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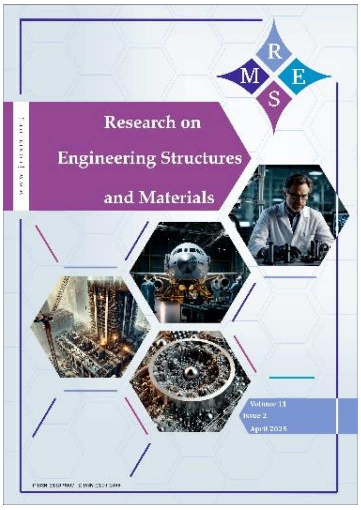

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Research Article

Evaluating the Ramberg–Osgood Model for nonlinear moment–curvature analysis of reinforced concrete beams

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Abstract

This paper improves the prediction of the nonlinear moment-curvature Characteristics of Reinforced concrete beams by using the Ramberg-Osgood model. Develop an integrated analytical (closed-form) moment–curvature framework based on the Ramberg–Osgood function, which provides a smooth (continuous) elastic-to-plastic transition for steel. At the same time, concrete degradation is captured progressively through cracking and crushing using section analysis with equilibrium and strain-compatibility conditions. The model was validated through two extensive investigations of ultimate moments measured in laboratory tests. The Ramberg-Osgood model provides better predictions of ultimate moments than IS 456-2000 and achieves more than 28% Improvement in the average difference between predicted and actual ultimate moments, with a coefficient of determination (R2) greater than 0.98. The results demonstrate that the Ramberg-Osgood model can accurately describe the significant phases of reinforced concrete section behavior-concrete cracking, steel yielding, and post-yield hardening-thereby providing a more reliable method of analyzing Reinforced Concrete Flexural Performance through a nonlinear analysis than the traditional IS 456:2000 approach.

1. Introduction

The moment-curvature (M-κ) relationship is the metric by which all reinforced concrete (RC) beams' nonlinear flexural behaviours can be determined. Accurate determination of the M-κ relationship provides both deflection and ductility, but predicting it remains very difficult. Many Simplified Design Codes (IS 456, Eurocode 2, etc.) [1,2] generally use a bilinear material behaviour and an empirical approach to effective stiffness. Both methods can lead to significant discrepancies in accurately representing the gradual stiffness losses in the beam due to cracking and yielding [3,4].

Advanced numerical techniques, such as fibre section analysis and finite element modelling, produce much more accurate results; however, specialized computer programs and the considerable time required to run them limit their use to occasional applications in routine design [5]. A smooth transition from elastic to plastic modelling is possible using the Ramberg-Osgood constitutive model, which has been successfully applied to metal plasticity [6]. While there has been increased interest in RC section analysis (particularly in direct comparisons with design code methods), little research exists on this topic. Therefore, there exists a large gap between the practical need for accurate analytical tools and available simplified design code methods.

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Recent research has sought to develop enhanced methods for modelling the behaviour of reinforced concrete elements using more sophisticated numerical models and constitutive-based methodologies. Research has also been conducted to investigate the ductility behaviour of RC elements subjected to static or dynamic (cyclic) loads using fibre-based numerical models in conjunction with detailed moment-curvature relationships [7],[8]. Other studies of refined constitutive models have also been conducted, in which researchers have attempted to create smooth (continuous) stress-strain transitions from steel reinforcement and the nonlinear behaviour of concrete (many derived from the Menegotto-Pinto model or its modifications) [9],[10]. These refined constitutive models are more accurate than their predecessors in capturing the cyclic and nonlinear behaviour of steel; however, they typically require advanced mathematical techniques and sophisticated computing systems.

Recent years have seen many studies investigate the development of simplified analytical techniques for predicting the nonlinear flexural behaviour of reinforced concrete (RC) beams while retaining adequate accuracy. Specific techniques include the use of analytical moment-curvature models that account for both nonlinear concrete compression behaviour and tension stiffening effects, thereby enhancing predictions of stiffness degradation and post-crack behaviour [11]. Researchers have examined the use of section equilibrium analyses in conjunction with refined material constitutive relationships to improve ductility and ultimate capacity prediction [12]. Despite these recent advances, most of the identified techniques remain dependent on complex numerical integration procedures or require excessive computational resources. Therefore, they continue to limit the practical application of these techniques in routine design calculations [13].

Due to increased interest in specific RC sections, the number of researchers proposing simplified analytical systems capable of producing moment-curvature relationships with significantly fewer computations than traditional methods has increased considerably in recent years. Such new approaches already exist, and there is a large disparity between the practical requirements for reliable analytical tools and the simplified designs of analytical systems described in present-day building codes. There also does not exist a single unified closed-form developed moment-curvature expression that (1) uses a smooth constitutive relationship for steel, (2) can be used or calculated without having to perform computationally complex numerical schemes, or (3) has been independently verified through extensive experimental testing with the use of established design code comparisons, such as IS 456.

To fill the existing gap, this research aims to provide an improved and verified analysis framework for the Ramberg-Osgood Constitutive Model. The specific aims of this research are to develop a closed-form moment-curvature relationship for reinforced concrete (RC) sections, validate quantitatively the findings of this research against two other independent experimental studies, conduct a complete statistical comparison between the predictions produced by this model and the design recommendations in IS 456:2000, and to demonstrate that the model is superior in its ability to model the three major behaviours of reinforced concrete: cracking, yielding and post-yield hardening is better than any other existing modelling techniques.

2. Methodology

2.1. Analytical Framework: A Ramberg-Osgood-Based Formulation

Through the integration of the Ramberg-Osgood model for steel and a nonlinear response for concrete using a framework of sectional equilibrium and strain compatibility, a closed-form relationship between the moment and curvature of the section will be established. The approach is divided into three major components: (1) a linear-elastic analysis of an uncracked section; (2) an analysis of a cracked section with an elastic-to-nonlinear transition of steel and the load-transfer capacity of the cracked reinforced concrete section, including tension stiffening effects; and (3) analysis of the behaviour of the section in the plastic range incorporating strain hardening. The most important contribution of this work is the use of the Ramberg-Osgood function:

$$\epsilon = \frac{\sigma}{E} + \alpha \left(\frac{\sigma}{f_y}\right)^n \tag{1}$$

to provide a smooth, continuous representation of steel behavior across all stages, eliminating the abrupt stiffness changes inherent in bilinear code models [1].

2.2. Sectional Curvature Determination

2.2.1 Uncracked Stage

Strain compatibility between steel and concrete dictates the flexural behaviour of uncracked sections. To ensure equal strain throughout the section, the reinforced concrete section can be idealized with homogeneous stiffness; that is, the equivalent area of the reinforcing steel in concrete is calculated using the modulus of elasticity ratio (n = $\frac{E_s}{E_c}$)[14]. The idealisation of a reinforced concrete section with homogeneous stiffness allows its stiffness to be evaluated from a single strain distribution.

While both materials exhibit almost linear responses up to the point of cracking, introducing the Ramberg-Osgood model provides a better understanding of the gradual departure from linear elasticity, especially for steel, by predicting deflection and early curvature more accurately [15]. The Moment-Curvature equation is written as follows:

$$\phi = \frac{M}{E_c I_t} \tag{2}$$

where I_t is the transformed moment of inertia. The neutral axis location follows:

$$y_{NA} = \frac{\int y \, dA_t}{\int dA_t} \tag{3}$$

The tensile fiber strain (expansion) when reaching the tensile strength f_{ct}. of the load-bearing material (concrete), shall start to induce cracking. Where classical elastic theory may reach a limit, the Ramberg-Osgood equation provides a smooth transition (best fit) into the nonlinear aspect of steel behaviour, allowing better calculation of curvature at cracking.

2.2.2. Cracked Section Behaviour

2.2.2.1. Concrete Cracked – Steel Elastic

Once concrete has developed a fracture, it can only support compressive forces, while its reinforcement carries tensile forces. The forces carried by the entire cross-section of a member must be analysed across both the compressed concrete zone and the steel reinforcement bars that support it [18]. The rate of change in steel behaviour in this phase can best be represented by the Ramberg-Osgood linearised relation between stress and strain [5]. At this stage, the curvature of the section is as follows:

$$\phi = \frac{M}{E_{eff} I_c} \tag{4}$$

where E_{eff} is the tangent stiffness from the R–O model and I_c is the cracked transformed inertia.

The neutral axis depth x follows from force equilibrium:

$$C_c(x) = T_s(\epsilon_s) \tag{5}$$

Moreover, steel strain is defined as:

$$\epsilon_s = \frac{\sigma_s}{E_s}\left[1+\alpha\left(\frac{\sigma_s}{f_y}\right)^{n-1}\right] \tag{6}$$

This approach captures early nonlinear steel behavior before yielding, improving post-cracking curvature and load–deflection prediction accuracy.

$$n = \frac{ln\left(\frac{\varepsilon_u}{0.002}\right)}{ln\left(\frac{f_u}{f_y}\right)} \tag{7}$$

Where $f_u$is the ultimate stress of the material, ε_u is uniform strain at f_u, and f_y is the yield stress of the material.

2.2.2.2. Concrete Cracked – Steel Plastic/Nonlinear

The full Ramberg-Osgood constitutive model is necessary as loading continues to increase on the member and steel stresses reach the yield point, or continue to exceed it; it provides an uninterrupted transition from elastic to inelastic behaviour, without any intermediate bilinear approximations [6]. When a steel member enters its nonlinear plastic behaviour, the analysis is performed using sectional equilibrium.:

$$C_c(x) = T_s(\epsilon_s^{RO}) \tag{8}$$

where

$$\epsilon_s^{\mathrm{RO}} = \frac{\sigma_s}{E_s} + \alpha\left(\frac{\sigma_s}{f_y}\right)^n \tag{9}$$

Curvature grows significantly with increasing moment and can be obtained from:

$$\phi = \frac{\epsilon_c + \epsilon_s^{RO}}{d} \tag{10}$$

where d is the effective depth between the compression strain layer and tension strain layer, serves to define how these two areas transfer load between one another. This section becomes the dominant part of the global Structural Engineering response (load-deflection) due to the nature of Stiffness Degradation in steel, as we have just discussed. The Stiffness Degradation, in combination with curvatures, creates more significant mid-span deflections than at other stages. The Ramberg-Osgood model captures this transition continuously rather than applying a sudden stiffness change, as is usual with simplified bilinear models.

2.2.3. Load–Curvature and Load–Deflection Representation

Once cracking of concrete is observed and the tensile reinforcement is experiencing plastic behaviour, then the sectional behaviour of the composite will be controlled by the nonlinear compressive zone of the concrete and the plastified portion of the tensile reinforcement (steel). The equilibrium and compatibility properties of such a section can be described by multiple-formula relationships that relate to the compression zone depth, the axial force at that point, and the bending moments generated [17]. The axial force that can be produced for an imposed state of curvature is:

$$N = \frac{1}{2\phi}\eta^2 bE_c - A_s f_{yd} + \phi(\eta - a)E_s A_s' \tag{11}$$

Where η is the neutral axis position, A_s is the area of steel in the tension fiber, A_s' is the steel area in the compression fiber, f_{yd} is the yield value for steel in tension fiber, and a is the depth of the compression zone in concrete. To find the neutral axis position η evaluated from:

$$\eta = -\frac{\alpha_1 A_s'}{b} + \left[\left(\frac{\alpha_1 A_s'}{b}\right)^2 + \frac{2}{bE_c\phi}\left(\phi a E_s A_s' + N + A_s f_{yd}\right)\right]^{1/2} \tag{12}$$

Where α_1The parameter to calculate (η)that is used to calculate the internal bending moment for the cracked–plastic stage is:

$$M^* = \frac{E_c b\eta^3}{3}.\phi - A_s f_{yd}(a - h + \eta) + \phi(\eta - a)^2 E_s A_s' \tag{13}$$

The bending moment referred to the concrete section centroid (mid-depth) is:

$$M = M^* + N\left(\frac{h}{2} - \eta\right) \tag{14}$$

The interactions among these relationships are used to define the cracked concrete section. At the same time, the transition from the nonlinear hardening of steel reinforcement to plastic behaviour has been incorporated into the Ramberg-Osgood constitutive model. By applying these relationships to determine the tangent modulus (Eeff) at several strain levels, we can directly obtain moment-curvature evaluations and accurately predict curvature growth and stiffness degradation with increasing load-deflection [18].

2.4. Evaluation of Tangent Stiffness Based on the Ramberg–Osgood Model

To determine a predictable load-deflection relationship for a reinforced concrete beam, the change in beam stiffness at all stress levels must be considered. Adding reinforcement to a beam changes its stiffness because it alters the range of stresses the beam experiences. The Ramberg-Osgood Model gives a continuous and differentiable relationship between stress and strain, and can therefore be used to determine Tangent Stiffness values for all key regions of Structural Response [18]

2.4.1. Tangent Elastic Stiffness (Point 1)

The tangent elastic stiffness characterize the final stage of the response of a cracked section when the steel reinforcement still behaves elastically; this stiffness is equal to the moment-curvature slope at the limit of the elastic portion of the Ramberg-Osgood curve for steel. Therefore, we can express it mathematically as the derivative:

$$k_{el} = \left(\frac{dM}{d\phi}\right)_{\phi=\phi_1} \tag{15}$$

Where; ϕ_1 is the curvature at which the steel strain reaches the boundary of the elastic Ramberg–Osgood domain, k_{el} corresponds to Point 1 in the equivalent diagram (Fig. 1). This point marks the onset of noticeable nonlinear behaviour in the reinforcement and the transition from linear to nonlinear stiffness reduction [6].

2.4.2. Tangent Plastic Stiffness (Point 2)

When the reinforcement is loaded, it will have progressed to the "nonlinear plastic" portion of its load-deformation response, which has a considerable reduction in the value of tangent stiffness that may be determined from the differential method mentioned before for the Ramberg-Osgood formulation at the curve corresponding to significant yield in the steel:

$$k_{pl} = \left(\frac{dM}{d\phi}\right)_{\phi=\phi_2} \tag{16}$$

Where; ϕ_2 is the curvature corresponding to Point 2 in Fig. 1. k_{pl} reflects the tangent stiffness in the plastic range of the Ramberg–Osgood model. This stiffness value governs the beam response during the large-deflection stage and is essential for accurately predicting the descending stiffness branch preceding the ultimate limit state [19].

2.4.3. Ultimate Bending Moment

The bending capacity associated with ultimate conditions, M_u, corresponds to the conventional flexural failure of the RC section, whether governed by compressive concrete crushing or tensile reinforcement rupture. In the Ramberg–Osgood-based framework, the moment at the ultimate state is evaluated through the strain profile extending to the ultimate concrete strain. ϵ_{cu}:

$$M_u = T_s(\epsilon_{s,u}) \cdot z \tag{17}$$

Where; $\epsilon_{s,u}$ is the reinforcement strain obtained from the full Ramberg–Osgood expression at ultimate curvature. z is the internal lever arm at failure. This ultimate moment, together with Points 1 and 2, is shown in the Figure. 1, forms a complete representation of the nonlinear stiffness degradation throughout the beam response [1].

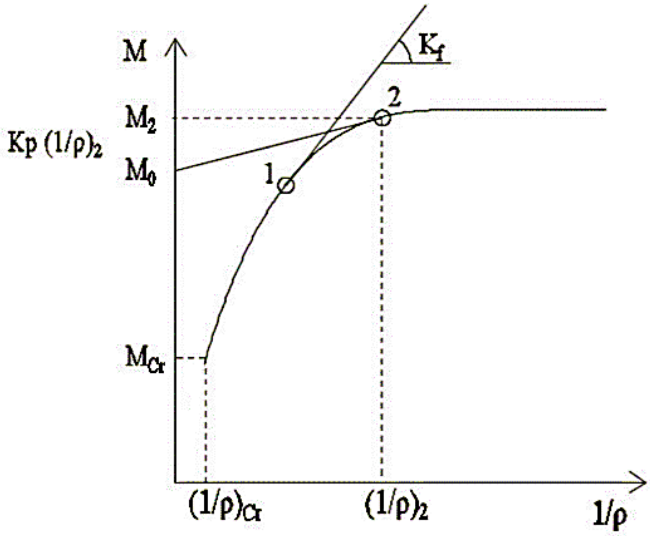


Fig. 1. General moment-curvature diagram for cracked concrete.

3. Ramberg–Osgood approximation of the Cracked Response

3.1. Curvature–Moment Representation Using the Ramberg–Osgood Model

After the formation of tensile cracks, the flexural response of reinforced concrete beams becomes highly nonlinear, primarily due to the combined effects of progressive concrete softening in compression and the nonlinear hardening of the reinforcing steel. To obtain a smooth and continuous representation of the curvature–moment relationship in this post-cracking regime (State II), the Ramberg–Osgood constitutive model provides a convenient analytical framework [5]. The curvature corresponding to a given bending moment can be expressed in a normalized Ramberg–Osgood form as:

$$\phi = \frac{M}{K_{el}}\left[1+\alpha\left(\frac{M}{M_y}\right)^{n_1-1}\right] \tag{18}$$

Where; ϕ is the section curvature, M is the applied bending moment. K_{el} is the initial post-cracking stiffness obtained from cracked-section analysis, Equ. 15, M_y is the moment corresponding to the onset of yielding in the tensile reinforcement. α is the Ramberg–Osgood shape parameter controlling the onset of nonlinearity. n_1 is an exponent governing the transition between elastic and plastic steel behavior.

The parameter n controls the steepness of the curvature increase as the reinforcement moves from elastic towards plastic response. Its value is selected best to match the nonlinear hardening characteristics of the reinforcement and is obtained from calibration against sectional moment–curvature analysis or experimental data:

$$n_1 = \frac{ln\,(\phi_2/\phi_1)}{ln\,(M_2/M_1)} \tag{19}$$

where (ϕ_1, M_1) and (ϕ_2, M_2) There are two reference points corresponding to the elastic–nonlinear transition and the plastic–hardening region, respectively.

3.2. Comparison of Curvature–Moment Curves for Different Values of n_1

Figure 2 presents a comparison between curvature–moment curves predicted by the Ramberg–Osgood formulation using different values of the transition between elastic and plastic steel behavior n_1 (e.g., 6, 20, 40) Furthermore, the reference analytical curves were obtained from sectional analysis under various levels of axial load.

The comparison highlights that:

- Lower values of n_1 produce smoother, more gradual transitions from elastic to plastic behavior.
- Higher values of n_1 yield a curve that approaches a bilinear response with a sharper yielding point.

The Ramberg–Osgood model can closely approximate the full nonlinear sectional response when the parameters (N, M, ϵ_s^{RO}, ф, and n) are properly calibrated [20]. This ability to reproduce a wide range of nonlinear behaviours with only two adjustable parameters makes the Ramberg–Osgood formulation particularly useful for predicting load–deflection responses of RC beams, where curvature plays a central role in determining midspan deflection through numerical integration along the member length [21].

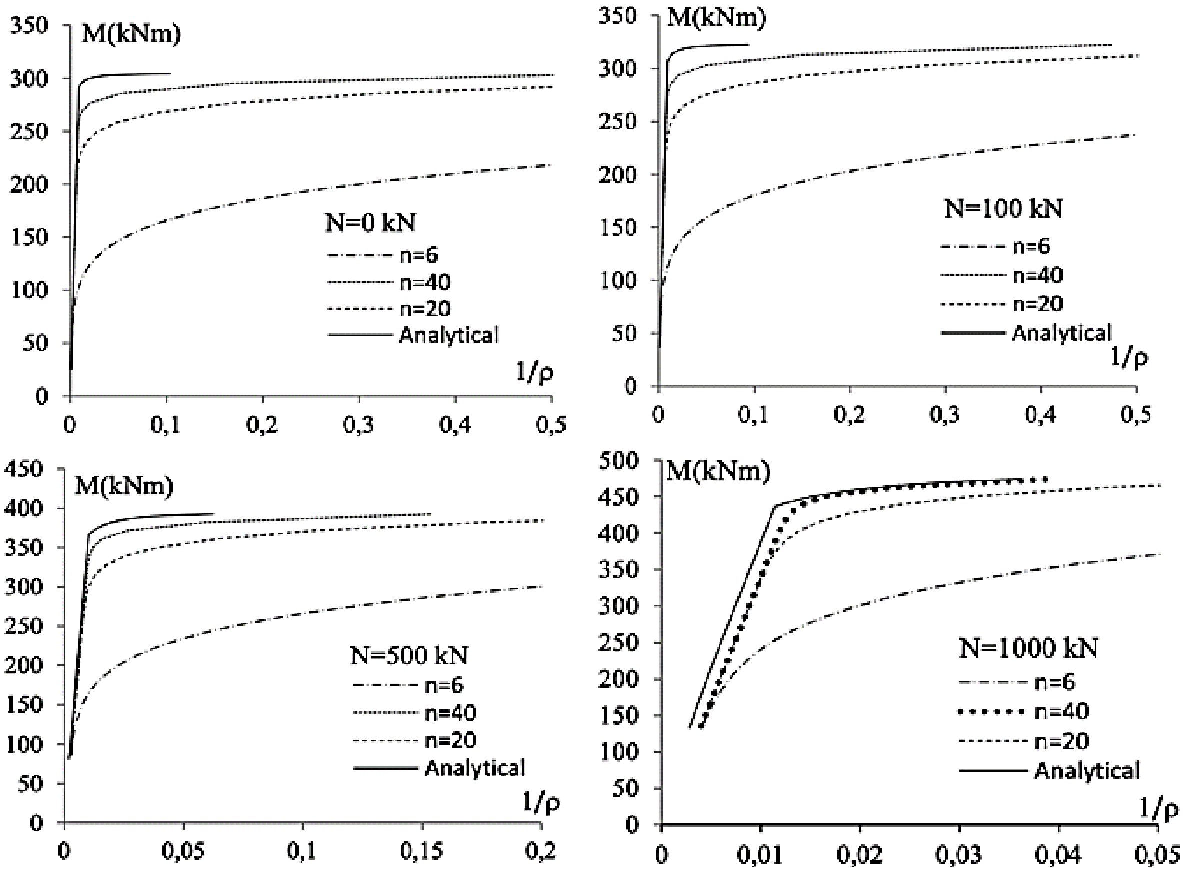


Fig. 2. Moment-curvature diagram by Ramberg-Osgood equation

3.3. Application of IS 456 Provisions In Analytical Modelling

In this study, the IS 456:2000 code provisions were employed to provide a reference analytical model for reinforced concrete beams. The effective moment of inertia approach was used to account for concrete cracking, and a bilinear representation of steel stress–strain behaviour was adopted to approximate the reinforcement's elastic and plastic ranges. This approach provides conservative estimates of flexural stiffness and ultimate moment capacity, serving as a benchmark to compare with both experimental measurements and the Ramberg–Osgood constitutive model. Utilising IS 456 in the methodology ensures that the analysis aligns with established engineering design practices and provides a practical context for evaluating the predictive accuracy of more advanced analytical formulations [1].

4. Numerical Procedure

4.1. Numerical Implementation Framework

The overall analysis follows these sequential steps, consistent with recent nonlinear sectional-analysis frameworks used in RC members:

- Define material properties: $E_c, f_{ct}, f_c, E_s, f_y, \epsilon_{cu}, f_u, and\ \epsilon_{su}$.
- Compute uncracked stiffness and evaluate M_{cr} following common RC tension-stiffening formulations [13].
- Generate reference (M,ϕ) points for the cracked–elastic and cracked–plastic stages using sectional analysis with fibre discretisation and nonlinear constitutive relationships, for calibration and validation purposes only.
- Use these reference cracked-section points to identify the parameters. $K_0, \alpha, n, M_y$of the Ramberg–Osgood-based closed-form relation. This relation provides a continuous transition between the elastic and plastic states of steel [21].
- Assemble the full moment–curvature curve:
- Uncracked branch
- R–O cracked branch
- Applying the average curvature method to compute global mid-span deflection is an accepted technique that has been shown in numerous Nonlinear beam model experiments to be efficient [22].

The methodology is therefore based primarily on mechanics-derived sectional analysis as a reference tool for parameter calibration and model verification, whereas the final outcome of the study is a smooth, closed-form analytical representation of RC flexural behaviour that can be applied without repeated fibre discretisation in routine applications.

4.2. Statistical Evaluation of Moment–Curvature Comparisons

Statistical metrics provide a way to quantitatively compare the moment-curvature responses of reinforced concrete beams predicted by analytical models to experimental results. Common statistical measures used for this comparison are:

- The mean absolute error (MAE) is the average absolute difference between the model's predictions and the experimental results, representing the overall magnitude of prediction error.
- The root mean square error (RMSE) is more sensitive to larger deviations between predicted and experimental results because it squares the errors, giving greater weight to discrepancies in curvature (magnitude) or moment values.
- The coefficient of determination (R^2) indicates the amount of variation in the experimental results that can be explained by the variations in the analytical model data, thereby providing a normalized measure of how well the analytical model fits the experimental results.

Statistical metrics allow objective comparisons of multiple analytical methods (e.g., Ramberg-Osgood formulations, IS 456 code predictions, experimental data) to identify the best model for describing the nonlinear behaviour of reinforced concrete beam flexure and to improve the reliability and repeatability of analytical assessments in structural engineering research [18].

$$MAE = \frac{1}{N}\sum_{n=1}^{N} \mid A_n - P_n \mid \tag{20}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N} (A_n - P_n)^2} \tag{21}$$

$$R^2 = 1 - \frac{\sum(A_n - P_n)^2}{\sum(A_n - S_n)^2} \tag{22}$$

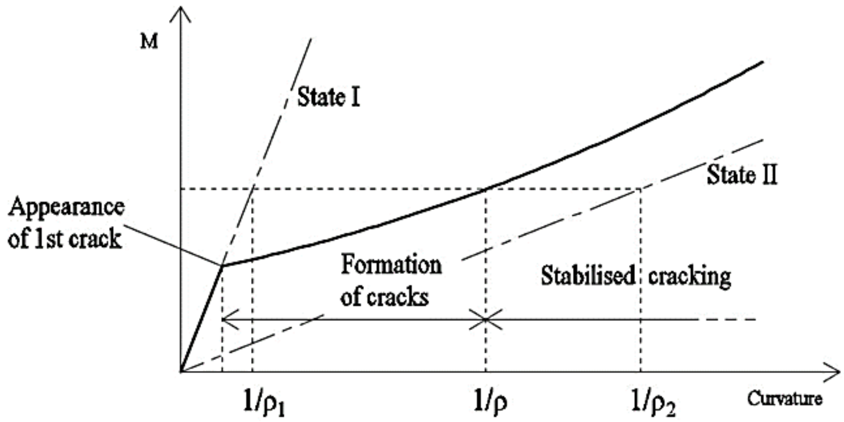
Where (A_n) actual and predicted (P_n) values are compared, and comparing the estimated (N) number of points within the dataset is compared.

4.3. Overall Behaviour of the Cracked Member

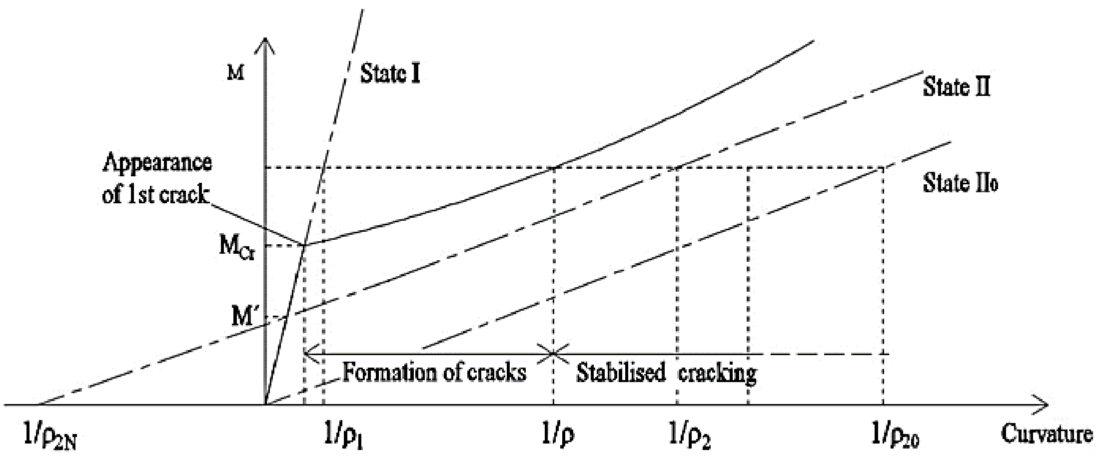
The structural response of a reinforced concrete member after the initiation of the first flexural crack may be idealised into three distinct behavioural stages:

- Uncracked stage, where the section responds linearly.
- Crack-formation stage, with decreasing sectional stiffness.
- Stabilised cracking stage, with nearly constant crack distribution and reduced stiffness.

These stages align with the multi-stage behaviour described by modern curvature-based RC models [4]. see (Figure 3a, b) [22].



(a)



(b)

Fig. 3. (a) Idealised moment curvature behaviour for bending moment [22], (b) Idealised moment curvature behaviour for bending moment and axial load [22]

In State I, curvature ϕ_I corresponds to an uncracked section. In State II_0, curvature $1/\rho_{II0}$ (from Equ. 16 and 18) represents the fully cracked state. Under combined bending and axial load, curvatures become ϕ_1 in State I and ϕ_2 in State II, respectively. To incorporate tension-stiffening, the mean curvature ϕ_m is evaluated as:

$$\phi_1 = \zeta(\phi_{II} - \phi_I) + \phi_I \quad (23)$$

Where the tension-stiffening parameter ζ depends on M_{cr}, the acting moment M, and the bond-dependent coefficient β, following CEB-FIP recommendations and commonly used in recent RC curvature studies [4]. The presence of axial force modifies ζ according to:

$$\zeta = \begin{cases} 1-\left(\frac{M_{cr}}{M}\right)^2, & N=0 \\ \beta\left(\frac{M'}{M}\right)^2, & N>0, M'<M \\ 1, & N>0, M'\geq M \end{cases} \tag{24}$$

Where M' is the intersection of State-I and State-II curves. In the present study, this formulation is integrated with the Ramberg–Osgood constitutive model, which provides a smooth nonlinear steel stress–strain curve capable of accurately representing cracked-section behaviour.

5. Experimental Benchmarking And Comparative Assessment Framework

To validate the proposed analytical formulation and demonstrate its applicability to real structural behavior, a robust comparison framework incorporating both experimental evidence and established design standards was established. The empirical data obtained from significant investigations provide independent standards for assessing the predictive fidelity of the Ramberg-Osgood-based model [23,24]. Examples of the experimental work performed in the series of studies included reinforced concrete beams tested under monotonic bending loads. The measured values obtained from these tests are necessary for establishing the validity of a modelling methodology to accurately predict the expected load-deflection behaviour of reinforced concrete beams, as well as to quantify the development of cracks, the evolution of curvature, and the increase in strain of the reinforcing steel up to failure.

Furthermore, the IS 456:2000 provisions serve as the basis for normative references to the code-based predictions made by engineers. The design framework of IS 456 was established as a widely accepted engineering standard through the use of simple, conservative assumptions about cracking in concrete, effective stiffness, tension stiffening, and the yield strength of steel. These assumptions are useful for providing estimates for practical design-level purposes. However, because the framework uses bilinear or idealised elastic–plastic curves to represent the material behaviour of steel, the actual material behaviour is likely more complex than what is represented by the curves in the framework, which do not accurately depict the smooth nonlinear transition between elastic and plastic behaviour of steel [1].

The Ramberg-Osgood model provides a continuous, differentiable representation of the transition from elastic to plastic behaviour, more accurately reflecting the degradation of reinforcing steel stiffness in a member and better able to predict the curvature of a reinforced concrete member under increasing moments due to loading. The combination of this fibre model and moment-curvature and load-deflection behaviour analysis will allow comparison of these results with the current conventional codes for behaviour and with the true behaviour of concrete members tested [5,25].

6. Results and Comparative Analysis

6.1. Validation Against Experimental Benchmark I [23]

The moment-curvature relationship for an RC beam examined in [23] is shown in Figure 4. The Ramberg-Osgood (R-O) model developed in this study fits the experimental data extremely well. It captures all the phases of behaviour identified in the experimental data: (i) a linear-elastic phase, (ii) a reduction in stiffness due to cracking, (iii) a smooth transition to yield at approximately 19.9 kN·m, and (iv) a plateau during post-yield hardening. In contrast, the IS 456 model exhibits significant differences from the experimental data, especially in overestimating the initial stiffness and failing to capture a gradual yield transition, as depicted by the smooth transition of the R-O model, instead showing an abrupt bilinear response.

Table 1 depicts that the R-O Model had a statistically significant reduction of 40% in MAE (mean absolute error) that was achieved using the R-O Model (1.76 kN·m) as compared to that achieved using IS 456 (2.89 kN·m), and explained 98.2% of the experimental variance (R^2=0.982). The results of a t-test indicate that there was no statistically significant difference between the R-O Model predictions and the experimental measurements (p-value=0.082). In contrast, the IS 456 Model showed a statistically significant difference (p-value = 0.022).

Table 1. Statistical performance metrics for Benchmark I [23]

Metric	Ramberg-Osgood Model	IS 456 Model
MAE (kN·m)	1.76	2.89
RMSE (kN·m)	2.15	3.42
R^2	0.982	0.934
Mean Relative Error	-11.2%	-15.7%
Ultimate Moment	21.50 kN·m (99% of exp.)	21.01 kN·m (97% of exp.)
Curvature(uncracked section)	0.000641 rad/m (3.6% of exp.)	0.000583 rad/m (5.8% of exp.)
Curvature (fully cracked)	0.00313 rad/m (3.3% of exp.)	0.00320 rad/m (4.7% of exp.)

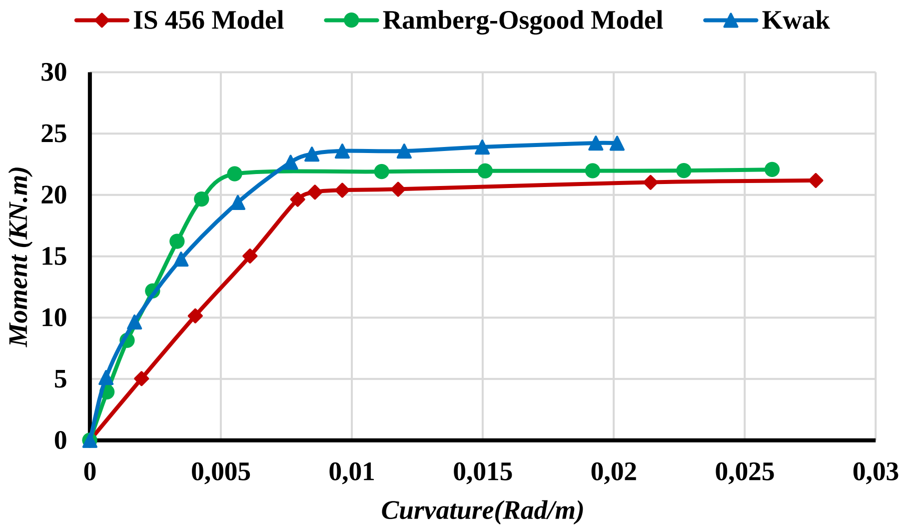


Fig. 4. Moment-curvature diagrams

6.2. Validation Against Experimental Benchmark II [24]

The R-O model continues to accurately predict the curvature development of concrete (M30) beams from an earlier independent data set, as shown in Figure 5, which compares the analytical predictions with the experimental results reported in [24]. Thus, the R-O model predicts the initiation (the cracking moment of 4.33 kN · m) and the final yield curvature (within 2.5% error).

Table 2. Accuracy metrics for Benchmark II [24]

Metric	Ramberg-Osgood Model	IS 456 Model
MAE (kN·m)	0.23	0.28
RMSE (kN·m)	0.31	0.36
R^2	0.994	0.987
Mean Relative Error	-2.8%	-3.3%
Ultimate Moment Prediction	10.26 kN·m (97% of exp.)	10.21 kN·m (97% of exp.)
Curvature(uncracked section)	0.034 rad/m (3.0% of exp.)	0.035 rad/m (6.0% of exp.)
Curvature (fully cracked)	0.095 rad/m (3.1% of exp.)	0.097 rad/m (5.4% of exp.)

The statistical analysis (Table 2) supports the consistency of the R-O Model with respect to the benchmark. Table (2) shows that the R-O model and IS 456 prediction made by both models yielded an ultimate predicting capacity within 3 per cent accuracy, with 97% predicted true value for both models at 10.26 kN·m for the R-O model and 10.21 kN·m for the IS 456 model. According to the MAE and RMSE results, the R-O model performed better than the IS 456 model, as evidenced by lower MAE and RMSE values.

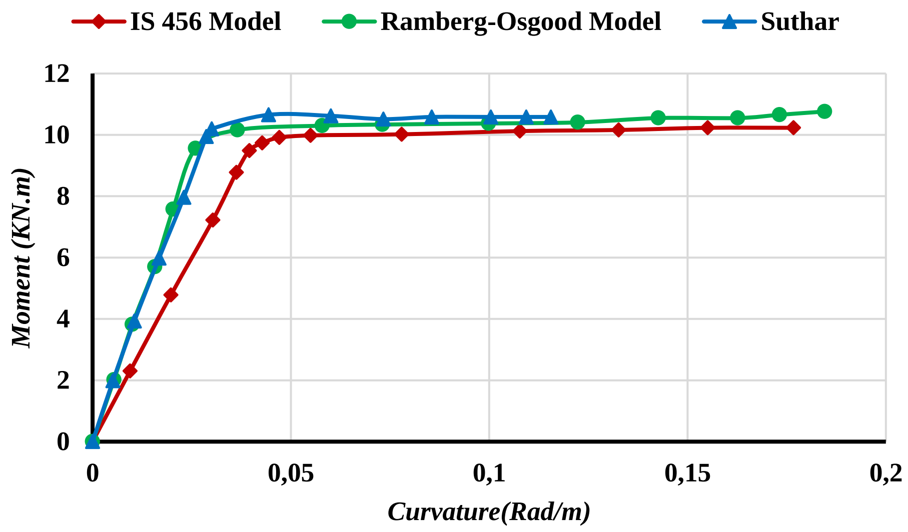


Fig. 5. Moment-curvature diagrams

7. Conclusion

This study has developed a validated integrated approach for nonlinear moment-curvature modelling using Ramberg-Osgood (R-O) constitutive modelling for reinforced concrete (RC) beams. The main findings from this research are:

- A continuous representation of the transition from elastic to plastic behaviour, which allows for accurate prediction of serviceability deflections.
- The R-O model can accurately predict the curvature-hardening phase, with post-yield error rates of less than 6% for ultimate curvatures, compared to traditional models.
- Although both methods provide reliable predictions of ultimate strength, the R-O model method produces more accurate predictions of moment than the IS 456 method based on lower MAE and RMSE values from test results from both benchmarks (Benchmark I: MAE of 1.76 versus 2.89 kN.m, RMSE of 2.15 versus 3.42 kN.m.; Benchmark II: MAE of 0.23 versus 0.28 kN.m, RMSE of 0.31 versus 0.36 kN.m).
- Compared to the IS 456 Method of predicting ultimate moments, the R-O Model gave a reduction of more than 28% to the mean absolute error of all the predicted moments across the entire response curve. Nevertheless, there was little difference in the accuracy of the two methods in predicting ultimate moments. The framework's ability to model the smooth transition in yield behaviour enables accurate predictions of post-cracking stiffness degradation and post-yield ductility, which bilinear models in current codes do not account for.
- In scope of the considered cases, engineers may use the framework as a method for predicting full-range flexural behaviour, rather than engaging in complex finite element modelling.
- The model has been validated on the basis of monotonic loading of rectangular reinforced concrete beams only. Therefore, there is no evidence to support the use of this model for cyclic loading, cross-sectional shapes, or members subjected to significant axial loading.

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