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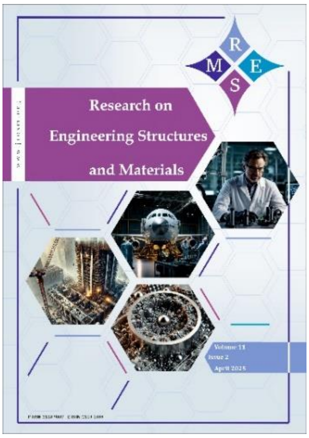

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Research Article

Numerical simulation, constructal design, and systematic search applied to the geometrical evaluation of hat-stiffened plates under bending

Mariana Alvarenga Alves [1,a], Eduarda Machado Rodrigues [2,b], Vinícius Torres Pinto [2,c], Luiz Alberto Oliveira Rocha [1,2,d], Elizaldo Domingues dos Santos [1,2,e], Liércio André Isoldi [*,1,2,f]

[1]*Graduate Program in Computational Modeling (PPGMC), Federal University of Rio Grande - FURG, Brazil*
[2]*Graduate Program in Ocean Engineering (PPGEO), Federal University of Rio Grande - FURG, Brazil*

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Abstract

Steel plates have wide structural applications in various engineering sectors due to their combination of lightness, efficiency, and high load-bearing capacity. When stiffeners are added to these structures, the deflections can be significantly minimized. In this study, hat-stiffened plates having box-girder stiffeners with a trapezoidal shape, positioned longitudinally along the plate, are considered to evaluate their influence on the maximum and central deflection of the structure. The analyses in this work are carried out using ANSYS Mechanical APDL software, employing a validated and verified computational model developed with the SHELL281 finite element. Starting from a reference plate without stiffeners, the Constructal Design method was applied, allowing to propose different geometric configurations for plates with longitudinal hat-stiffeners, keeping the total steel volume constant. All these cases were simulated numerically and their results were compared through a Systematic Search technique. Among these 100 different cases investigated, the geometric configuration with 1 hat-stiffener of 250 mm in height and 9.53 mm in thickness achieved the best performance, resulting in a reduction of approximately 86% in maximum and central deflections compared to the reference plate. This highlights its effectiveness in minimizing out-of-plane displacements.

1. Introduction

Steel plates with stiffeners are widely used in the naval and offshore industries, aerospace industry, automotive sector, and civil construction [1]. These are versatile structural components that combine lightness and efficiency with high load-bearing capacity [2]. In this context, the computational modeling has assumed an important role in this field, as commercial software like ANSYS, which is based on the Finite Element Method (FEM), enabling highly accurate numerical analysis of real structures.

On the other hand, the Constructal Design method has proven efficient for the geometric evaluation of engineering problems, including structural engineering. Constructal Design is based on the Constructal theory proposed by Adrian Bejan in 1996, which, in turn, is supported by the Constructal Law, stated as: "For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that provides greater and greater access to the currents that flow through it" [3]. Therefore, this theory suggests that the generation of flow structures, which are observed throughout nature, can be reasoned

*Corresponding author: liercioisoldi@furg.br
[a]orcid.org/0009-0003-4630-5782; [b]orcid.org/0009-0002-1470-4271; [c]orcid.org/0000-0002-0977-5086; [d]orcid.org/0000-0003-2409-3152; [e]orcid.org/0000-0003-4566-2350; [f]orcid.org/0000-0002-9337-3169
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through an evolutionary principle of increasing access to flow over time. Its application requires the definition of constraints, degrees of freedom, and performance indicators, aiming not only to achieve superior configurations but also to understand the effect of the degrees of freedom variation over the performance indicator [3]. From Constructal theory, the shape of the system and the architecture of internal flow do not develop randomly but result from the ongoing pursuit of better performance and, therefore, should evolve over time [3-4]. In the analysis of structural engineering, Bejan and Lorente [3] shows that the structures are minimal volume connections through which stresses 'flow' from components to their neighbors, and that optimal use of mechanical support material is achieved when the maximum allowable stresses are uniformly distributed throughout the available material.

Among the studies that associate computational modeling and Constructal Design for the geometric evaluation of stiffened plates subjected to bending, the following can be highlighted: the application of the Constructal Design method combined with a systematic search technique to optimize I-stiffened plates under bending due to uniformly distributed loading, aiming to minimize the central deflection in these structures [5]; the minimization of maximum and central deflections in stiffened plates by varying the heights of I-shaped stiffeners in the transverse and longitudinal directions [6]; the evaluation of the influence of I-shaped stiffeners oriented at 0° and 45° on the minimization of maximum and central deflection by varying the number of transverse and longitudinal stiffeners (which have the same height) [7]; and the geometric analysis of stiffened steel plates to assess the influence of I or T-shaped transverse reinforcements on mechanical behavior in terms of maximum deflection and von Mises stress [8]."

Most studies addressing stiffened plates use rectangular open-section stiffeners (such as I and T-shaped), which are the most traditional types. However, in the present study, an analysis of plates with trapezoidal box-beam-type stiffeners, also known as hat-stiffened plates, will be conducted. Tharian and Nandakumar [9] emphasize that hat stiffeners have numerous advantages compared to open-section stiffeners, mainly due to their high strength-to-weight ratio and torsional rigidity. Considering the analysis of plates with hat-stiffeners, the following works can be highlighted: Tharian and Nandakumar [10] presented a numerical analysis using FEM to quantify the structural advantages of trapezoidal box-beam stiffeners compared to the open-section stiffeners that are typically used; Pal et al. [11] analyzed the maximum deflection and stress of plates with trapezoidal box-beam stiffeners, adopting different boundary conditions for the plates and varying geometries for the stiffeners while maintaining constant volume; Virág and Szirbik [12] conducted a modal analysis of plates with optimized trapezoidal reinforcements to identify any potentially dangerous frequencies and eliminate failure possibilities in the structure; Tharian and Nandakumar [9] presented a study on the application of a super element for the structural analysis of bunkers made from plates with hat-stiffeners; and Filippatos et al. [13] developed a design for composite aircraft components, in which the structure includes a hat-stiffener to evaluate the structural performance of the composite material, as well as its overall impact on sustainability.

In this sense, the present study purposes the geometric evaluation and optimization of simply supported steel plates with trapezoidal hat-stiffeners under the incidence of a uniformly distributed loading. To do so, computational modeling (FEM by means ANSYS Mechanical APDL 2024 R2), Constructal Design method, and Systematic Search technique were employed, aiming to reduce maximum and central deflections of the hat-stiffened plates. A reference plate with no stiffeners is adopted, from which 30% of its volume is transformed into hat-stiffeners. Since it is kept constant its length and width, the hat-stiffened plates are formed by reducing the thickness of the reference plate and incorporating the stiffeners. With this, it was possible to evaluate geometric configurations of hat-stiffened plates with up to 5 longitudinal stiffeners. The trapezoidal cross section of the box-beam type hat-stiffeners have an angle of 120°. The thickness of the hat-stiffeners varies from 4.75 mm (3/16 in) to 12.7 mm (1/2 in), while its high ranges between 25 mm and 300 mm.

2. Material and Methods

In this study, it was applied the Constructal Design method together with a Systematic Search technique for the evaluation and optimization of the geometric configurations of the hat-stiffened plates. It is worth mentioning that the Constructal Design is not an optimization method, but a geometric evaluation method. However, when associated with an optimization method, such as a Systematic Search, it becomes responsible for generating the search space (composed of possible geometric configurations), and the optimization method identifies the geometry of the system that leads to its best performance [14]. To use Constructal Design, it is necessary to define restrictions (global or local), degrees of freedom (that vary, respecting the restrictions, to generate the geometries), and performance indicators (that must be maximized or minimized, searching the superior performance) [15].

To numerically simulate the mechanical behavior of the reference and hat-stiffened plates, the ANSYS Mechanical APDL software (version 2024 R2) was used, which is based on the FEM. The finite element SHELL281 was adopted both in the plate as in the stiffeners (such as in Nogueira et al. [6], Pinto et al. [7], and Kucharski et al. [8]), being indicated for thin to moderately-thick plate/shell structures. It has eight nodes with 6 degrees of freedom per node: 3 translations (Ux, Uy, Uz) and 3 rotations (θx, θy, θz) [16]. The FEM is typically employed in its displacement formulation for structural analysis, where the unknown displacements are obtained by solving a system of algebraic equations [17], which can be represented as;

$$[K]\{\,U\} = \{F\} \tag{1}$$

where $[K]$ is the global stiffness matrix, $\{U\}$ is the vector of unknown nodal displacements, and $\{F\}$ is the vector of equivalent nodal forces applied to the system. To ensure that the computational model is adequately defined, validation and verifications were carried out, always comparing the results obtained with those in the literature.

2.1. Case Study

As already mentioned, this study is based on the application of the Constructal Design associated to a Systematic Search technique to analyze simply supported rectangular steel plates with trapezoidal box-beam type hat-stiffeners longitudinally positioned, when submitted to a uniformly distributed out-of-plane load. The goal is to minimize the maximum and central deflections of these hat-stiffened plates. To do so, a reference plate (without stiffeners) is adopted, having length a = 2000 mm, width b = 1000 mm, and thickness t_{rp} = 20 mm. From the reference plate a volumetric fraction ϕ = 0.30 [6-8], i.e. 30% of the total steel volume, is transformed into hat-stiffeners. As the values of a and b are kept constants, the thickness of the hat-stiffened plates is t = 14 mm, which when incorporating the stiffeners, maintain the same total volume of material as the reference plate. As in Refs. [6-8], the A-36 steel is employed, having elasticity modulus E = 200 GPa and Poisson's ratio ν = 0.3. The four edges of the plate and the ends of the stiffeners are considered simply supported. The magnitude of the lateral uniformly distributed loading is 10 kPa. In turn, the geometry of the hat-stiffener is defined in agreement with Anej et al. [18] that indicated an angle of 120° as ideal to improve its mechanical strength. In addition, the stiffeners thickness vary: t_s = 4.75 mm (3/16 in); t_s = 6.35 mm (1/4 in); t_s = 8.00 mm (5/16 in); t_s = 9.53 mm (3/8 in); and t_s = 12.7 mm (1/2 in). Likewise, the stiffeners high assumes a minimal value of 25 mm with maximum values of 300 mm (1 stiffener), 150 mm (2 and 3 stiffeners), 125 mm (4 stiffeners), and 100 mm (5 stiffeners). From the Constructal Design application, it was generated a total of 100 different geometries of hat-stiffened plates that have the same volume of material. Appendix A presents the dimensions of the stiffeners used in each of these analyses, as well as the maximum and central deflection results obtained in each proposed case.

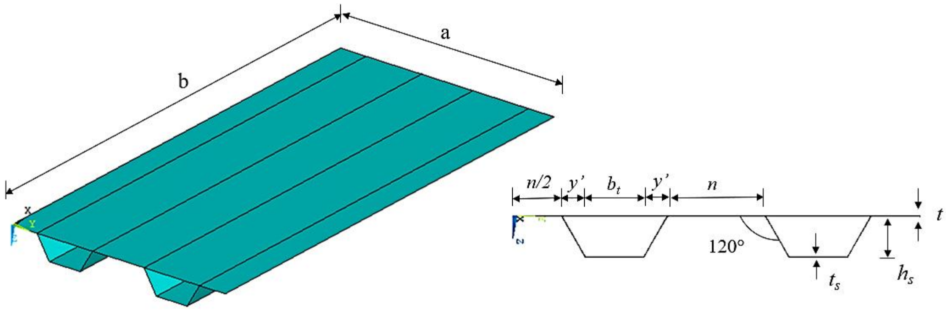

Fig. 1. Schemtaic representation of a hat-stiffened plates with 2 longitudinal stiffeners

Considering a structure composed of multiple hat-stiffened plates, when a plate has only one stiffener, the distance between the stiffener and the plate edge will always be the same on both sides. Starting from two stiffeners, the distance between the stiffener and the plate edge equals half the distance between two stiffeners, as shown in Fig. 1. This is because, when joining multiple plates, the spacing between the stiffeners is always uniform. From Fig. 1, h_s and t_s represent the heightand thickness of the hat-stiffener, respectively; n is the spacing between 2 stiffeners; b_t is the size of the shorter base of the trapezoidal cross-section; and y' is the projection of the stiffener's side onto the y-axis.

3. Results and Discussions

3.1. Computational Model Validation

The computational model validation consisted in the numerical simulation of a stiffened plate experimentally studied by Carrijo et al. [19], as illustrated in Fig. 2.

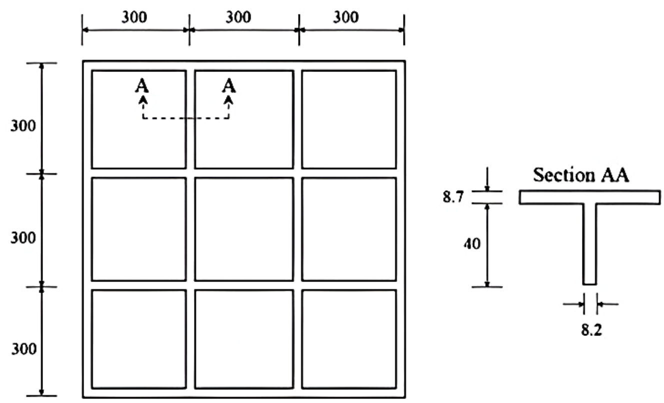

Fig. 2. Geometry of the square plate used in the validation

The square steel plate with eight I-shaped stiffeners equally distributed in both directions across the structure has boundary conditions with all 4 corners simply supported. The plate was subjected to a uniformly distributed load of 0.96 kPa, and the structure's material has E = 2.5 GPa and ν = 0.36. For this analysis, a converged regular mesh of square SHELL281 finite elements was generated, each with a side length of 2.436 mm. The numerical result obtained for the central deflection was Uz = 6.5075 mm (Fig. 3), while the experimental results from Carrijo et al. [19] is Uz = 6.2200 mm, achieving a relative error of 4.58%, validating the computational model.

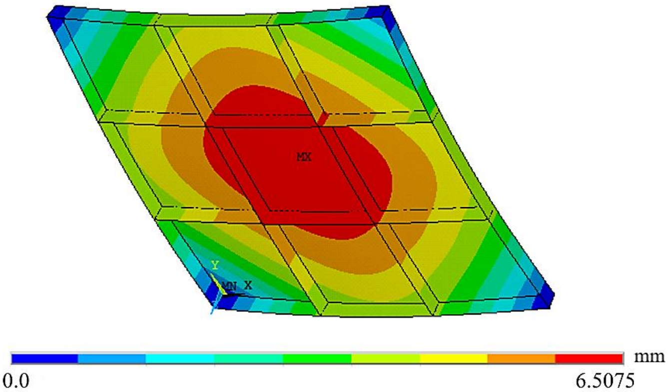

Fig. 3. Displacement distribution for the validation

3.2. Computational Model Verification

First verification was carried out by Tharian and Nandakumar [9], analyzing a rectangular simply supported steel plate with 2 longitudinal hat-stiffeners (Fig. 4). Plate is under an out-of-plane uniformly distributed loading of 10 kPa and has E = 210 GPa and ν = 0.3. For that, the current numerical solution obtained with SHELL281 finite element was compared with those presented by Tharian and Nandakumar [10] with SHELL63 and SHELL93 finite elements.

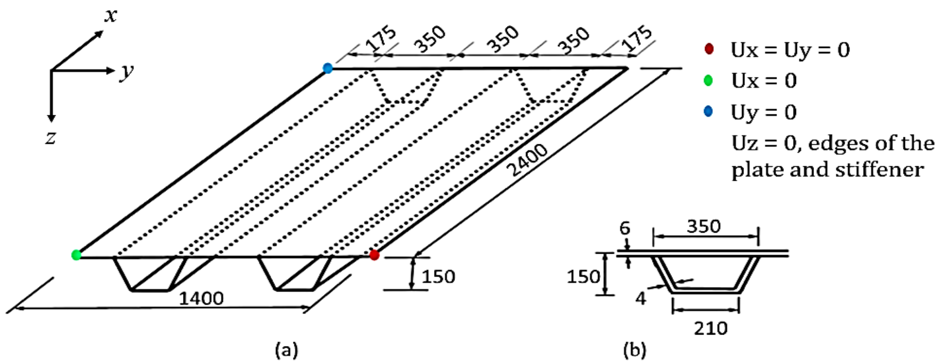

Fig. 4. First verification: (a) hat-stiffened plate dimensions, in mm; and (b) detail of hat-stiffener dimensions, in mm [10]

The spatial discretization was defined through a mesh convergence test, considering square finite elements with 7 different sizes (varying from 5 mm to 150 mm). In Fig. 5 one can note that from mesh with 20 mm (15,600 finite elements) occurs a stabilization for the maximum deflection value. Hence, to guarantee a solution independent of the spatial discretization, it was adopted the mesh with size of 10 mm (60,240 finite elements). A maximum deflection of 0.7295 mm was reached in the present study, while values of

0.7198 mm (SHELL63) and 0.7214 mm (SHELL93) was presented by Tharian and Nandakumar [10].

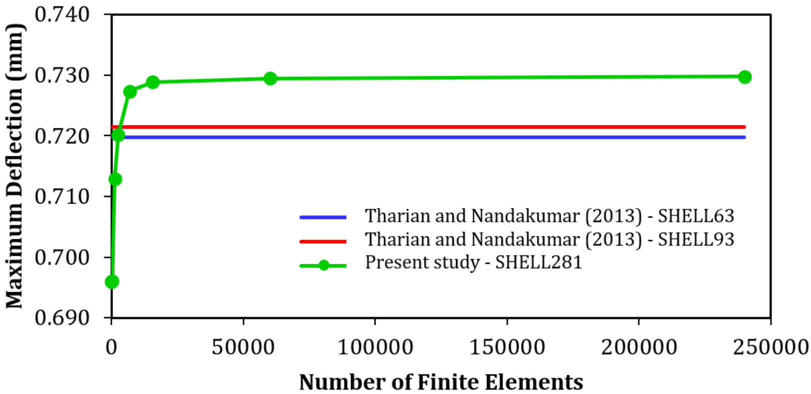

Fig. 5. Mesh convergence test for the first verification

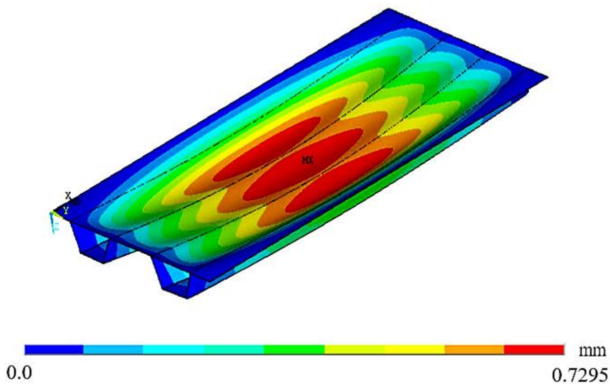

Fig. 6. Displacement distribution for the first verification

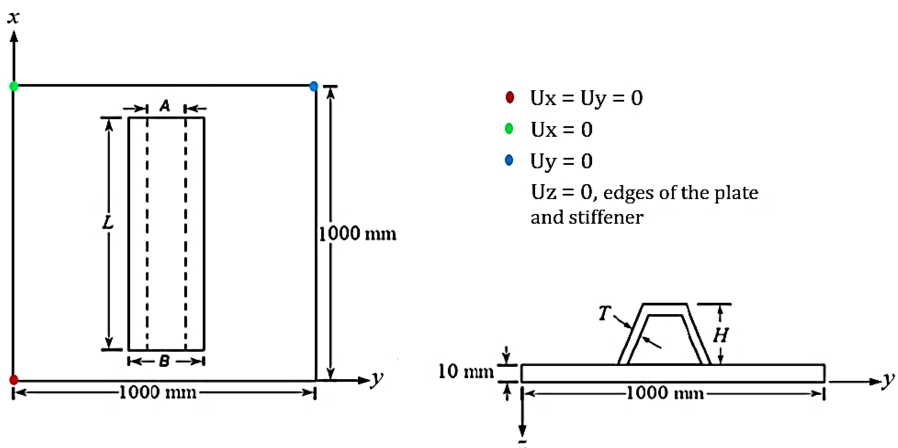

Fig. 7. Hat-stiffened dimensions for the second verification [11]

Therefore, the computational model was verified by relative differences of 1.35% and 1.12%, respectively. Figure 6 depicted the displacement distribution obtained in the present study, in which it is possible to identify that the maximum structural deflection occurs in the center of the hat-stiffened plate.

Table 1. Hat-stiffener dimensions for the second verification, in mm [11]

Case	Thickness (T)	Shorter Base (A)	Longer Base (B)	Height (H)	Length (L)
1	2	35	60	65	955.90
2	3	35	60	65	637.26
3	4	35	60	65	477.95
4	5	35	60	65	382.36
5	6	35	60	65	318.63
6	2	40	60	65	932.78
7	3	40	60	65	621.86
8	4	40	60	65	466.39
9	5	40	60	65	373.11
10	6	40	60	65	310.93
11	2	40	65	70	878.09
12	3	40	65	70	585.39
13	4	40	65	70	439.04
14	5	40	65	70	351.23
15	6	40	65	70	292.70
16	2	40	60	70	881.92
17	3	40	60	70	587.95
18	4	40	60	70	440.96
19	5	40	60	70	352.77
20	6	40	60	70	293.97
21	2	35	65	65	950.02
22	3	35	65	65	633.35
23	4	35	65	65	475.01
24	5	35	65	65	380.01
25	6	35	65	65	316.67

The second verification was performed with a simply supported square steel plate having a centered hat-stiffener (Fig. 7), earlier investigated by Pal et al. [11]. A uniform distributed loading of 1 kPa is applied perpendicularly to the plate, which in turn has E = 200 GPa and ν = 0.3 as its material mechanical properties. It is important to explain that the hat-stiffener has variable dimensions but keeping constant its total volume of 320,000 mm^3. Table 1 describes the hat-stiffener dimensions for the 25 geometric configurations analyzed. As in the first verification, a mesh convergence test was developed with square finite elements with 7 different sizes (varying from 5 mm to 150 mm) and taken into account the Case of Table 1. In Fig. 8 it is possible to observe a stabilization of the maximum deflection value from the mesh with size of 40 mm (796 finite elements), being chosen for the numerical simulations the mesh with size of 10 mm (11,928 finite elements).

The obtained results for the maximum deflection in the present study (with SHELL281 finite element) for the 25 cases of Table 1 are shown in Table 2 and Fig. 9, in comparison to those obtained by Pal et al. [11] that used SHELL181 and BEAM3 finite elements, respectively, for the plate and stiffener. One can infer in Fig. 9 a good agreement in a qualitative way between the maximum deflection result of the present study and that of Pal et al. [11], for each case. In general, from Table 2, quantitatively an average relative difference of 3.30% was achieved for the 25 cases and maximum relative differences around 5% were identified. Therefore, it is possible to consider that the computational model was proper verified.

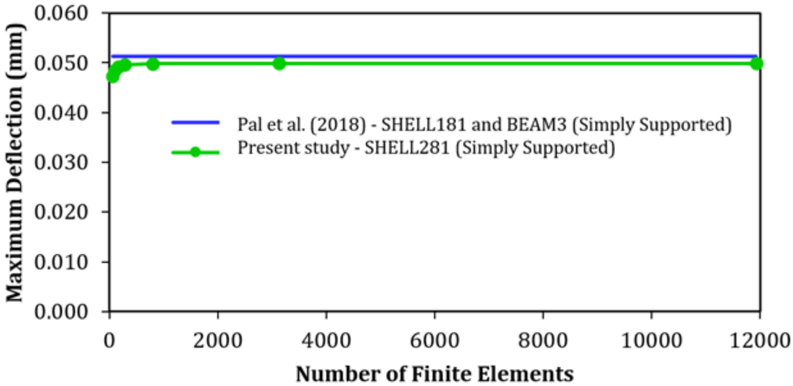

Fig. 8. Mesh convergence test for the second verification

Table 2. Maximum deflection, in mm, for cases of Table 1

Case	Pal *et al.* [11]	Present Study	Relative Difference (%)
1	0.0513	0.0499	-2.73
2	0.0908	0.0936	3.08
3	0.1279	0.1338	4.61
4	0.1506	0.1580	4.91
5	0.1647	0.1732	5.16
6	0.0508	0.0498	-1.97
7	0.0939	0.0970	3.30
8	0.1306	0.1367	4.67
9	0.1527	0.1603	4.98
10	0.1664	0.1750	5.17
11	0.0505	0.0501	-0.79
12	0.1008	0.1046	377
13	0.1365	0.1432	4.91
14	0.1572	0.1653	5.15
15	0.1700	0.1789	5.24
16	0.0503	0.0499	-0.80
17	0.1004	0.1043	3.88
18	0.1364	0.1430	4.84
19	0.1572	0.1653	5.15
20	0.1700	0.1789	5.24
21	0.0514	0.0500	-2.72
22	0.0915	0.0942	2.95
23	0.1284	0.1342	4.52
24	0.1508	0.1583	4.97
25	0.1648	0.1732	5.10

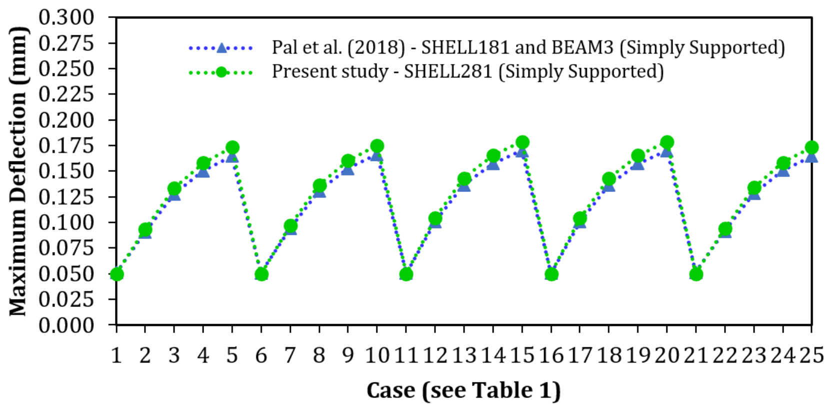

Fig. 9. Maximum deflection for cases of Table 1

3.3 Case Study

As earlier mentioned, all the proposed hat-stiffened plate geometric configurations were derived from the reference plate (with no stiffeners). Based on the structural characteristics outlined in the case study description, it was possible to insert a maximum of five stiffeners in the longitudinal direction of the plate while maintaining the total material volume of the structure constant. The limitation imposed by adopting a 120° angle prevents the addition of more stiffeners, even with variations in height and thickness, without altering the total volume of the structure. As in the validation and verification procedures, it was also necessary here to perform a mesh convergence test considering the structure with the greatest geometric complexity, represented by the plate with 5 stiffeners. Considering the maximum deflection of the hat-stiffened plate, regular meshes formed by square SHELL281 finite elements with 6 different sizes (ranging from 5 to 40 mm) were tested, as shown in Fig. 10.

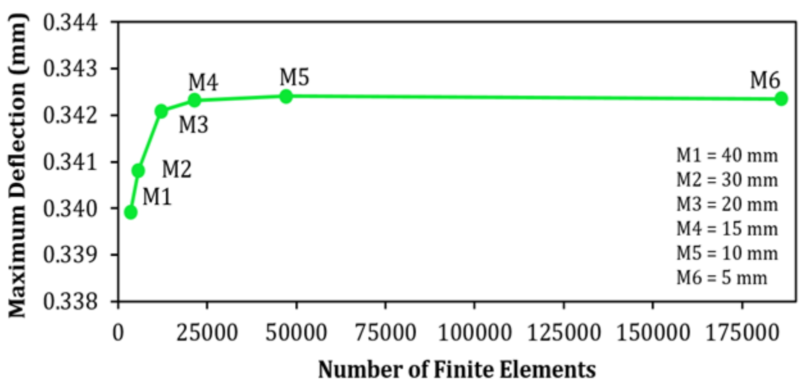

Fig. 10. Convergence test mesh for the case study

It is possible to visualize in Fig. 10 a stabilization of the maximum deflection value from mesh M4. In order to guarantee a mesh-independent solution the mesh M5 (finite elements with a size of 10 mm) was adopted for all numerical simulations of the case study. Therefore, the maximum and central deflections of the reference plate was 0.69758 mm. The main purpose regarding the transformation of 30% of the material volume in hat-stiffeners is to reduce the maximum and central deflections. The results of maximum and central deflections for the different stiffener height and stiffener thickness of the hat-stiffened plates having 1, 2, 3, 4, and 5 longitudinal hat-stiffeners are plotted, respectively, in Figures 11, 12, 13, 14, and 15.

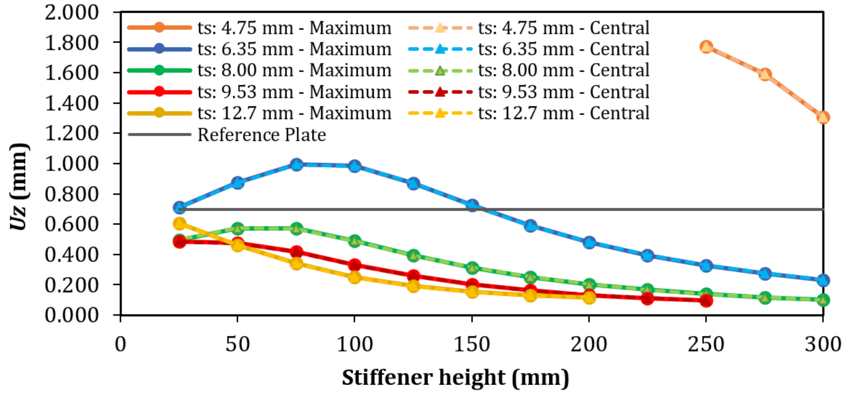

Fig. 11. Results for maximum and central deflections for the plates with 1 hat-stiffener

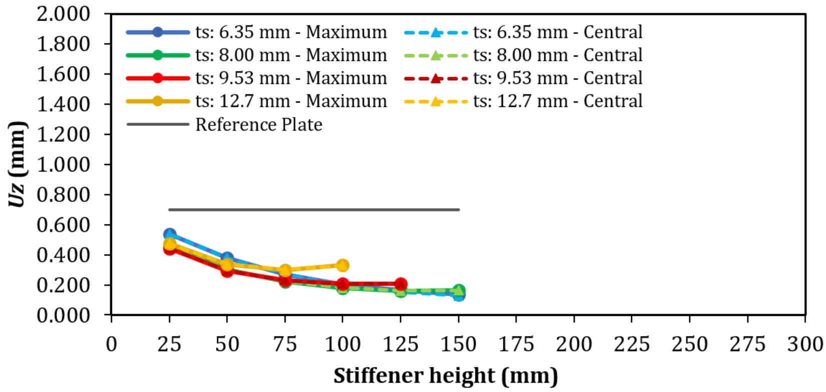

Fig. 12. Results for maximum and central deflections for the plates with 2 hat-stiffeners

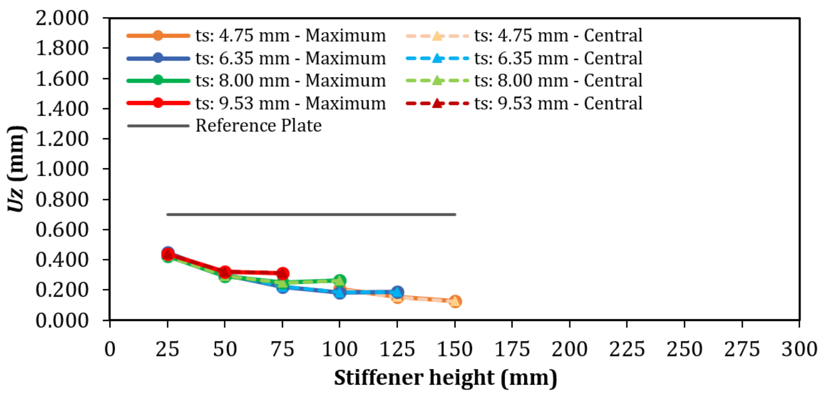

Fig. 13. Results for maximum and central deflections for the plates with 3 hat-stiffeners

Figures 11 to 15 demonstrate that applying Constructal Design to convert 30% of the reference plate into hat-stiffeners effectively reduces out-of-plane displacements. All hat-stiffened plates exhibit maximum and central deflections lower than those of the reference plate. Furthermore, in a general way, from Figs. 11 to 15 it is noticed that the maximum and central deflections coincide for each analyzed case. Moreover, it is also observed that the greater the height of the stiffener, the smaller the resulting displacements, except for the plate with a single hat-stiffener where thicknesses of 6.35 mm and 8.00 mm are used (see Fig. 11). In this case, an initial increase in deflections occurs before they decrease as the height increases.

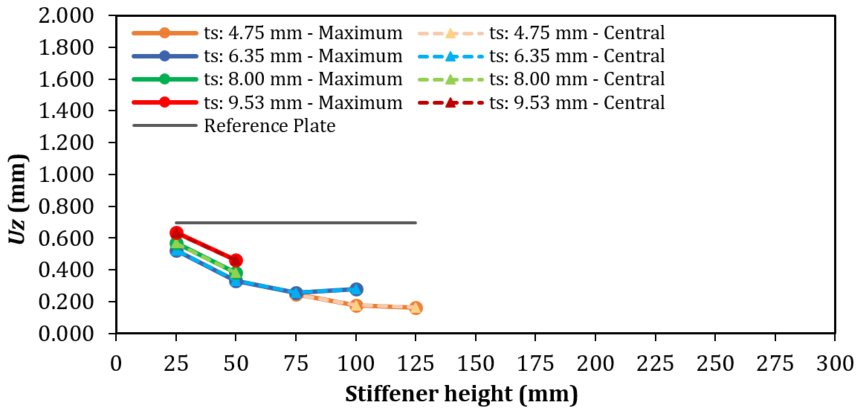

Fig. 14. Results for maximum and central deflections for the plates with 4 hat-stiffeners

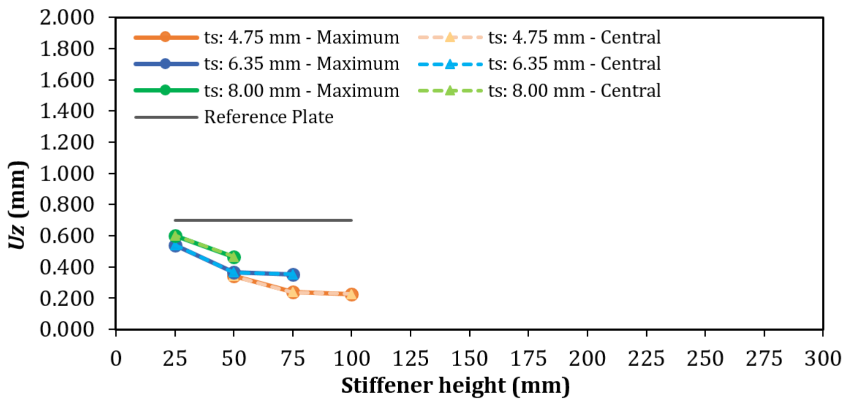

Fig. 15. Results for maximum and central deflections for the plates with 5 hat-stiffeners

For the plates with 1 or 2 hat-stiffeners, the smallest deflections occur at the largest thicknesses. However, for the plates having 3 to 5 hat-stiffeners, the smallest deflections are observed at the smallest thicknesses. As the number of stiffeners increases, the heights and thicknesses become limited because the portion of the plate's material volume converted into stiffeners cannot be fully distributed as the stiffener dimensions change in each proposed geometry.

Table 3 presents the best and worst results according to the number of hat-stiffeners in the longitudinal direction of the plate, as well as the relative difference (RD) in maximum and central deflections compared to the reference plate for these analyzed cases.

Table 3. Best and worst results for the maximum and central deflections according to the number of hat-stiffeners

Number of hat-stiffeners	h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%) U_z^{max}	RD (%) $U_z^{central}$
1	250	9.53	0.0958	0.0958	-86.26	-86.26
	250	4.75	1.7735	1.7735	154.23	154.23
2	150	6.35	0.1381	0.1266	-80.20	-81.84
	25	6.35	0.5371	0.5371	-23.01	-23.01
3	150	4.75	0.1244	0.1244	-82.17	-82.17
	25	6.35	0.4497	0.4497	-35.53	-35.53
4	125	4.75	0.1650	0.1650	-76.34	-76.34
	25	9.53	0.6344	0.6345	-9.06	-9.04
5	100	4.75	0.2259	0.2259	-67.61	-67.61
	25	8.00	0.6014	0.6014	-13.79	-13.79

According to Table 3, the worst deflections occur with the smallest stiffener height (25 mm) when the plate has 2 to 5 stiffeners. However, the thickness associated with these results varies: for 1 to 3 stiffeners, the worst deflections happen at the smallest thicknesses, whereas for 4 and 5 stiffeners, the largest deflections occur with the thickest stiffener. For the plate with only 1 stiffener (see Fig. 11), the worst deflection is observed with a height of 250 mm and a thickness of 4.75 mm. This configuration represents the smallest feasible height for this thickness in the proposed geometric arrangement. Moreover, this deflection is, on average, 2.5 times greater than the result for the reference plate, making the use of this thickness unviable. The objective is to minimize maximum and central deflections with the help of stiffeners, thereby increasing the structural rigidity.

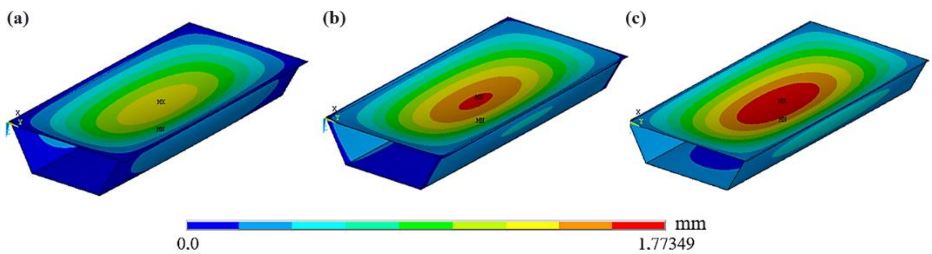

Fig. 16. Displacement distributions of the hat-stiffened plate having 1 stiffener for t_s = 4.75 mm, with: (a) h_s = 300 mm; (b) h_s = 275 mm; and (c) h_s = 250 mm

With only one stiffener on the plate (see Figure 11), 45 analyses were conducted using different geometric arrangements. The maximum stiffener height was 300 mm, and 5 stiffener thicknesses were employed. The only thickness that did not yield satisfactory results was 4.75 mm. For the 3 heights that allowed this thickness, deflection values exceeded 1.3 mm, surpassing the results obtained for the reference plate and making its use not recommended. This can be attributed to the stiffener nearly spanning the entire 1000 mm width of the plate, as shown in Figure 16. For a thickness of 6.35 mm, heights below 150 mm also resulted in displacements exceeding Uz = 0.69758 mm of the reference plate. However, for the other thicknesses, regardless of the adopted height, all results showed deflections lower than the maximum and central deflections of the reference plate. According to Table 3, the smallest maximum and central deflection was 0.0958 mm, achieved with a stiffener height of 250 mm and thickness of 9.53 mm, which is the maximum height allowed for this thickness. This represents a reduction of 86.26% in maximum and central deflections if compared to the reference plate.

In turn, 21 geometric arrangements were analyzed with two longitudinal stiffeners (see Fig. 12), considering four stiffener thicknesses, with a maximum stiffener height of 150 mm. Among these configurations, the highest and lowest maximum and central deflection results occurred with a thickness of 6.35 mm, being the smallest displacement corresponding to a height of 150 mm and the largest displacement at a height of 25 mm. Regarding the reference plate, the best result achieved reductions of 80.20% and 81.74% in maximum and central deflections, with displacements of 0.1381 mm and 0.1266 mm (see Table 3), respectively. The worst result was 0.5371 mm for both maximum and central deflections. For the 6.35 mm thickness, the maximum and central deflections coincide up to a height of 75 mm. Above this, a slight variation occurs due to the position of the stiffeners, which tend to shift closer to the plate edges as the stiffener height decreases. This is similar to the behavior observed with a single stiffener (see Fig. 16). It is noteworthy that for thicknesses of 12.70 mm, 9.53 mm, and 8.00 mm, there is a slight increase in deflection results at greater heights, where the stiffeners feature very narrow bases. For instance, in the 12.70 mm thickness the stiffeners occupy less than 50% of the plate width, regardless of height, as illustrated in Fig. 17.

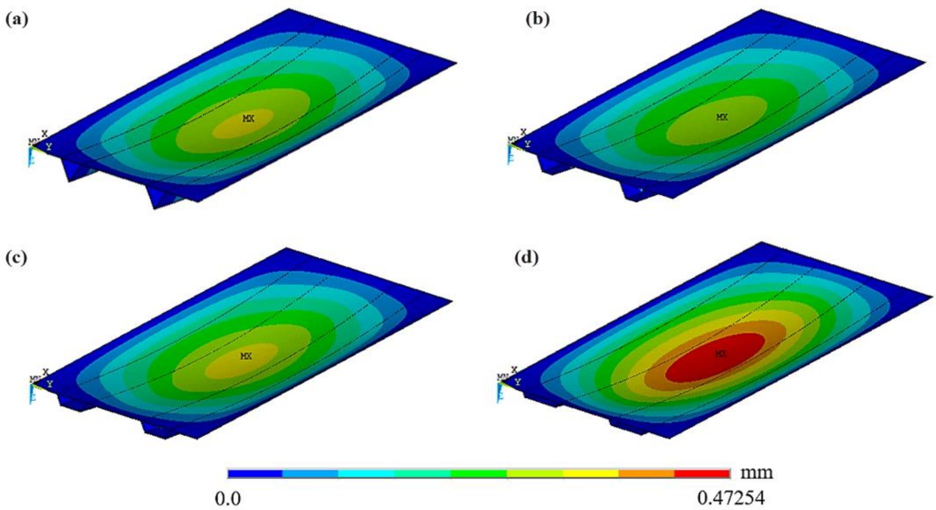

Fig. 17. Displacement distributions of the hat-stiffened plate having 2 stiffener for t_s = 12.70 mm, with: (a) h_s = 100 mm; (b) h_s = 75 mm; (c) h_s = 50 mm; and (d) h_s = 25 mm

In all 15 analyses conducted on the plate with 3 stiffeners (see Fig. 13), the distribution of resulting displacements is similar to that of the reference plate. In this geometric arrangement, the 12.70 mm thickness can no longer be applied, and the available height options are also limited for each analyzed thickness. The maximum height of 150 mm is only achievable with a thickness of 4.75 mm, which also results in the lowest maximum and central deflection of 0.1244 mm, representing an 82.17% reduction compared to the reference plate.

Analyzing the hat-stiffened plate with 4 stiffeners (see Fig. 14), 11 geometric arrangements were studied, considering a minimum thickness of 4.75 mm and a maximum of 9.53 mm. The maximum stiffener height reached 125 mm for the smallest thickness, resulting in the best maximum and central deflection value of 0.1650 mm, which is 4.23 times smaller than the result for the reference plate. On the other hand, the worst displacements occurred at the smallest height (25 mm) with the largest thickness (9.53 mm), yielding maximum and central deflections of 0.6344 and 0.6345 mm, respectively, still reaching 9% better than the reference plate. Moreover, with 5 hat-stiffeners on the plate (see Fig. 15), the increased number of stiffeners meant that the portion of the reference plate volume converted into stiffeners only allowed for 8 analyses, with 3 stiffener thicknesses and a maximum height of 100 mm. The best results were achieved with a thickness of 4.75 mm, while the worst were observed with a thickness of 8.00 mm. Compared to the reference plate, the smallest displacement showed a reduction of 67.61% in maximum deflection, with maximum and central deflections of Uz = 0.2259 mm. The worst result occurred with the smallest stiffener height and thickness, corresponding to 25 mm and 4.75 mm, respectively, yielding maximum and central deflections of Uz = 0.6014 mm, about 13.79% lower than the values obtained for the reference plate.

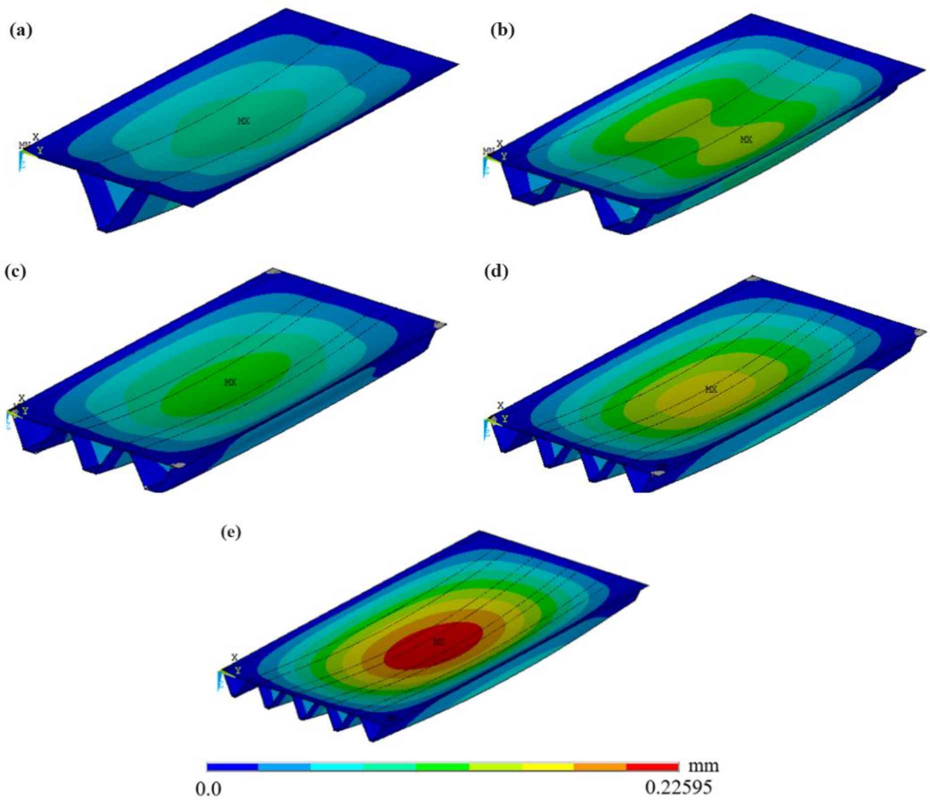

Fig. 18. Displacement distributions with the best result (with minor maximum deflection) for: (a) 1 hat-stiffener; (b) 2 hat-stiffeners; (c) 3 hat-stiffeners; (d) 4 hat-stiffeners; and (e) 5 hat-stiffeners

Figure 18 depicts the displacements distribution for the best result, i.e. with small deflection, for each number of hat-stiffeners investigated (see Figures 11 to 15 and Table 3). It can be observed in Fig. 18 that only in the case with two stiffeners do the maximum and central deflections differ. This occurs because the geometry with 2 hat-stiffeners caused the plate to be divided into two regions with lower maximum displacement, whereas in the other cases, the resulting displacements are concentrated in the central region of the plate (with a similar behavior than reference plate, i.e. having an overall structure deflection). Additionally, the smallest deflection values are observed when the plate has 1 or 3 hat-stiffeners. In these configurations, there is always a stiffener crossing the central region of the plate, which helps reduce displacements at this point. However, the worst case among those presented in Fig. 18 is the one with five stiffeners, which also includes a central stiffener. Nevertheless, the combination of the stiffeners' height (100 mm) and thickness (4.75 mm) resulted in the poorest mechanical performance among the five best cases with longitudinal stiffeners.

One can also discuss the obtained results in the light of Constructal theory. As earlier mentioned, the Constructal theory is based on the evolution of design over time. When applying Constructal Design, this temporal evolution can be understood as the progression that occurs while maintaining a constant material volume and allowing the degrees of freedom to vary in pursuit of improved performance indicator. Consequently, as material redistribution occurs due to the variation in degrees of freedom, the geometric configuration adapts to achieve superior performance, respecting the defined constraints and proposed search space. Thus, the Constructal principle of the optimal distribution of imperfections is fulfilled, leading to the ideal geometric configuration for the problem under investigation [20]. Specifically, solid mechanic systems can be understood as flow systems in which occurs a flow of stresses [3]. This flow of stresses changes as geometric configurations take different shapes due to variations in the degrees of freedom. From the perspective of Constructal theory and in agreement with Da Silveira et al. [15], the optimal geometry is the one that facilitates the flow of stress, distributes imperfections better, and achieves lower displacements. That said, it can be inferred from Fig. 18 that the optimal global geometry was the one that facilitated the flow of stresses, best distributed the mechanical imperfections, and achieved the smallest displacements.

4. Conclusions

The present study applied the Constructal Design method combined with a Systematic Search technique to analyze trapezoidal box beam hat-stiffeners in the longitudinal direction of a simply supported steel plate. Starting from a non-stiffened reference plate, 100 geometric configurations were proposed and numerically simulated using a FEM computational model, which was properly validated and verified. This approach allowed for a systematic investigation into the influence of the geometry on the mechanical performance of plates with hat-stiffener.

The primary objective of the study was to minimize the structural out-of-plane displacements. Results demonstrated that most of the hat-stiffened plates proposed showed significant improvements in mechanical behavior when compared to the reference plate. For example, the optimized geometric configuration (featuring 1 stiffener with a height of 250 mm and a thickness of 9.53 mm) reduced the maximum and central deflections by approximately 86% if compared to the reference plate. This highlights the potential of strategically designed stiffeners to enhance the stiffness and load-bearing capacity of the structure.

It can also be observed that, maintaining a constant total volume of material, an increase in the number of stiffeners does not necessarily conduct the reduction of the out-of-plane displacements of the structural component. In addition, it is important to highlight the influence of the thickness and height of the stiffeners on the results. As expected, the greater the height of the stiffener, the smaller the resulting deflections in the structure.

The application of Constructal Design allows the evolution of the geometric configuration through the variation of degree of freedoms until the optimized one is defined, i.e., among the proposed geometries that compose the search space, identify the one that facilitates the flow of stresses, best distributed the mechanical imperfections, and reached the smallest displacements.

Future research will expand on these findings by analyzing deflections in plates with trapezoidal box beam hat-stiffeners oriented transversely to the plate. This will help identify the most effective geometric configurations to further enhance structural rigidity and minimize displacements. Additional studies will also consider varying material volume fractions, different aspect ratios between plate dimensions, and the inclusion of von Mises stress as a performance indicator to provide a more comprehensive understanding of the structure's behavior.

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Appendix A

Tables A1 to A5 present the geometric characterization based on the height and thickness of the stiffener, as well as the maximum and central deflection results, along with the relative percentage differences between these results for each hat-stiffened plate generated through the application of the Constructal Design method and investigated in this study.

Table A1. Plates with 1 longitudinal hat-stiffener

h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%)
300	4.75	1.30951	1.30950	−0.001
275	4.75	1.58995	1.59010	0.009
250	4.75	1.77349	1.77350	0.001
300	6.35	0.23210	0.23208	−0.008
275	6.35	0.27515	0.27519	0.015
250	6.35	0.32857	0.32854	−0.009
225	6.35	0.39571	0.39575	0.011
200	6.35	0.48157	0.48153	−0.008
175	6.35	0.59097	0.59102	0.008
150	6.35	0.72568	0.72563	−0.006
125	6.35	0.87134	0.87142	0.010
100	6.35	0.98356	0.98367	0.011
75	6.35	0.99490	0.99488	−0.002
50	6.35	0.87462	0.87474	0.014
25	6.35	0.71122	0.71122	0.000
300	8.00	0.09970	0.09970	0.004
275	8.00	0.11669	0.11666	−0.024
250	8.00	0.13870	0.13871	0.004
225	8.00	0.16708	0.16705	−0.017
200	8.00	0.20354	0.20351	−0.017
175	8.00	0.25087	0.25088	0.005
150	8.00	0.31295	0.31291	−0.014
125	8.00	0.39295	0.39297	0.005
100	8.00	0.48827	0.48822	−0.009
75	8.00	0.57142	0.57146	0.008
50	8.00	0.57404	0.57401	−0.006
25	8.00	0.49597	0.49604	0.015
250	9.53	0.09582	0.09581	−0.010
225	9.53	0.11087	0.11087	−0.003
200	9.53	0.13234	0.13231	−0.023
175	9.53	0.16187	0.16186	−0.003
150	9.53	0.20240	0.20236	−0.018
125	9.53	0.25773	0.25773	0.000

100	9.53	0.33128	0.33123	−0.016
75	9.53	0.41662	0.41663	0.003
50	9.53	0.47598	0.47595	−0.007
25	9.53	0.48389	0.48393	0.008
200	12.7	0.11700	0.11697	−0.022
175	12.7	0.12942	0.12939	−0.025
150	12.7	0.15282	0.15279	−0.022
125	12.7	0.19061	0.19059	−0.009
100	12.7	0.24956	0.24952	−0.016
75	12.7	0.33913	0.33911	−0.004
50	12.7	0.46219	0.46216	−0.007
25	12.7	0.60353	0.60355	0.003

Table A2. Plates with 2 longitudinal hat-stiffeners

h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%)
150	6.35	0.13812	0.12665	−8.306
125	6.35	0.16315	0.15477	−5.139
100	6.35	0.20409	0.20027	−1.871
75	6.35	0.27030	0.27026	−0.014
50	6.35	0.37879	0.37880	0.003
25	6.35	0.53705	0.53705	0.000
150	8.00	0.16236	0.16232	−0.024
125	8.00	0.16076	0.16074	−0.015
100	8.00	0.18045	0.18043	−0.013
75	8.00	0.22377	0.22375	−0.009
50	8.00	0.30227	0.30229	0.008
25	8.00	0.45519	0.45518	−0.001
125	9.53	0.20960	0.20959	−0.006
100	9.53	0.20547	0.20546	−0.006
75	9.53	0.23113	0.23111	−0.010
50	9.53	0.29423	0.29426	0.010
25	9.53	0.44266	0.44266	0.001
100	12.7	0.32981	0.32978	−0.009
75	12.7	0.29945	0.29942	−0.009
50	12.7	0.33666	0.33669	0.009
25	12.7	0.47255	0.47254	−0.001

Table A3. Plates with 3 longitudinal hat-stiffeners

h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%)
150	4.75	0.12436	0.12437	0.011
125	4.75	0.15318	0.15317	−0.007
100	4.75	0.20518	0.20519	0.006
125	6.35	0.18738	0.18737	−0.005
100	6.35	0.18473	0.18473	−0.002
75	6.35	0.22168	0.22171	0.012
50	6.35	0.30289	0.30289	−0.001
25	6.35	0.44971	0.44975	0.009
100	8.00	0.26199	0.26198	−0.005
75	8.00	0.24723	0.24725	0.010
50	8.00	0.29335	0.29334	−0.002
25	8.00	0.42348	0.42351	0.008
75	9.53	0.31217	0.31219	0.005
50	9.53	0.32200	0.32200	−0.001
25	9.53	0.43755	0.43758	0.006

Table A4. Plates with 4 longitudinal hat-stiffeners

h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%)
125	4.75	0.16503	0.16502	−0.005
100	4.75	0.17824	0.17824	0.002
75	4.75	0.24494	0.24495	0.003
100	6.35	0.27831	0.27833	0.006
75	6.35	0.25812	0.25814	0.009
50	6.35	0.33317	0.33317	0.001
25	6.35	0.52355	0.52364	0.017
50	8.00	0.38249	0.38253	0.010
25	8.00	0.56959	0.56959	0.000
50	9.53	0.46197	0.46196	−0.001
25	9.53	0.63441	0.63451	0.016

Table A5: Plates with 5 longitudinal hat-stiffeners

h_s (mm)	t_s (mm)	U_z^{max} (mm)	$U_z^{central}$ (mm)	RD (%)
100	4.75	0.22595	0.22595	0.000
75	4.75	0.23852	0.23855	0.013
50	4.75	0.34241	0.34241	0.001
75	6.35	0.35397	0.35400	0.008
50	6.35	0.36385	0.36385	0.000
25	6.35	0.53978	0.53982	0.007
50	8.00	0.46638	0.46638	0.000
25	8.00	0.60137	0.60140	0.005

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