |
| 121 | 30 | 5 | 50 | 0.6 | 3 | 3 | 0.8936 | 176 | 45 | 6 | 70 | 0.6 | 3 | 3 | 1.4051 |
| 122 | 45 | 5 | 50 | 0.6 | 3 | 3 | 1.3582 | 177 | 60 | 6 | 70 | 0.6 | 3 | 3 | 1.9230 |
| 123 | 60 | 5 | 50 | 0.6 | 3 | 3 | 1.9135 | 178 | 15 | 6 | 60 | 0.6 | 3 | 3 | 0.4510 |
| 124 | 15 | 5 | 40 | 0.6 | 3 | 3 | 0.4406 | 179 | 30 | 6 | 60 | 0.6 | 3 | 3 | 0.9466 |
| 125 | 30 | 5 | 40 | 0.6 | 3 | 3 | 0.9449 | 180 | 45 | 6 | 60 | 0.6 | 3 | 3 | 1.4603 |
| 126 | 45 | 5 | 40 | 0.6 | 3 | 3 | 1.4362 | 181 | 60 | 6 | 60 | 0.6 | 3 | 3 | 1.9986 |
| 127 | 60 | 5 | 40 | 0.6 | 3 | 3 | 2.0233 | 182 | 15 | 6 | 50 | 0.6 | 3 | 3 | 0.4720 |
| 128 | 15 | 5 | 30 | 0.6 | 3 | 3 | 0.4734 | 183 | 30 | 6 | 50 | 0.6 | 3 | 3 | 0.9908 |
| 129 | 30 | 5 | 30 | 0.6 | 3 | 3 | 1.0171 | 184 | 45 | 6 | 50 | 0.6 | 3 | 3 | 1.5284 |
| 130 | 45 | 5 | 30 | 0.6 | 3 | 3 | 1.5433 | 185 | 60 | 6 | 50 | 0.6 | 3 | 3 | 2.0918 |
| 131 | 45 | 5 | 70 | 0.7 | 3 | 3 | 1.2159 | 186 | 15 | 6 | 40 | 0.6 | 3 | 3 | 0.4991 |
| 132 | 60 | 5 | 70 | 0.7 | 3 | 3 | 1.7121 | 187 | 30 | 6 | 40 | 0.6 | 3 | 3 | 1.0476 |
| 133 | 45 | 5 | 60 | 0.7 | 3 | 3 | 1.2637 | 188 | 45 | 6 | 40 | 0.6 | 3 | 3 | 1.6161 |
| 134 | 60 | 5 | 60 | 0.7 | 3 | 3 | 1.7794 | 189 | 60 | 6 | 40 | 0.6 | 3 | 3 | 2.2118 |
| 135 | 45 | 5 | 50 | 0.7 | 3 | 3 | 1.3226 | 190 | 15 | 6 | 30 | 0.6 | 3 | 3 | 0.5363 |
| 136 | 60 | 5 | 50 | 0.7 | 3 | 3 | 1.8624 | 191 | 30 | 6 | 30 | 0.6 | 3 | 3 | 1.1257 |
| 137 | 45 | 5 | 40 | 0.7 | 3 | 3 | 1.3985 | 192 | 45 | 6 | 30 | 0.6 | 3 | 3 | 1.7366 |
| 138 | 60 | 5 | 40 | 0.7 | 3 | 3 | 1.9692 | 193 | 45 | 6 | 70 | 0.7 | 3 | 3 | 1.3542 |
| 139 | 45 | 5 | 30 | 0.7 | 3 | 3 | 1.5028 | 194 | 60 | 6 | 70 | 0.7 | 3 | 3 | 1.8566 |
| 140 | 15 | 6 | 70 | 0.4 | 3 | 3 | 0.5899 | 195 | 45 | 6 | 60 | 0.7 | 3 | 3 | 1.4075 |
| 141 | 30 | 6 | 70 | 0.4 | 3 | 3 | 1.1811 | 196 | 60 | 6 | 60 | 0.7 | 3 | 3 | 1.9295 |
| 142 | 45 | 6 | 70 | 0.4 | 3 | 3 | 1.8003 | 197 | 45 | 6 | 50 | 0.7 | 3 | 3 | 1.4731 |
| 143 | 60 | 6 | 70 | 0.4 | 3 | 3 | 2.4584 | 198 | 60 | 6 | 50 | 0.7 | 3 | 3 | 2.0195 |
| 144 | 15 | 6 | 60 | 0.4 | 3 | 3 | 0.6131 | 199 | 45 | 6 | 40 | 0.7 | 3 | 3 | 1.5576 |
| 145 | 30 | 6 | 60 | 0.4 | 3 | 3 | 1.2275 | 200 | 60 | 6 | 40 | 0.7 | 3 | 3 | 2.1354 |
| 146 | 45 | 6 | 60 | 0.4 | 3 | 3 | 1.8711 | 201 | 45 | 6 | 30 | 0.7 | 3 | 3 | 1.6737 |
| 147 | 60 | 6 | 60 | 0.4 | 3 | 3 | 2.5549 | 202 | 15 | 7 | 70 | 0.4 | 3 | 3 | 0.6458 |
| 148 | 15 | 6 | 50 | 0.4 | 3 | 3 | 0.6417 | 203 | 30 | 7 | 70 | 0.4 | 3 | 3 | 1.2821 |
| 149 | 30 | 6 | 50 | 0.4 | 3 | 3 | 1.2847 | 204 | 60 | 5 | 90 | 0.7 | 3 | 3 | 1.1400 |
| 150 | 45 | 6 | 50 | 0.4 | 3 | 3 | 1.9583 | 205 | 45 | 7 | 70 | 0.4 | 3 | 3 | 1.9654 |
| 151 | 15 | 6 | 40 | 0.4 | 3 | 3 | 0.6785 | 206 | 60 | 7 | 70 | 0.4 | 3 | 3 | 2.6429 |
| 152 | 30 | 6 | 40 | 0.4 | 3 | 3 | 1.3585 | 207 | 15 | 7 | 60 | 0.4 | 3 | 3 | 0.6711 |
| 153 | 45 | 5 | 70 | 0.7 | 6 | 3 | 0.8800 | 208 | 30 | 7 | 60 | 0.4 | 3 | 3 | 1.3325 |
| 154 | 15 | 6 | 30 | 0.4 | 3 | 3 | 0.7291 | 209 | 45 | 7 | 60 | 0.4 | 3 | 3 | 1.9803 |
| 155 | 30 | 6 | 30 | 0.4 | 3 | 3 | 1.4598 | 210 | 15 | 7 | 50 | 0.4 | 3 | 3 | 0.7024 |
| 156 | 15 | 6 | 70 | 0.5 | 3 | 3 | 0.4847 | 211 | 30 | 7 | 50 | 0.4 | 3 | 3 | 1.3947 |
| 157 | 30 | 6 | 70 | 0.5 | 3 | 3 | 0.9921 | 212 | 45 | 7 | 50 | 0.4 | 3 | 3 | 2.0726 |
| 158 | 45 | 6 | 70 | 0.5 | 3 | 3 | 1.5212 | 213 | 15 | 7 | 40 | 0.4 | 3 | 3 | 0.7427 |
| 159 | 60 | 6 | 70 | 0.5 | 3 | 3 | 2.0806 | 214 | 30 | 7 | 40 | 0.4 | 3 | 3 | 1.4747 |
| 160 | 15 | 6 | 60 | 0.5 | 3 | 3 | 0.5038 | 215 | 15 | 7 | 70 | 0.5 | 3 | 3 | 0.5162 |
| 161 | 30 | 6 | 60 | 0.5 | 3 | 3 | 1.0311 | 216 | 30 | 7 | 70 | 0.5 | 3 | 3 | 1.0452 |
| 162 | 45 | 6 | 60 | 0.5 | 3 | 3 | 1.5810 | 217 | 45 | 7 | 70 | 0.5 | 3 | 3 | 1.5632 |
| 163 | 60 | 6 | 60 | 0.5 | 3 | 3 | 2.1623 | 218 | 60 | 7 | 70 | 0.5 | 3 | 3 | 2.1723 |
| 164 | 15 | 6 | 50 | 0.5 | 3 | 3 | 0.5273 | 219 | 15 | 7 | 60 | 0.5 | 3 | 3 | 0.5364 |
| 165 | 30 | 6 | 50 | 0.5 | 3 | 3 | 1.0792 | 220 | 30 | 7 | 60 | 0.5 | 3 | 3 | 1.0862 |
| 166 | 45 | 6 | 50 | 0.5 | 3 | 3 | 1.6547 | 221 | 45 | 7 | 60 | 0.5 | 3 | 3 | 1.6246 |
| 167 | 60 | 6 | 50 | 0.5 | 3 | 3 | 2.2632 | 222 | 60 | 7 | 60 | 0.5 | 3 | 3 | 2.2576 |
| 168 | 15 | 6 | 40 | 0.5 | 3 | 3 | 0.5575 | 223 | 15 | 7 | 50 | 0.5 | 3 | 3 | 0.5615 |
| 169 | 30 | 6 | 40 | 0.5 | 3 | 3 | 1.1411 | 224 | 30 | 7 | 50 | 0.5 | 3 | 3 | 1.1369 |
| 170 | 45 | 6 | 40 | 0.5 | 3 | 3 | 1.7496 | 225 | 45 | 7 | 50 | 0.5 | 3 | 3 | 1.7003 |
| 171 | 60 | 6 | 40 | 0.5 | 3 | 3 | 2.3930 | 226 | 60 | 7 | 50 | 0.5 | 3 | 3 | 2.3629 |

Table 4. Continued

| Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) | Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 227 | 15 | 7 | 40 | 0.5 | 3 | 3 | 0.5937 | 285 | 30 | 4 | 60 | 0.7 | 3 | 9 | 0.5080 |
| 228 | 30 | 7 | 40 | 0.5 | 3 | 3 | 1.2021 | 286 | 30 | 4 | 70 | 0.7 | 3 | 9 | 0.4890 |
| 229 | 45 | 7 | 40 | 0.5 | 3 | 3 | 1.7979 | 287 | 30 | 4 | 80 | 0.7 | 3 | 9 | 0.4730 |
| 230 | 60 | 7 | 40 | 0.5 | 3 | 3 | 2.4985 | 288 | 30 | 4 | 90 | 0.7 | 3 | 9 | 0.4590 |
| 231 | 15 | 7 | 70 | 0.6 | 3 | 3 | 0.4540 | 289 | 30 | 4 | 30 | 0.6 | 3 | 9 | 0.7140 |
| 232 | 30 | 7 | 70 | 0.6 | 3 | 3 | 0.9398 | 290 | 30 | 4 | 40 | 0.6 | 3 | 12 | 0.6640 |
| 233 | 45 | 7 | 70 | 0.6 | 3 | 3 | 1.4144 | 291 | 30 | 4 | 50 | 0.6 | 3 | 12 | 0.6280 |
| 234 | 60 | 7 | 70 | 0.6 | 3 | 3 | 1.9661 | 292 | 30 | 4 | 60 | 0.6 | 3 | 12 | 0.6000 |
| 235 | 15 | 7 | 60 | 0.6 | 3 | 3 | 0.4718 | 293 | 30 | 4 | 70 | 0.6 | 3 | 12 | 0.5780 |
| 236 | 30 | 7 | 60 | 0.6 | 3 | 3 | 0.9767 | 294 | 30 | 4 | 80 | 0.6 | 3 | 12 | 0.5590 |
| 237 | 45 | 7 | 60 | 0.6 | 3 | 3 | 1.4699 | 295 | 30 | 4 | 90 | 0.6 | 3 | 12 | 0.5420 |
| 238 | 60 | 7 | 60 | 0.6 | 3 | 3 | 2.0434 | 296 | 45 | 5 | 30 | 0.8 | 3 | 12 | 0.9500 |
| 239 | 15 | 7 | 50 | 0.6 | 3 | 3 | 0.4938 | 297 | 45 | 5 | 40 | 0.8 | 3 | 12 | 0.8800 |
| 240 | 30 | 7 | 50 | 0.6 | 3 | 3 | 1.0223 | 298 | 45 | 5 | 50 | 0.8 | 3 | 15 | 0.8300 |
| 241 | 45 | 7 | 50 | 0.6 | 3 | 3 | 1.5385 | 299 | 45 | 5 | 60 | 0.8 | 3 | 15 | 0.8000 |
| 242 | 60 | 7 | 50 | 0.6 | 3 | 3 | 2.1386 | 300 | 45 | 5 | 70 | 0.8 | 3 | 15 | 0.7700 |
| 243 | 15 | 7 | 40 | 0.6 | 3 | 3 | 0.5221 | 301 | 45 | 5 | 80 | 0.8 | 3 | 15 | 0.7400 |
| 244 | 30 | 7 | 40 | 0.6 | 3 | 3 | 1.0809 | 302 | 45 | 5 | 90 | 0.8 | 3 | 15 | 0.7200 |
| 245 | 45 | 7 | 40 | 0.6 | 3 | 3 | 1.6268 | 303 | 45 | 5 | 30 | 0.7 | 3 | 15 | 1.0900 |
| 246 | 60 | 7 | 40 | 0.6 | 3 | 3 | 2.2613 | 304 | 45 | 5 | 40 | 0.7 | 3 | 15 | 1.0100 |
| 247 | 15 | 7 | 70 | 0.7 | 3 | 3 | 0.4178 | 305 | 45 | 5 | 50 | 0.7 | 3 | 15 | 0.9600 |
| 248 | 30 | 7 | 70 | 0.7 | 3 | 3 | 0.8870 | 306 | 90 | 5 | 60 | 1.2 | 3 | 3 | 1.3900 |
| 249 | 45 | 7 | 70 | 0.7 | 3 | 3 | 1.3450 | 307 | 45 | 5 | 90 | 0.7 | 6 | 3 | 0.8300 |
| 250 | 60 | 7 | 70 | 0.7 | 3 | 3 | 1.8710 | 308 | 45 | 5 | 30 | 0.6 | 6 | 3 | 1.3000 |
| 251 | 15 | 7 | 60 | 0.7 | 3 | 3 | 0.4342 | 309 | 45 | 5 | 40 | 0.6 | 6 | 3 | 1.2000 |
| 252 | 30 | 7 | 60 | 0.7 | 3 | 3 | 0.9219 | 310 | 45 | 5 | 50 | 0.6 | 6 | 3 | 1.1400 |
| 253 | 45 | 7 | 60 | 0.7 | 3 | 3 | 1.3978 | 311 | 45 | 5 | 60 | 0.6 | 6 | 3 | 1.0900 |
| 254 | 60 | 7 | 60 | 0.7 | 3 | 3 | 1.9445 | 312 | 45 | 5 | 70 | 0.6 | 9 | 3 | 1.0500 |
| 255 | 45 | 5 | 80 | 0.7 | 6 | 3 | 0.8500 | 313 | 45 | 5 | 80 | 0.6 | 9 | 3 | 1.0100 |
| 256 | 15 | 7 | 50 | 0.7 | 3 | 3 | 0.4545 | 314 | 45 | 5 | 90 | 0.6 | 9 | 3 | 0.9900 |
| 257 | 30 | 7 | 50 | 0.7 | 3 | 3 | 0.9648 | 315 | 3 | 5 | 30 | 0.3 | 9 | 3 | 0.2440 |
| 258 | 45 | 7 | 50 | 0.7 | 3 | 3 | 1.4630 | 316 | 3 | 5 | 40 | 0.3 | 9 | 3 | 0.2300 |
| 259 | 60 | 7 | 50 | 0.7 | 3 | 3 | 2.0352 | 317 | 3 | 5 | 30 | 0.25 | 9 | 3 | 0.3300 |
| 260 | 15 | 7 | 40 | 0.7 | 3 | 3 | 0.4806 | 318 | 3 | 5 | 40 | 0.25 | 9 | 3 | 0.3050 |
| 261 | 30 | 7 | 40 | 0.7 | 3 | 3 | 1.0202 | 319 | 15 | 5 | 30 | 0.8 | 9 | 3 | 0.3000 |
| 262 | 45 | 7 | 40 | 0.7 | 3 | 3 | 1.5469 | 320 | 15 | 5 | 40 | 0.8 | 12 | 3 | 0.2800 |
| 263 | 60 | 7 | 40 | 0.7 | 3 | 3 | 2.1520 | 321 | 15 | 5 | 50 | 0.8 | 12 | 3 | 0.2600 |
| 264 | 90 | 6 | 70 | 0.8 | 3 | 3 | 2.7410 | 322 | 15 | 5 | 60 | 0.8 | 12 | 3 | 0.2500 |
| 265 | 90 | 6 | 70 | 0.9 | 3 | 3 | 2.7200 | 323 | 15 | 5 | 70 | 0.8 | 12 | 3 | 0.2400 |
| 266 | 90 | 6 | 80 | 0.9 | 3 | 3 | 2.6310 | 324 | 15 | 5 | 80 | 0.8 | 12 | 3 | 0.2300 |
| 267 | 90 | 6 | 90 | 1 | 3 | 3 | 2.5580 | 325 | 15 | 5 | 90 | 0.8 | 12 | 3 | 0.2250 |
| 268 | 90 | 6 | 60 | 0.7 | 3 | 3 | 2.8060 | 340 | 15 | 5 | 30 | 0.7 | 12 | 3 | 0.3400 |
| 269 | 90 | 6 | 90 | 1.2 | 3 | 3 | 2.5970 | 341 | 15 | 5 | 40 | 0.7 | 12 | 3 | 0.3200 |
| 270 | 90 | 6 | 90 | 1 | 3 | 3 | 2.5150 | 342 | 15 | 5 | 50 | 0.7 | 15 | 3 | 0.3000 |
| 271 | 90 | 4 | 70 | 0.7 | 3 | 3 | 2.6550 | 343 | 60 | 5 | 60 | 0.8 | 3 | 3 | 1.1100 |
| 272 | 90 | 4 | 50 | 0.7 | 3 | 3 | 2.9800 | 344 | 60 | 5 | 70 | 0.8 | 3 | 3 | 1.0700 |
| 273 | 90 | 4 | 40 | 0.7 | 3 | 3 | 3.1600 | 345 | 60 | 5 | 80 | 0.8 | 3 | 3 | 1.0300 |
| 274 | 90 | 4 | 40 | 0.6 | 3 | 6 | 3.1800 | 346 | 60 | 5 | 90 | 0.8 | 3 | 3 | 1.0000 |
| 275 | 30 | 4 | 30 | 0.8 | 3 | 6 | 0.5290 | 347 | 60 | 5 | 30 | 0.7 | 3 | 3 | 1.5000 |
| 276 | 30 | 4 | 40 | 0.8 | 3 | 6 | 0.4920 | 348 | 60 | 5 | 40 | 0.7 | 3 | 3 | 1.4000 |
| 277 | 30 | 4 | 50 | 0.8 | 3 | 6 | 0.4660 | 349 | 60 | 5 | 50 | 0.7 | 3 | 3 | 1.3200 |
| 278 | 30 | 4 | 60 | 0.8 | 3 | 6 | 0.4450 | 350 | 60 | 5 | 60 | 0.7 | 3 | 3 | 1.2600 |
| 279 | 30 | 4 | 70 | 0.8 | 3 | 6 | 0.4280 | 351 | 60 | 5 | 70 | 0.7 | 3 | 3 | 1.2200 |
| 280 | 30 | 4 | 80 | 0.8 | 3 | 6 | 0.4140 | 352 | 90 | 5 | 70 | 1.2 | 3 | 3 | 1.3400 |
| 281 | 30 | 4 | 90 | 0.8 | 3 | 6 | 0.4000 | 353 | 90 | 5 | 80 | 1.2 | 3 | 3 | 1.2900 |
| 282 | 30 | 4 | 30 | 0.7 | 3 | 9 | 0.6040 | 354 | 90 | 5 | 90 | 1.2 | 3 | 3 | 1.2600 |
| 283 | 30 | 4 | 40 | 0.7 | 3 | 9 | 0.5630 | 355 | 90 | 5 | 30 | 1 | 3 | 3 | 1.8400 |
| 284 | 30 | 4 | 50 | 0.7 | 3 | 9 | 0.5320 | 356 | 90 | 5 | 40 | 1 | 3 | 3 | 1.7100 |

Table 4. Continued

| Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) | Mod. | H (m) | L (m) | F'c (MPa) | D (m) | Bx (m) | By (m) | T (sec.) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 357 | 90 | 5 | 50 | 1 | 3 | 3 | 1.6200 | 370 | 90 | 5 | 40 | 0.8 | 3 | 3 | 2.0300 |
| 358 | 90 | 5 | 60 | 1 | 3 | 3 | 1.5500 | 371 | 90 | 5 | 50 | 0.8 | 3 | 3 | 1.9200 |
| 359 | 90 | 5 | 70 | 1 | 3 | 3 | 1.4900 | 372 | 90 | 5 | 60 | 0.8 | 3 | 3 | 1.8300 |
| 360 | 90 | 5 | 80 | 1 | 3 | 3 | 1.4400 | 373 | 90 | 5 | 70 | 0.8 | 3 | 3 | 1.7600 |
| 361 | 90 | 5 | 90 | 1 | 3 | 3 | 1.4000 | 374 | 90 | 5 | 80 | 0.8 | 3 | 3 | 1.7000 |
| 362 | 90 | 5 | 30 | 0.9 | 3 | 3 | 1.9800 | 375 | 90 | 5 | 90 | 0.8 | 3 | 3 | 1.6500 |
| 363 | 90 | 5 | 40 | 0.9 | 3 | 3 | 1.8500 | 376 | 90 | 5 | 30 | 0.7 | 3 | 3 | 2.4500 |
| 364 | 90 | 5 | 50 | 0.9 | 3 | 3 | 1.7500 | 377 | 90 | 5 | 40 | 0.7 | 3 | 3 | 2.2800 |
| 365 | 90 | 5 | 60 | 0.9 | 3 | 3 | 1.6700 | 378 | 90 | 5 | 50 | 0.7 | 3 | 3 | 2.1600 |
| 366 | 90 | 5 | 70 | 0.9 | 3 | 3 | 1.6100 | 379 | 90 | 5 | 60 | 0.7 | 3 | 3 | 2.0600 |
| 367 | 90 | 5 | 80 | 0.9 | 3 | 3 | 1.5500 | 380 | 90 | 5 | 70 | 0.7 | 3 | 3 | 1.9800 |
| 368 | 90 | 5 | 90 | 0.9 | 3 | 3 | 1.5100 | 381 | 90 | 5 | 80 | 0.7 | 3 | 3 | 1.9200 |
| 369 | 90 | 5 | 30 | 0.8 | 3 | 3 | 2.1800 | 382 | 90 | 5 | 90 | 0.7 | 3 | 3 | 1.8600 |

## 4. Proposed Formula

The proposed fundamental time model in this study incorporates the main design parameters that affect the fundamental period of buildings. The considered parameters in the proposed model include buildings height, spans length and columns stiffness (size and material properties). The proposed fundamental time model is presented as:

$$T_f = \alpha_1 \, (H)^{\alpha_2} (L)^{\alpha_3} (f_c')^{\alpha_4} (D)^{\alpha_5} \tag{1}$$

In which, D, $f_c'$ and L are the column side dimension, compressive strength of the concrete and span length, respectively. The constants $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ and $\alpha_5$ are the proportional factors that determined by adopting sensitive analyses on 382 modal analysis results.

## 5. Sensitivity Investigations

Sensitivity analyses are conducted to evaluate the effect of different design parameters on the vibration period of the building models. The sensitivity investigations are conducted considering the numerical results of 382 models of RC MRF buildings presented in this study. The parameters that considered in this study are illustrated in Table 4. The results of the sensitivity analyses are presented in the following subsections.

### 5.1. Effect of Buildings Height

A relation between the building height and the fundamental period of the buildings was presented in which the best fit curve was obtained and illustrated in Fig. 5. The fundamental time period to building height relation becomes;

$$T_f = 0.0537 \, H^{0.828} \tag{2}$$

Fig. 5 shows that the vibration period $T_f$ is directly proportional to the height of the building with a power of 0.828. In comparison with previous simple models, it is shown that equation 2 is closest to the model specified by FEMA [3] that presented in Table 1.

### 5.2. Effect of Span Length

To investigate the effect of span length L on the vibration period of the buildings, the vibration periods are normalized by ( $H^{0.828}$) to exclude the effect of building height. The resulted normalized vibration period is given by;

$$T_{f1} = T_f / H^{0.828} \tag{3}$$

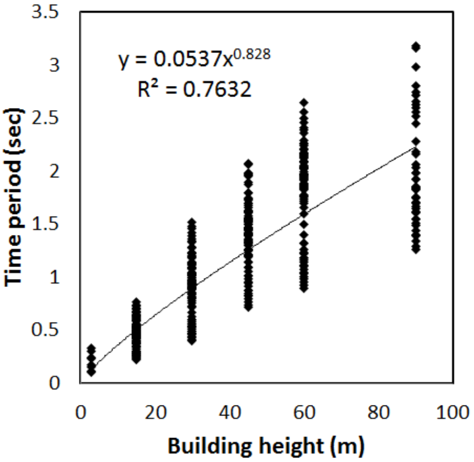


Fig. 5 Varying the fundamental period of the buildings with building height

Fig. 6 shows that the normalized vibration period $T_1$ increased with increasing span length with a power of 0.55.

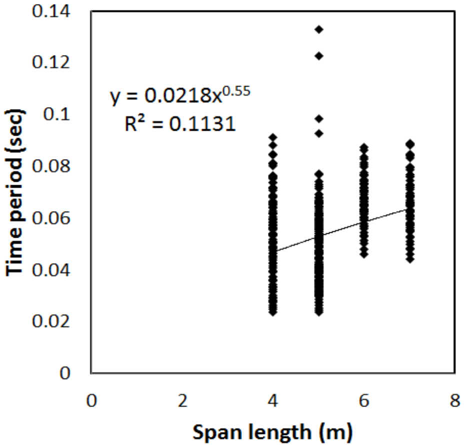


Fig. 6 Varying the normalized fundamental period of the buildings with spans length

### 5.3. Effect of Members Elasticity

In modeling reinforced concrete members, the elasticity of concrete is considered for defining members elasticity in which it is directly proportional to compressive strength of the concrete. In order to investigate the effect of concrete compressive strength on the vibration period of the buildings, the vibration periods $(T_{f1})$ are normalized by $(L^{-0.55})$ to exclude the effect of column size. The resulted normalized vibration period is given by;

$$T_{f2} = Tf_1 / L^{-0.55} \tag{4}$$

Fig. 7 shows that the normalized vibration period $T_{f2}$ decreased with increasing compressive strength with a power of - 0.358.

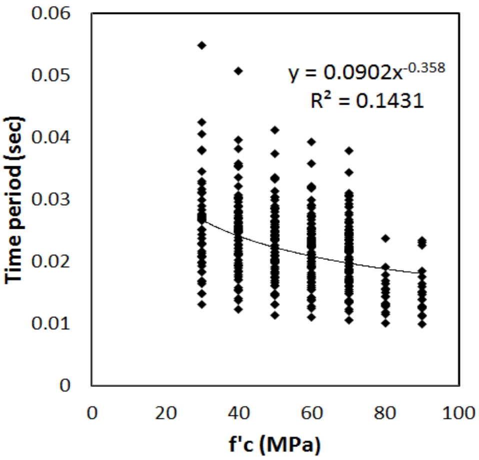


Fig. 7 Varying of normalized fundamental period of the buildings with concrete compressive strength

### 5.3 Effect of Columns Size

To investigate the effect of columns size on the vibration period of the buildings, the vibration periods ($T_{f2}$) are normalized by (${f_c'}^{-0.358}$) to eliminate the effect of concrete compressive strength. The resulted normalized vibration period is given by;

$$T_{f3} = \frac{T_{f2}}{{f_c'}^{-0.358}} \tag{5}$$

Fig. 8 shows that the normalized vibration period $T_{f3}$ decreased with increasing columns size with a power of - 0.665.

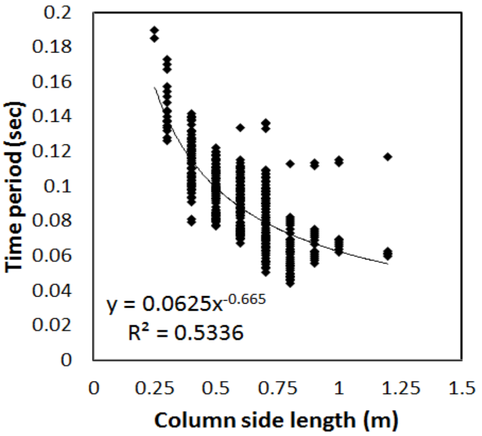


Fig. 8 Varying of normalized fundamental period of the buildings with columns size

### 5.4 Proportional Factor $\alpha_1$

The obtained powers 0.828, 0.55, -0.358 and -0.665 of the parameters H, L, f'c and D, respectively that obtained from curves of Figs. 5-8 are substituted in equation 1 in which becomes;

$$T_{f4} = \alpha_1 \, (H)^{0.828} (L)^{0.55} (f_c')^{-0.358} \, (D)^{-0.665} \tag{6}$$

In order to obtain the value of the coefficient a₁, a relation between numerical period $T_f$ and $T_{f4}$ is presented in which the best fit curve is obtained. A value of 0.0314 for the coefficient $\alpha_1$ and a normalizes period with a power of 1.2459 are obtained with R² equal to 0.953. By substituting the coefficient $\alpha_1$and refining equation 6 by multiplying the parameters powers by the obtained new power from Fig. 9 resulted in new powers for parameters H, L, D and f'c that equals to 1.032, 0.69, -0.83 and -0.45, respectively. Substituting the new powers in equation 1 resulted in;

$$T_f = 0.0314 \quad (H)^{1.023} (L)^{0.69} \, (f_c')^{-0.45} \, (D)^{-0.83} \tag{7}$$

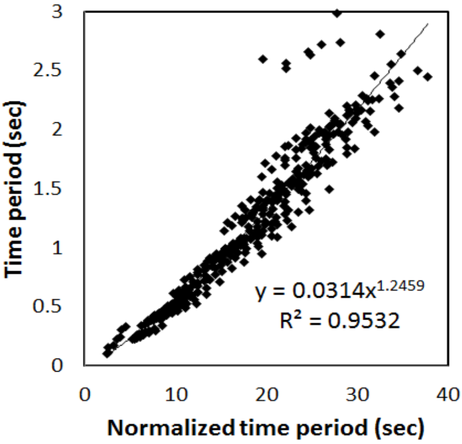


Fig. 9 Varying of fundamental period of the buildings with normalized time period

## 6. Validation of the Proposed Model

The calculated fundamental period using the developed simple equation is verified against the results of modal analysis of the 382 numerical models of buildings. Verification results in terms of the ratio of proposed model to modal analysis predictions demonstrated very good performance of the developed model. A mean value of the prediction's ratio (proposed model predictions to modal analysis predictions) equal to $1.00 \pm 0.15$ and a coefficient of variation of 15.38 are obtained. On the other hand, statistical measures are compared with those corresponding to predictions using the models available in the literature as illustrated in Table 5. It is obvious that the proposed model results in the most accurate predictions compared with other models as it provides the least standard of deviation and coefficient of variation as well as the closest results between the modal analysis and model predictions.

Additional ten models were considered for comparison in which haven't used in the sensitivity analyses and verification of the developed model. Table 6 illustrates the comparison results of the predicted fundamental vibration period of the ten models by using modal analysis and the proposed model. Comparison results have demonstrated very good agreement between the predictions using the proposed model and the modal analysis. The mean value of the proposed model to modal analysis ratio of $1.00 \pm 0.12$ and a coefficient of variation of 11.79 were obtained.

Table 5. Comparison of the predicted fundamental periods using the proposed model and other available models

| Researcher | Mean | St. Deviation | CoV % |
|---|---|---|---|
| ASCE/SEI 7-16 [1] | 0.936 | 0.289 | 30.89 |
| Eurocode 8 [2] | 0.991 | 0.290 | 29.22 |
| FEMA 450 [3] | 0.832 | 0.257 | 30.89 |
| Hong and Hwang [6] | 1.462 | 0.453 | 31.01 |
| Michel et al. [43] | 2.366 | 0.834 | 35.25 |
| Chiauzzi et al. [7] | 1.938 | 0.565 | 29.17 |
| Guler et al. [8] | 1.677 | 0.518 | 30.89 |
| Velani and Ramancharla [11] | 2.424 | 1.029 | 42.45 |
| Inel et al. [9] | 2.075 | 0.605 | 29.17 |
| Proposed model | 1.00 | 0.155 | 15.38 |

Table 6. Comparison of the predicted fundamental periods using the proposed model and numerical modal analysis

| Span (m) | Col. width (m) | $f_c'$ (MPa) | Building Height (m) | Time period | | |
|---|---|---|---|---|---|---|
| | | | | Modal analysis | Proposed model | Prop. /Modal |
| 4 | 0.4 | 30 | 30 | 1.1089 | 1.2317 | 1.1108 |
| 4 | 0.4 | 50 | 30 | 0.9760 | 0.9788 | 1.0029 |
| 4 | 0.4 | 70 | 30 | 0.8972 | 0.8412 | 0.9376 |
| 4 | 0.6 | 70 | 30 | 0.7301 | 0.6008 | 0.8230 |
| 5 | 0.4 | 30 | 30 | 1.2845 | 1.4367 | 1.1185 |
| 5 | 0.5 | 70 | 30 | 0.8851 | 0.8154 | 0.9213 |
| 6 | 0.4 | 30 | 30 | 1.4598 | 1.6293 | 1.1162 |
| 6 | 0.4 | 70 | 30 | 1.1811 | 1.1128 | 0.9422 |
| 7 | 0.5 | 70 | 30 | 0.8870 | 1.0284 | 1.1595 |
| 7 | 0.7 | 70 | 45 | 1.3450 | 1.1782 | 0.8760 |
| | | | Mean | | | 1.00 |
| | | | St. Deviation | | | 0.12 |
| | | | CoV % | | | 11.79 |

## 7. Conclusions

Fundamental vibration period of buildings is an essential factor for the analysis and design of buildings subjected to lateral loads. Design codes and standards recommended either conducting modal analysis on numerical models or using a specified simple formula that relates the fundamental vibration period to either building height or number of floors. The specified formulas by design standards and guidelines to predict the fundamental vibration period of buildings were empirically derived based on limited actual records of buildings vibration in different regions around the world. In contrast, modal analyses entail careful and time consuming numerical modelling process that include defining specific design parameters. The aim of this study is to propose a simple but more accurate model to estimate fundamental vibration period of reinforced concrete moment frame buildings. The first part of this study includes collecting some of the published recorded vibration period of buildings in different regions in the world. Some of the collected records were adopted by previous researcher in regression analyses for developing vibration time to building height relations. Also, several simple models available in the literature were explored. The collected data were compared with predictions of the available models in which shows large scatters in actual building vibrations compared to the estimations of the simple formulas. Also, investigations show different and diverse estimations of vibration periods using the available simple models. The large discrepancy between predictions using different model and the large scatters of actual values compared with empirical predictions stem from neglecting the main design parameters in the developed models. In this paper, fundamental periods of moment frame buildings have extensively investigated considering 382 building models by using numerical modal analyses. A sensitivity analysis considering the modal analysis results is conducted in which a model is developed to estimate the fundamental period of the buildings considering the main influencing design parameters. The main design parameters include building height, spans length, members stiffness. Then, the effect of each design parameter is evaluated using sensitivity analyses in which assist in developing the proposed model. Sensitivity analysis showed that fundamental vibration period was directly proportional to the height of the building and span length with powers equal to 1.032 and 0.69, respectively. On the other hand, the fundamental period was inversely proportional to the lateral size of the columns and compressive strength of the concrete with powers equal to 0.83 and 0.45, respectively. Finally, a simple model is proposed based on modal analyses results and considering the effect of main design parameters. The proposed model is validated against modal analysis results of the 382 numerical models of buildings in which results in modal analysis to proposed model ratio equal to $1.00 \pm 0.155$ with coefficient of variation of 15.38. Additional ten models haven't used in the sensitivity analyses and verification of the developed model shows mean value of the proposed model to modal analysis ratio equal to $1.00 \pm 0.12$ and a coefficient of variation of 11.79. Also, the predictions of the proposed model were compared with the predictions of the available simple models and showed the most accurate results. Accordingly, adopting the developed model by practicing engineers rather than using simple models or complex modal analysis for predicting time of fundamental vibration provides efficient solution.

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