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Research Article

Empirical equations for prediction of split tensile and flexural strength of high strength concrete including effect of steel fiber

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Abstract

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The current Indian Standard IS: 456-2000 has the equation for prediction of flexural strength which is valid only up to concrete strength of 55 MPa and with introduction of High Strength Concrete (HSC) in the standard, there is need to develop equation and relationship which can predict flexural and split tensile strength of HSC including the effect of steel fibres. This study is aimed to develop relationship between the ratios of split tensile to flexural strength which is applicable to concrete having strength range from 15 MPa to 150 MPa. The study is done by analyzing experimentally obtained test results of a total 120 specimens for flexural strength and 120 specimens for split tensile strength. Further study is also conducted on steel fibre reinforced concrete samples by analyzing experimentally obtained test results of a total 24 specimens for flexural strength and 30 specimens for split tensile strength. The results obtained from the test are compared with empirical equations given by International standards and literatures. On the basis of analysis of results, ratio of split tensile strength to flexural strength is recommended for concrete with and without steel fibre having wide strength range from 100 to 120 MPa. Empirical equations for prediction of split tensile and flexure strength of normal to high strength concrete is proposed and applicability of proposed equation for evaluation for normal to high strength steel fiber reinforced concrete is also discussed.

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1. Introduction

Concrete is generally not designed to sustain tensile forces, understanding and prediction of tensile behavior becomes crucial as it is applicable for estimating stress level under which initiation of cracking may take place. From the structural design point of view of unreinforced concrete for application in structures such as dams or pavements, the role of tensile properties are more important than compressive properties [1-3]. The accurate and realistic value of in situ tensile and compressive strengths of concrete distressed during service period provide an important base for the structural assessment and better decision on repair or rehabilitation. In addition to high degree of variability in results, complexity, cost etc. involved in correlation of concrete tensile strength, it is important to develop realistic constitutive relationship between concrete tensile and compressive strength [4-16]. The concrete as a construction material has a very low tensile strength as compared to its compressive strength which leads to its low resistance to tensile crack which in turn affects safety and durability performance thereby reducing service life of reinforce concrete structures [17-21]. Tensile strength is measured in terms of direct or splitting tension or flexural tensile strength wherein split tensile strength test gives a lower coefficient of variation [22]. The tensile properties of concrete is mainly influenced by

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water to binder ratio, characteristics and quantum of fine and coarse aggregates, concrete age, loading rate, addition of fibers etc. [21-27]. High strength concrete (HSC) is generally defined as concrete with a specified characteristic cube strength between 60 and 100 MPa, Major applications for HSC are in offshore structures, columns for tall buildings, long-span bridges and highway structures. Advantage of HSC is the reduction in size of compression elements and amount of longitudinal reinforcement required. Apart from conventional hydraulic cement and supplementary cementitious materials, Silica fume (microsilica) or metakaoline can be used to enhance the strength at high levels (75 MPa and above). In concrete mix proportioning, the role of impact, crushing or abrasion value of aggregate is generally not a significant parameter for normal concrete strength except in case of lightweight aggregate as aggregate is stronger and failure occurs in paste and not from aggregate [25]. However, it is well established that aggregate properties apart from strength, such as size, shape, surface texture, grading, and mineralogy affects concrete strength in different degrees [26-27]. Further, since the interfacial transition zone properties has key role on concrete tensile properties compared to compressive strength, aggregates characteristics has impact on the constitutive relationship between tensile to compressive strength [27-30].

Many international standards and researchers have proposed empirical equations earlier based on experimentation and therefore applicability of these equations may not be suitable for different conditions and concrete ingredients particularly with addition of fibers [4–16]. This study is aimed to develop relationship between the ratios of split tensile to flexural strength which is applicable to concrete having strength range from 20 MPa to 120 MPa. The study is done by analyzing experimentally obtained test results of a total 120 specimens for flexural strength and 120 specimens for split tensile strength. Further study is also conducted on steel fibre reinforced concrete samples by analyzing experimentally obtained test results of a total 30 specimens for flexural strength and 24 specimens for split tensile strength. The results obtained from the test are compared with empirical equations given by international standards and literatures. On the basis of analysis of results, ratio of split tensile strength to flexural strength is recommended for concrete with and without steel fibre having wide strength range from 100 to 120 MPa. Apart from this empirical equation developed are for certain strength range and in this study an attempt has been made to develop empirical equation for strength range of 15 to 150 MPa. Study also includes the comparison of flexural and split tensile strength of fibre reinforced concrete with proposed equations.

2. Materials

Details of Ordinary Portland Cement (OPC) cement, Coarse and Fine aggregate, Fly ash, ultrafine GGBS, Silica Fumes, Superplasticizer, water and steel fiber used for producing concrete mixes are given hereunder.

2.1. Cementitious Materials

OPC 53 grade as per IS 269-2015 [31] is used along with fly ash, ultrafine GGBS and silica fume in mixes based on strength requirement of concrete mix. Detailed physical and chemical characteristics of cement, fly ash, ultrafine GGBS and silica fume are given in Table-1 & 2. The 3-, 7- & 28-days compressive strength of OPC 53 Grade cement are 36.50 MPa, 45.00 MPa and 56.50 MPa respectively. The 28 days compressive strength of controlled concrete (OPC based concrete without fly ash) and concrete sample with fly ash are 38.83 MPa and 31.94 MPa respectively. The 7 days compressive strength of controlled concrete (OPC based concrete without silica fume) and concrete sample with silica fume are 12.66 MPa and 14.56 MPa respectively, when tested as per IS: 1727.

Table 1. Physical properties of materials [31]

S. No.	Properties	Cement	G.G.B.S	Fly ash	UFGGBS	Silica Fume
1.	Fineness(m ² /kg)	323	400	310	2026	16701
2.	Specific Gravity	3.15	2.93	2.28	2.88	2.28

Table 2. Chemical properties of materials

Sl. No.	Properties	Cement	G.G.B.S	Fly ash	UFGGBS	Silica Fume
1	Loss of Ignition (LOI)	2.3	0.33	0.4	0.17	2.73
2	Silica (SiO ₂)	20.71	34.41	60.95	33.05	85.03
3	Iron oxide (Fe ₂ O ₃)	4.08	1.18	5.7	0.58	-
4	Aluminium oxide (Al ₂ O ₃)	5.15	18.45	26.67	20.40	-
5	Calcium oxide (CaO)	59.96	36.46	2.08	33.14	-
6	Magnesium oxide (MgO)	4.57	7.00	0.69	7.62	-
7	Sulphate (SO ₃)	1.84	0.097	0.29	0.19	-
8	Na ₂ O	0.42	0.30	0.06	0.19	0.73
9	K ₂ O	0.56	0.37	1.46	0.58	2.96
10	Chlorides	0.012	0.022	0.009	0.016	-
11	Insoluble Residue	1.25	0.40	-	0.86	-

2.2. Aggregates

Granite type coarse aggregate having maximum size of 20 mm is utilized and crushed fine aggregate (Zone II) as per IS: 383-2016 [32] is used in study for strength level up to 90 MPa. Physical properties of both aggregate is presented in Table-3. To obtain enhancement in packing density of concrete making materials, three different types of aggregates were used namely Fine Quartz Sand (FQS), Ground Quartz (GQ) and River sand for concrete strength level between 90 MPa to 120 MPa. No coarse aggregate was used in concrete mix produced to achieve strength level of 90 MPa to 120 MPa as coarse aggregate was failing before the cement paste and hence strength could not be achieved. Ground Quartz having finer size particles was used as micro filler to increase the packing density of cement aggregate matrix. Its particle size ranges from 0.5 to 63 microns. Particle size distribution are shown in Figure 1. Majority of quartz grains are having size range of 20-30 μ m. High strength concrete mixes were produced using fine quartz sand having particle size ranging from 150 to 996 microns. Majority of quartz grains are having size range of 300-600 μ m. Quartz sand used in this study have particle size ranging from 1mm to 3 mm.

2.3. Superplasticizer and Water

A polycarboxylate-based superplasticizer complying with IS: 9103 [34] is used in concrete mixes having w/c ratios as 0.16, 0.18, 0.20, 0.27, 0.30 and 0.36. A naphthalene-type superplasticizer complying with IS: 9103 [34] is used in concrete mixes having w/c ratios as 0.47, 0.50, 0.57 & 0.60. Water meeting IS: 456-2000 limits for construction was utilized in concrete mixing.

2.4. Steel Fiber

The trough and hooked end shaped steel fibers are used for the study. Past studies suggest that these fibers are more efficient with improved pull out resistance and toughness as compared to straight end fibers [4]. The fibers used are 0.55 mm (diameter) and 35 mm (length) having aspect ratio of 63 which meets requirement of ASTM A-820. The photograph of trough and hooked end shaped steel fibers and length of fibres measured

using scale are shown in Figure-2. Tensile strength of the fibers as per the manufacturer’s test certificate is 1486.99 MPa.

Table 3. Properties of aggregates

Property	Granite		Fine Aggregate
	20 mm	10 mm	
Specific gravity	2.83	2.83	2.65
Water absorption (%)	0.3	0.3	0.59
Sieve Analysis	20mm	98	100
	10 mm	1	68
	4.75 mm	0	2
	Cumulative	0	0
	Percentage	0	0
	Passing (%)	0	0
	600 μ	0	0
	300 μ	0	0
	150 μ	0	0
	Pan	0	0
Abrasion, Crushing & Impact Value	19,19,13	-	-
Flakiness % & Elongation %	29, 25	-	-

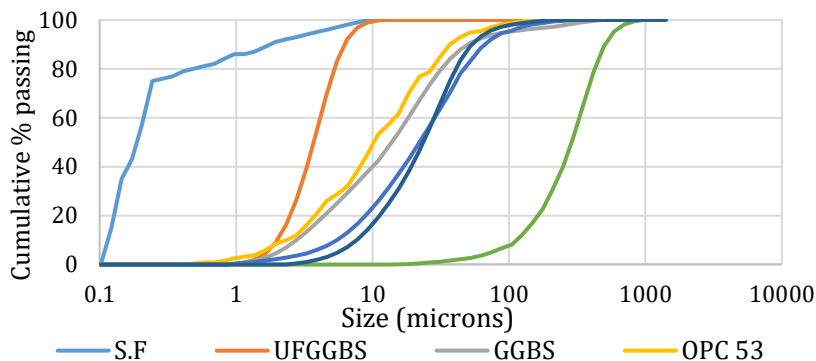


Fig. 1 Materials particle size distribution

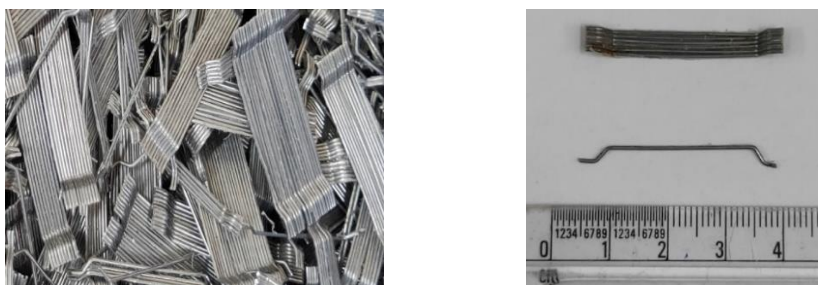


Fig. 2 Trough and hooked end shaped steel fibers

3. Mix Proportion and Compressive Strength of Concrete

The mixes from w/b ratio of 0.60 to 0.16 were adopted to achieve the compressive strength range of 20 to 120 MPa for determining flexural and split tensile strength of concrete. The mix details and 28 days compressive strength are presented in Table-4. The slump of concrete was kept between 80-100 mm. Amount of steel fiber added to the concrete is 1 % of steel fibers by volume of the concrete and it is adopted based on past studies suggesting it as an economical and optimum dose for maximum improvement in overall mechanical performance of concrete [35], [36].

To propose an equation for prediction of the flexural tensile strength; for one mixture, twelve specimens were cast and for total ten mixture without steel fibre referred in the Table-4A, total 120 specimens were tested. To propose an equation for prediction of the split tensile strength; for one mixture, twelve cylindrical specimens were cast and for total ten mixture without steel fibre referred in the Table-4A, total 120 specimens were tested. The experimental results of split tensile and flexural tensile of concrete for strength range between 20 to 110 MPa obtained from testing samples of 3 concrete mixes (Table-4B (Mix:M11-M13)) with steel fibre addition of 1 percent by volume on 30 specimens and 24 specimens respectively are given in section 8. 10 specimens for each of three mixtures were cast for split tensile strength and 8 specimens for each of three mixture were cast for flexural tensile strength. This was done to compare the variation in experimental results with the model developed based on experimental data on flexural tensile and split tensile strength of concrete without steel fibre. For conducting experimental studies, mixes were made in pan type mixer for strength level 20 to 90 MPa and in planetary mixer for strength level above 90 MPa. The moulds were adequately coated with oil before usage, and casting was done in layers, each of which was vibrated on a vibration table to compact. Specimens were removed from moulds after 24 hours. The laboratory conditions of temperature and relative humidity were monitored during the ambient curing i.e. $27\pm 2^{\circ}\text{C}$ and relative humidity 65 ± 5 as per IS: 516. The specimens were allowed to become surface dry before being testing in a saturated surface dried state. The compressive strength of hardened concrete were determined using 150 mm size cube as per the procedure given in IS 516. The 28 days average compressive strength of concrete determined from three cube specimens are tabulated in Table-4. Three cylindrical specimens of diameter 150 mm and length 300 mm were also cast for 10 concrete mixtures given in Table-4A (M1-M10) to generate experimental data on compressive strength of cylinder and cubes for comparing empirical equations of different international standards and experimental results.

4. Experimental Study for Determining Split Tensile Strength and Flexural Strength

The split tensile strength was carried out as per IS: 5816 and set up is shown in Figure 3. For split Tensile Strength as per IS: 5816, the cores of 150 mm diameter and 300 mm long complying with the requirements given in IS 516 were used. The 28 days flexural strength of the concrete mixes was determined through beam specimens of size 150 x 150 x 700 mm as per IS: 516 wherein beam was kept on two rollers and load was applied in center and test set up is shown in Figure 4. The clear span of beam used for test were kept as 600 mm. For one mixture, twelve specimens (4 samples with each sample consisting of three specimens) were cast and for total ten mixture without steel fibre referred in the Table-4A, total 120 specimens (40 samples with 4 samples for each mixture) were tested. The findings of the results of split tensile strength and flexural strength of concrete including comparison of proposed models with empirical models given in international standards and literature are given in subsequent sections.

Table 4. Mix design details of concrete mix without steel fibre

Mixture Details	Mix ID						
	M1	M2	M3	M4	M5	M6	M7
w/b	0.60	0.57	0.50	0.47	0.36	0.30	0.27
Cement, kg/m ³	200	250	290	290	334	400	400
Fly Ash, kg/m ³	50	65	60	72	83	75	75
Silica fume, kg/m ³	--	--	--	--	--	25	50
Ultrafine GGBS, kg/m ³	--	--	--	--	--	--	--
Total cementitious materials, kg/m ³	250	315	350	362	417	500	525
Coarse aggregate 10 mm, kg/m ³	795	780	775	777	730	729	754
Coarse aggregate 20 mm, kg/m ³	520	525	520	518	487	486	406
Fine aggregate (River Sand), kg/m ³	690	685	660	650	726	725	692
Fine aggregate (Quartz Sand), kg/m ³	--	--	--	--	--	--	--
Fine aggregate (Ground Quartz Sand), kg/m ³	--	--	--	--	--	--	--
Fine aggregate (Coarse Quartz Sand), kg/m ³	--	--	--	--	--	--	--
Water, kg/m ³	150	180	175	170	150	150	140
Water-reducing admixture, kg/m ³	0.60	0.50	0.35	0.40	0.35	0.60	1.00
28 Days average Compressive Strength of Concrete, MPa	24.85	27.85	40.35	45.72	65.14	78.93	89.60

Table 4 (Con). Mix design details of concrete mix without steel fibre

Mixture Details	Mix ID					
	M8	M9	M10	M11	M12	M13
w/b	0.20	0.18	0.16	0.47	0.36	0.20
Cement Quantity, kg/m ³	563	480	635	290	334	563
Fly Ash, kg/m ³	112	--	95	72	83	112
Silica fume, kg/m ³	75	165	139	--	--	75
Ultrafine GGBS, kg/m ³	--	165	109	--	--	--
Total cementitious materials, kg/m ³	750	810	978	362	417	750
Coarse aggregate 10 mm, kg/m ³	640	1033	--	777	730	640
Coarse aggregate 20 mm, kg/m ³	427	--	--	518	487	427
Fine aggregate (River Sand), kg/m ³	536	640	--	650	726	536
Fine aggregate (Quartz Sand), kg/m ³	--	--	516	-	-	-
Fine aggregate (Ground Quartz Sand), kg/m ³	--	--	290	-	-	-
Fine aggregate (Coarse Quartz Sand), kg/m ³	--	--	371	-	-	-
Steel Fibre % by volume of concrete	150	135	156	1.00	1.00	1.00
Water, kg/m ³	1.16	1.20	1.8	170	150	150
Water-reducing admixture, kg/m ³	103.5	107.5	127.5	0.40	0.35	1.16
	5		0			
28 Days average Compressive Strength of Concrete, MPa				48.72	68.14	111.55

5. Comparison of Experimental Values of Compressive Strength of Cylinder and Cubes with Different International Standards

The conversion factor for cube to cylindrical compressive strength for the ten concrete mixes without steel fibre (M1-M10) have been worked out (Figure-5). The results indicate

that as compressive strength of concrete increases the ratio of cube to cylindrical strength decreases. IS: 456-2000 recommends fix value of 1.25 up to concrete grade M55 and considering the significant decrease in ratio for grade above M55, values needs to be modified for high grades of concrete. This is also important in assessment of structure where concrete core test is recommended for in-situ verification of strength and cylindrical strength is needed to be converted to concrete equivalent cube compressive strength. The values obtained are used for validating models developed in past which uses either cylindrical or cubical compressive strength in empirical equations. In the present study the cube to cylindrical strength ratio is adopted to be 1.20 upto cube compressive strength of 90 MPa and 1.10 for cube compressive strength above 90 MPa. The assumed ratio is based on the experimental findings and the comparison between the experimental and assumed values are given in Figure 5.



Fig. 3 Test set-up for split tensile strength

Fig. 4 Test set up for flexural strength

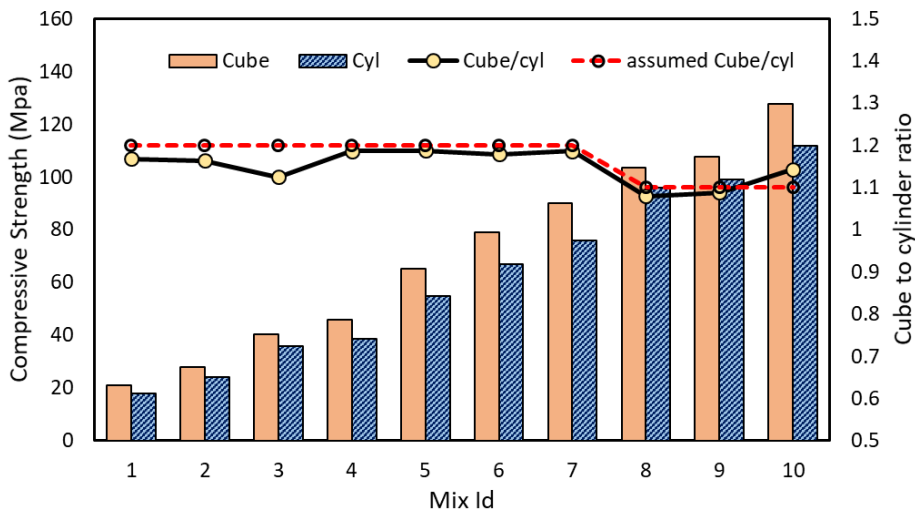


Fig. 5 Comparison of cube to cylinder compressive strength of 20-120 MPa strength

6. Comparison of Proposed Model for Split Strength of Concrete with International Codes and Literature

The experimental results of split tensile of concrete for strength range between 15 to 150 MPa obtained from testing samples of 10 concrete mixes without steel fibre are shown in Figure 6 and 7. In case of experimental results of split tensile strength, for HSC both matrix aggregate separation and transgranular failure was observed. The studies done in past [5 & 19] has shown that ratios of flexural strength to compressive strength increases with increase in the compressive strength of concrete. The ratio of split tensile strength to compressive strength of concrete is about 10 % of compressive strength for normal strength concrete. However, for the high strength concrete it goes down to about 5 % of compressive strength. To propose an equation for prediction of the split tensile strength; for one mixture, twelve cylindrical specimens were cast and for total ten mixture without steel fibre referred in the Table-4A, total 120 specimens were tested. The cylindrical specimen cast from 10 mixes (referred in Table-4A) has compressive strength ranging from 15 to 150 MPa which has been plotted and shown in Figures 6 and 7.

The correlation between the split tensile strength and compressive strength is plotted and proposed equations are having high coefficient of correlation ($R^2 = 0.90$) using regression analysis. Using regression analysis, it is noted that for both one parameter analysis and two parameter analysis similar level of coefficient of correlation is achieved as shown in Figures 6 and 7. In Figure-6, based on two parameter analysis, the proposed equation for estimating split tensile strength of concrete for compressive strength ranging from 15 to 150 MPa can be $0.42 f_{ck}^{0.6}$ where F_{ck} is the cube compressive strength of concrete. In two parameter analysis, both functions i.e. multiplying factor with F_{ck} and power factor to F_{ck} has been varied. Whereas in single parameter analysis, power function to F_{ck} has been kept constant and only multiplying factor analysis has been carried out to develop a simplified equation for incorporation in Indian Standard which is similar to equations given in other international codes for normal strength concrete. Keeping this in view for simplicity, the proposed equation for estimating split tensile strength of concrete considering single parameter analysis (Figure-7) for compressive strength ranging from 15 to 150 MPa can be $0.63 f_{ck}^{0.5}$ where F_{ck} is the cube compressive strength of concrete.

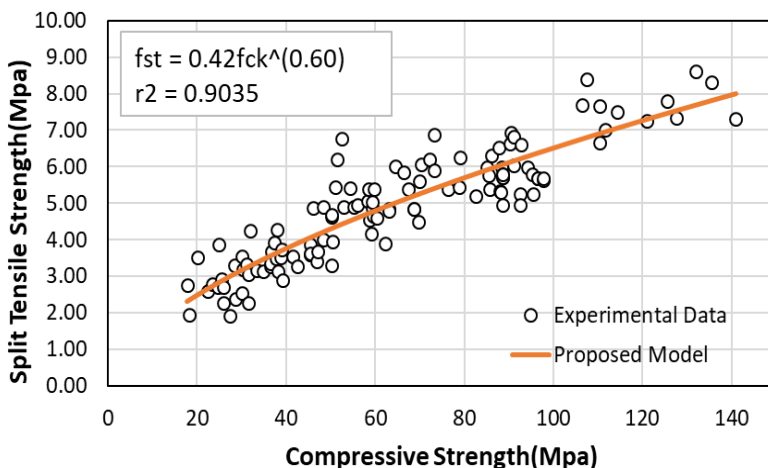


Fig. 6 (a) Derived model for split tensile strength vs compressive strength of concrete considering two parameters

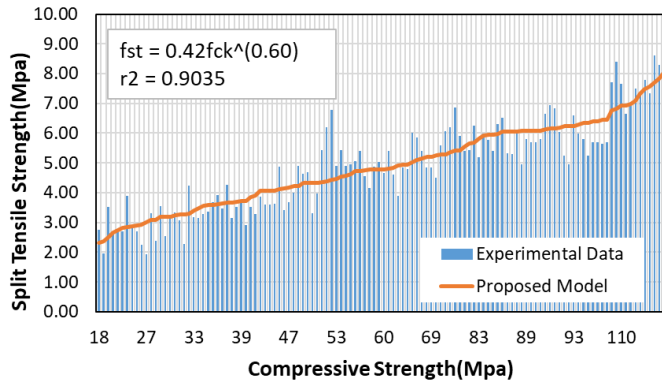


Fig. 6 (b) Comparison of proposed model for split tensile strength vs compressive strength of concrete considering two parameters

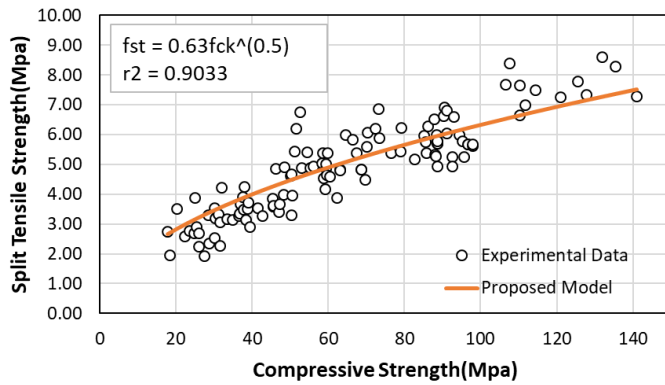


Fig. 7 (a) Proposed model for split tensile strength vs compressive strength of concrete considering one parameter

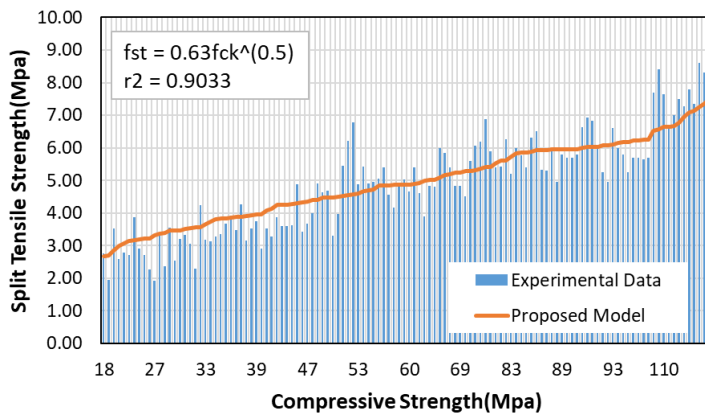


Fig. 7 (b) Comparison of proposed model for split tensile strength vs compressive strength of concrete considering one parameter

Table 5. Previous equation for estimation of split tensile strength of concrete at ambient temperature

Sl. No.	Reference	Previous Equations for estimation of split tensile strength of concrete [MPa]
1	CEB_FIP (1990)	$f_t = 0.3 f_c^{0.67}$
2	ACI 363R-92 (1984)	$f_t = 0.59 f_c^{0.5}$
3	ACI318-99 (2005)	$f_t = 0.56 f_c^{0.5}$
4	Perumal (2014)	$f_t = 0.188 f_c^{0.84}$
5	Rashid et al. (2002)	$f_t = 0.47 f_c^{0.56}$
6	Hueste et al. (2004)	$f_t = 0.55 f_c^{0.5}$
7	Ramados (2014)	$f_t = 0.12 f_c^{0.95}$
8	Xu and Shi (2009)	$f_t = 0.21 f_c^{0.83}$
9	Thomas and Ramasamy (2007)	$f_t = 0.57 f_c^{0.5}$
10	Singh et.al. (2021)	$f_t = 0.5 f_c^{0.5}$

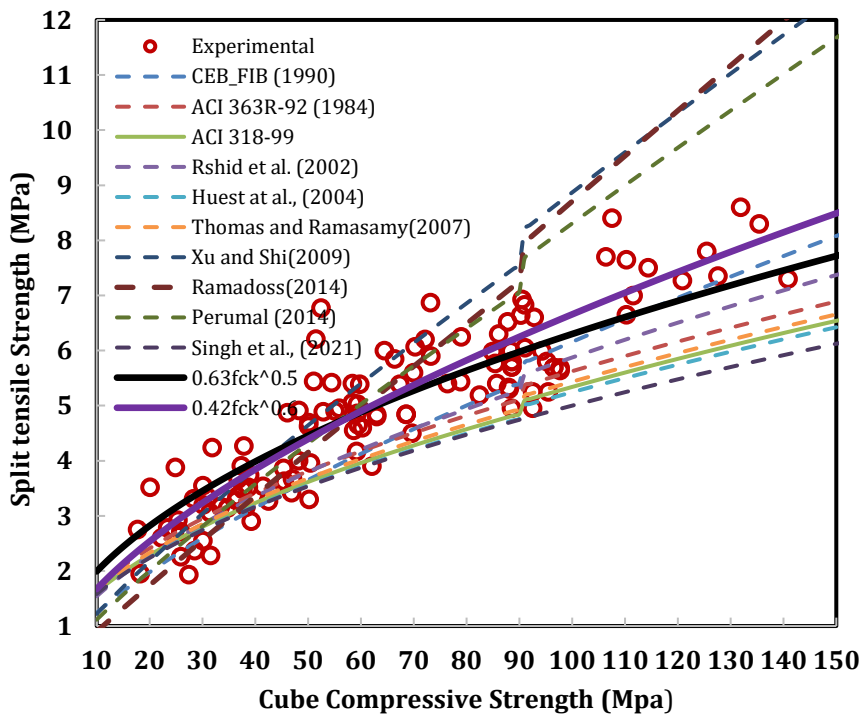


Fig. 8 Comparison of proposed model based on experimental results for split tensile strength of concrete in comparison with models proposed in international standards and literature

7. Comparison of Proposed Model for Flexural Strength of Concrete with International Codes and Literature

The experimental results of flexural tensile of concrete for strength range between 15 to 150 MPa obtained from testing samples of 10 concrete mixes without steel fibre are shown in Figures 9 and 10. To propose an equation for prediction of the flexural tensile strength; for one mixture, twelve cylindrical specimens were cast and for total ten mixture without steel fibre referred in the Table-4A, total 120 specimens were tested. The cylindrical specimen cast from 10 mixes (referred in Table-4A) has compressive strength ranging from 15 to 150 MPa which has been plotted and shown in Figures 9 and 10. The correlation between the flexure tensile strength and compressive strength is plotted and an equation is proposed with a high coefficient of correlation ($R^2 = 0.94$) using regression analysis. Using regression analysis, it is noted that for both one parameter analysis and two parameter analysis similar level of coefficient of correlation is achieved as shown in Figures 9 and 10. In Figure-9, based on two parameter analysis, the proposed equation for estimating flexural tensile strength of concrete for compressive strength ranging from 15 to 150 MPa can be $0.31 f_{ck}^{0.7}$ where F_{ck} is the cube compressive strength of concrete. In two parameter analysis, both functions i.e. multiplying factor with F_{ck} and power factor to F_{ck} has been varied. Whereas in single parameter analysis, power function to F_{ck} has been kept constant and only multiplying factor analysis has been carried out to develop a simplified equation for incorporation in Indian Standard which is similar to equations given in other international codes for normal strength concrete. Keeping this in view for simplicity, the proposed equation for estimating flexural tensile strength of concrete considering single parameter analysis (Figure-10) for compressive strength ranging from 15 to 150 MPa can be $0.83 f_{ck}^{0.5}$ where F_{ck} is the cube compressive strength of concrete.

Comparison of proposed model with previous equations (Table-6) for estimation of flexure tensile strength of concrete at ambient temperature is given in Figure-11. The cubic compressive strength were converted to cylindrical compressive strength for the comparison with previous models wherever applicable based on cube to cylinder ratio determined in section 4.

From Figure 11, it can be seen that ACI: 318, New Zealand code (NZS: 3101), Canadian code CSA: A23.3-14 (2014) and IS-456-2000 estimates flexural strength value on lower side than the actual experimental values for normal and high strength concrete. The difference between the estimated and experimental value increases significantly with increase in concrete strength. The values of flexural tensile strength of concrete estimated through equation proposed by Singh et.al holds good for even higher grades but the study conducted by them was upto 90 MPa only. In the current study the equation has been proposed upto 120 MPa compressive strength of concrete which estimates flexural strength with a reasonable level of consistency.

Table 6. Previous equation for estimation of tensile strength of concrete at ambient temperature

Sl. No.	Reference	Previous Equations for estimation of flexural tensile strength of concrete [MPa]
1	ACI: 318 (2014)	$f_t = 0.62 f_c^{0.5}$
2	NZS: 3101 (2006)	$f_t = 0.6 f_c^{0.5}$
3	CSA: A23.3-14 (2014)	$f_t = 0.6 f_c^{0.5}$
4	IS 456 (2000)	$f_t = 0.7 f_c^{0.5}$
5	Singh et. al. (2021)	$f_t = 0.8 f_c^{0.5}$

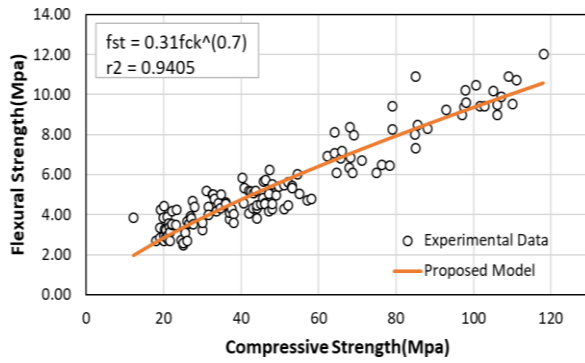


Fig. 9 (a) Proposed model for flexural tensile strength vs compressive strength of concrete considering two parameters

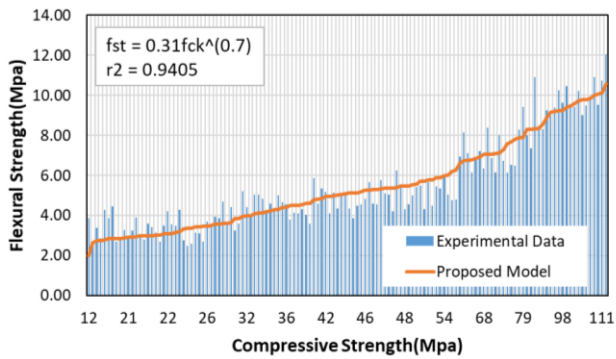


Fig. 9 (b) Comparison of proposed model for flexural tensile strength vs compressive strength of concrete considering two parameters

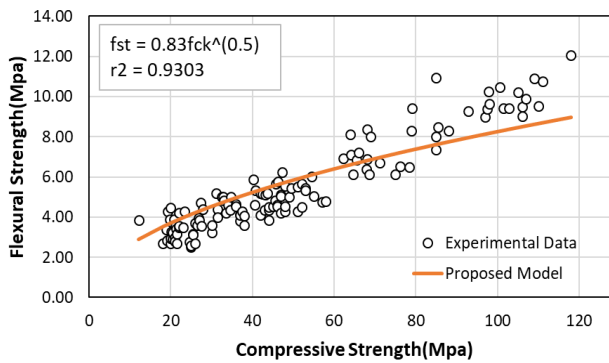


Fig. 10 (a) Proposed model for flexural tensile strength vs compressive strength of concrete considering two parameters

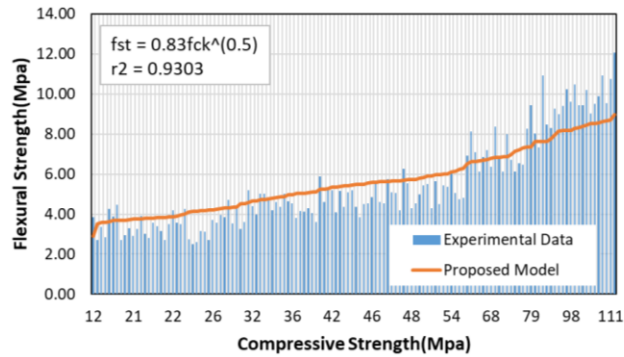


Fig. 10 (b) Comparison of proposed model for flexural tensile strength vs compressive strength of concrete considering one parameter

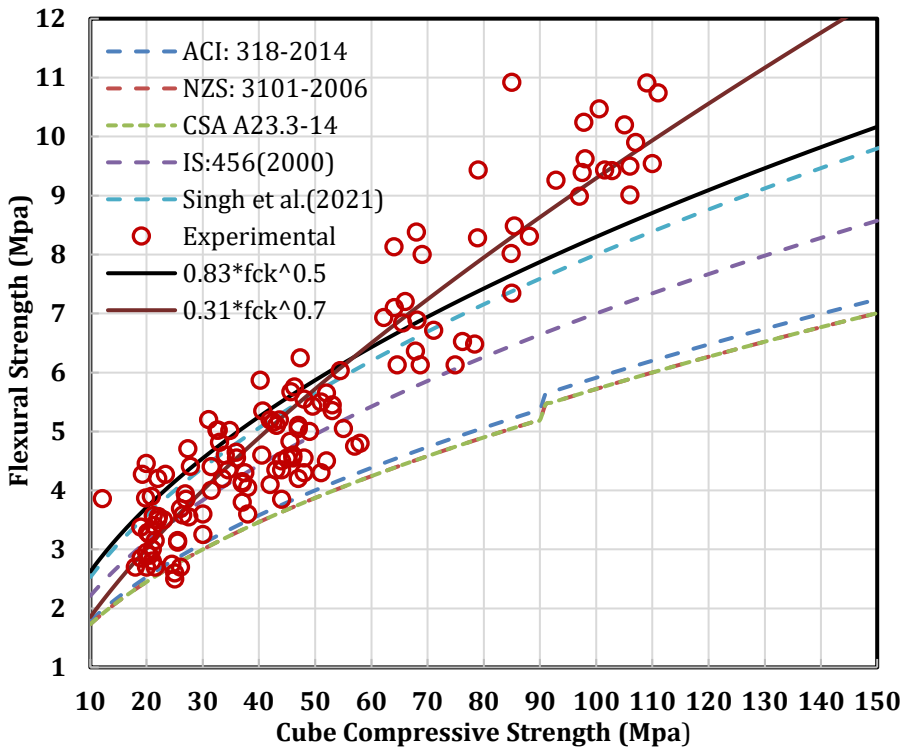


Fig. 11 Comparison of proposed model based on experimental results for split tensile strength of concrete in comparison with models proposed in international standards and literature

8. Comparison of Experimental Results of Flexural and Split Tensile Strength of Steel Fibre Reinforced Concrete with Proposed Models

The experimental results of split tensile of concrete for strength range between 20 to 110 MPa obtained from testing samples of 3 concrete mixes with steel fibre addition of 1

percent by volume on 30 specimens are shown in Figure 12. Total 10 specimens for each of three mixture were cast and results are analyzed.

In case of experimental results of split tensile strength of steel fibre reinforced concrete, for normal and high-strength concretes matrix aggregate separation was similar to that of concrete without fibre but there is improvement in split tensile strength of steel fibre reinforced concrete by 23% on average. The correlation between the split tensile strength of steel fibre reinforced concrete and compressive strength is plotted and when validated with proposed equation high coefficient of correlation ($R^2=0.90$) using regression analysis has been found. However, the proposed model is underestimating the split tensile strength of steel fibre reinforced concrete and it is suggested to use same equation as variation in split tensile strength of concrete is on average 23 percent. The experimental results of flexural tensile of concrete for strength range between 20 to 110 MPa obtained from testing samples of 3 concrete mixes with steel fibre addition of 1 percent by volume on 24 specimens are shown in Figure 13. Total 08 specimens for each of three mixture were cast and results are analyzed. Contrary to split tensile strength of fiber reinforced concrete, the experimental values for the flexural strength of steel fiber reinforced concrete shows high deviation from the behavior of plane concrete. The average variation in flexural strength values for steel fibre reinforced concrete with respect to plain concrete was found to be about 107 percent.

As presented in Figure 13, the proposed model for plain concrete depicts a very weak coefficient of correlation between the experimental and modelled values of the flexural strength. The findings indicates a significant improvement in the flexural strength of the concrete with addition of steel fiber. The fiber action also depends on the packing density of concrete used. It was also observed that the mix with intermediate strength shows a higher flexural strength after the addition of steel fiber than the mix of higher strength with significantly improved packing density and wider inter transition zone. The reason for the observed trend can be attributed to higher number of voids for the steel fiber in the intermediate strength mix as compared to highly optimized mix.

9. Evaluation of Ratio of Flexural Strength to Split Tensile Strength of Concrete for Strength Range 20 to 120 MPa

Based on the experimental results of 120 specimens tested for flexural tensile strength and 120 specimens tested for split tensile strength of concrete on the concrete prepared with 10 concrete mixes (Table-4), a relationship between flexural and split tensile strength of concrete has been proposed. The correlation between the flexural tensile strength and split tensile strength when plotted (Figure-14) has coefficient of correlation ($R^2 = 0.775$) using regression analysis has been found. The obtained equation is $F_{ft}=1.27 F_{spt}$ and the proposed equation can be $F_{ft}=1.25 F_{spt}$ where F_{ft} is flexural tensile strength of concrete and F_{spt} is split tensile strength of concrete for the strength range of 20 to 120 MPa. Figure 14 shows the comparison between the experimental values and the derived relation between the split and flexural tensile strength.

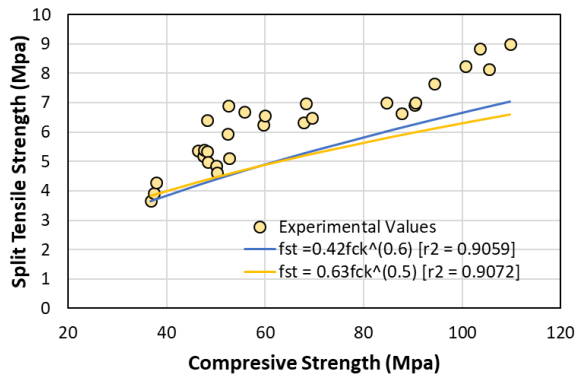


Fig. 12 Comparison of proposed model for split tensile strength vs compressive strength of plain concrete and experimental results of steel fibre reinforced concrete considering both models

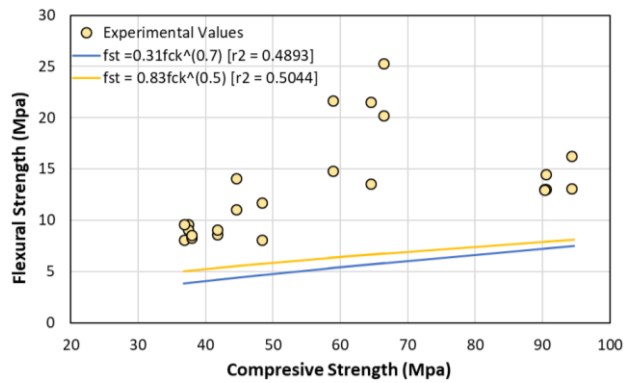


Fig. 13 Comparison of proposed model for flexural tensile strength vs compressive strength of plain concrete and experimental results of steel fibre reinforced concrete considering both models

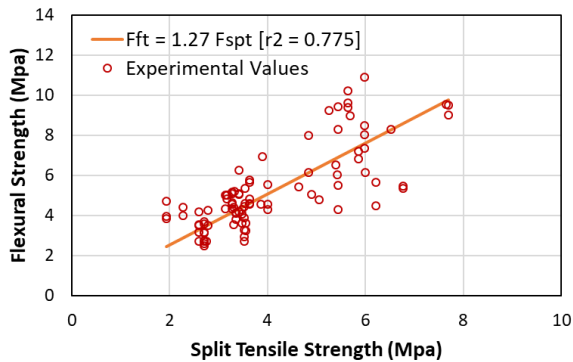


Fig.14 Comparison of proposed model for estimating flexural strength from split tensile strength of concrete

10. Conclusions

The equation for prediction of flexural and split tensile strength from the cube compressive strength in the current Indian Standard IS: 456-2000 is only valid up to concrete strength of 55 MPa. With the inclusion of high strength concrete to the standard, it is necessary to develop a relationship that can predict the approximated flexural and split tensile strength of high strength concrete. Along with this the impact of steel fibers on these two properties needs consideration to check the suitability of empirical equations developed for estimation of flexural and split tensile strength of concrete. The present study attempted to establish a correlation among the splitting tensile strength and flexural strength with compressive strength that is suitable to concrete with strengths between 20 MPa and 120 MPa. A total of 120 test results for split tensile strength and 120 test results for flexural strength were used in the investigation with a total of approximately 24 test results for split tensile strength and about 30 test results for flexural strength of concrete with steel fibers. Findings can be concluded as follow:

- The ratio of cube to cylindrical compressive strength varies with increase in concrete compressive strength. From the experimental investigation, the cube to cylindrical compressive strength is found to be approximately 1.20 upto 90 MPa and 1.10 for 90 MPa to 120 MPa cube compressive strength.
- The relation between the cube compressive strength and the split tensile strength was established as $f_{spt}=0.42*f_{ck}^{(0.60)}$ for two parameter optimisation and $f_{spt}=0.63*f_{ck}^{(0.5)}$ for single parameter optimisation wherein both equations depicts high coefficient of correlation. For simplicity the obtained equation $f_{spt}=0.63*f_{ck}^{(0.5)}$ is recommended for inclusion in codal provisions.
- Similarly the Correlation equation obtained for the estimation of flexural strength from the cube compressive strength is $F_{ft}=0.31*f_{ck}^{(0.7)}$. For simplicity keeping the power 0.5, the equation becomes $F_{ft}=0.83*f_{ck}^{(0.5)}$. Both the equations show a high coefficient of correlation in regression analysis. For simplicity the obtained equation $F_{ft}=0.83*f_{ck}^{(0.5)}$ is recommended for inclusion in codal provisions.
- Addition of steel fibers in concrete mix have different effects on the split and flexural tensile strength of the concrete. With the addition of steel fibers the split tensile strength increases marginally and the equation for the plain concrete can be used for predicting the approximate split tensile strength of fiber reinforced concrete also.
- The fiber action of steel fibers in steel fiber reinforced concrete affects the flexural behaviour significantly. The increase in the flexural strength observed is as high as 107% on average. The proposed equation in this study for plain concrete will underestimate the flexural strength for fiber reinforced concrete and further study is needed to establish the effect of steel fibre strength, shape, percentage, aspect ratio and composition of concrete mix on flexural tensile strength of concrete.
- The ratio between the flexural and split tensile strengths of plain concrete is estimated to be 1.27. The obtained coefficient of correlation is 0.78. Although, the experimental and predicted values does not show a very high coefficient of correlation, flexural strength still can be approximately estimated from the split tensile strength using the equation, $F_{ft} = 1.27F_{spt}$.

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