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Research Article

A novel differential quadrature method for transient heat transfer analysis in multilayered cylindrical structures: Numerical validation and parametric investigation

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Abstract

This study addresses the challenge of accurately modeling transient heat transfer in complex multilayered cylindrical structures, which are critical in aerospace and industrial applications. The primary objective is to develop and validate a novel numerical approach using the Differential Quadrature Method (DQM) to solve the time-dependent Fourier heat conduction equation in composite cylinders with distinct material properties. While conventional methods like Finite Element Method (FEM) are widely used, they often require computationally expensive meshes. In this work, the governing equations in cylindrical coordinates were discretized using Chebyshev-Gauss-Lobatto grid points for spatial derivatives, coupled with an efficient temporal discretization scheme. This DQM formulation allows for the precise handling of interface conditions between layers without complex mesh generation. The proposed model was validated against ANSYS finite element simulations for a four-layer cylinder under constant boundary temperatures (200 °C and 300 °C). Results demonstrate excellent agreement with maximum relative errors ranging from 0% to 2.4%, proving the method's superior accuracy and computational efficiency. Parametric investigations revealed that variations in thermal conductivity and heat capacity significantly alter the transient temperature profiles, offering key insights for the thermal design of composite insulation and piping systems.

1. Introduction

Transient heat transfer involves changes in temperature distribution within a system over time. This occurs when there are variations in temperature, thermal properties, or boundary conditions. Multilayered structures, also known as composite structures, are materials or systems that consist of multiple layers of different materials bonded together. These structures are designed to combine the beneficial properties of each layer to achieve enhanced performance, tailored for specific applications. Multilayered structures are widely used in various fields, including mechanics, aerospace engineering, civil engineering, and materials science. The transient analysis of heat transfer in a multi layered cylinder is attractive to many researchers, but literature review shows that the transient heat transfer in a multilayered cylinder using the basic Fourier heat conduction equation has not been investigated; thus, it's the innovative aspect of this study [1,2].

Saadatfar [3] presented a transition solution for heat and moisture distribution in a finite-length hollow cylinder made of a gradient material under the influence of interconnected thermophilic and hydromechanical conditions. The heat transfer and moisture diffusion equations were solved

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using the Fourier series method on the longitudinal axis, the radial quadratic numerical differential method, and the Newmark time method. The results showed the effect of the ocular correlation model versus the discrete model, and the negative and positive pressure differences of the gradient index, confirming the importance of the correlated model for more accurate analysis. Heydarpour [4] presented a three-dimensional analysis of temporary heat transfer in functionally graded multilayer cone shells reinforced with graphene plates, using a non-furrier heat transfer law. The spatial field transforming DQM and the NURBS-based multistep method of time were applied, and the results showed the effect of plate distribution and geometric properties on heat wave velocity and accuracy of calculations.

Pourasghar [5] provided a three-dimensional analysis of the thermoplastic behavior of cylindrical plates under transient heat load, using the hyperbolic heat transfer equation to reflect the velocity of a final heat wave. They were linked to three-dimensional elasticity equations and digitally solved via the DQM and Newmark diagram, with studies investigating and calibration of the effect of heat wave properties on stresses and deviations. Heydarpour [6] analyzed non-furoretic heat transfer in sandwich cone shells with face layers reinforced with graphene plates and a porous core, under the influence of a moving axial heat flow. It was based on the hyperproli heat transfer equation and the DQM with time integration in NURBS. The results showed the effect of flow velocity and graphene percentage on heat distribution and cochlear performance. Zhou [7] presented an analytical model of heat transfer in a multilayer cylinder with internal heat generation, taking the thermogenetic transfer coefficients at the outer surface. A chained solution method was applied to distribute heat in each layer, taking into account the thermal contact resistors and the properties of the disparate layers. It also examined the effect of changing the refrigerant and analyzed the sensitivity of heat distribution. Mubaraki [8] provided an analytical formulation of static heat transfer in a heterogeneous multilayer composite cylinder using the variable separation method to derive a general solution in series form for a multilayer body. The approach is based on a simple and efficient solution away from complex numerical and transformational methods. The model has been validated across three initial configurations, enhancing its applications in engineering, materials science and solid mechanics.

Tokovyy [9] presented a two-dimensional analytical technique of the heat field in hollow cylinders composed of thermally regressive layers, which addresses the continuous change in properties through direct integration to reformulate the conduction equation into second-order integration equations. The solution is built across the solvability core to reflect the explicit relationship between heat load and the gradual distribution of properties. The method allows you to manipulate an unlimited number of layers without the need for complex global differentiators. Chen [10] introduced a shell conduction model to simulate heat transfer within a thin, multi-layer wall in thermoelectric anti-snow systems. The model creates a virtual volumetric grid based on a surface grid, corrects cell areas according to convexity, and applies thermal conductivity equations using the finite volume method. The model has verified its accuracy in annular, cylindrical, and wing-length shapes.

DQM has emerged as a powerful numerical technique for solving complex heat transfer problems, particularly in multilayer and functionally graded material systems. Recent advances demonstrate its effectiveness in various engineering applications with high accuracy and computational efficiency. Afridi et al., presented a generalized differential quadrature framework for modeling heat transfer in boundary layer flows with nonlinear convection effects. The research demonstrates the versatility of DQM in handling complex boundary conditions and nonlinear phenomena, making it particularly suitable for practical engineering applications, including heat exchangers and thermal management systems. The method shows superior convergence properties compared to traditional finite difference approaches [11]. Afridi et al., again demonstrated the method's capability in handling coupled thermal phenomena, including frictional heating and radiation effects, using the implementation of generalized DQM for entropy generation analysis. This work established DQM as an effective tool for optimization studies in thermal systems, providing insights into energy efficiency and thermal performance enhancement [12]. Zhang et al., focused specifically on cylindrical geometries with functionally graded materials, presenting a mathematical model and numerical technique using DQM for heat conduction analysis. The research validated the effectiveness of DQM in handling variable material properties in cylindrical coordinates, which is directly relevant to multilayer cylinder applications [13].

The development of gear-generalized DQM represents a significant advancement in handling time-dependent problems with variable material properties. This approach is particularly valuable for transient heat transfer analysis in multilayer systems where material properties vary with temperature and time, offering improved stability and accuracy for oscillatory phenomena [14]. An article by Ghadimi et al., demonstrated the application of advanced numerical methods including finite element analysis for transient thermal problems in multilayer cylindrical structures. The study provided validation frameworks that can be adapted for DQM implementations, particularly for cryogenic applications where accurate thermal modeling is critical [15]. Das et al., provided benchmark solutions that can validate DQM implementations using the comprehensive analysis of functionally graded material cylinders with multiple layers. This work established analytical frameworks in polar coordinates that complement numerical approaches, offering verification standards for DQM applications in multilayer systems [16]. These recent publications (2023-2025) provided a strong foundation for DQM applications in heat transfer analysis, particularly for:

- Nonlinear and coupled phenomena - Advanced DQM formulations for complex physics
- Functionally graded materials - Specialized techniques for variable property systems
- Transient analysis - Time-dependent formulations with improved stability
- Validation frameworks - Benchmark solutions and experimental validation approaches
- Cylindrical geometries - Specific applications to multilayer cylinder configurations

The literature demonstrates the continued evolution and validation of DQM as a premier numerical technique for sophisticated heat transfer problems in engineering applications. This study is concerned with transient heat transfer in multilayer cylinders, a topic that has been studied using the DQM method rarely. This method offers an accurate and quick solution, making it ideal for analyzing heat distribution in these complex systems. This research also contributes to improving the design of multilayer materials, especially in applications that require accurate and reliable heat transfer, such as heat transfer pipes and insulating materials.

Despite the versatility of DQM in various domains, its specific application to transient heat conduction in multilayered cylindrical geometries with distinct thermophysical properties remains underexplored in recent literature. Previous studies [11–16] have largely focused on single-layer functionally graded materials or steady-state analyses. This research bridges this gap by formulating a DQM-based solution that explicitly handles the discontinuity of properties at layer interfaces in the time domain. The primary aim of this study is to establish a robust numerical framework based on DQM for predicting transient temperature distributions in multilayered cylinders. The specific objectives are: (1) to derive the DQM formulation for the transient Fourier heat conduction equation in cylindrical coordinates; (2) to validate the proposed method against standard Finite Element benchmarks (ANSYS); and (3) to conduct a comprehensive parametric study analyzing the effects of layer thickness, heat capacity, and thermal conductivity on thermal performance.

2. Methodology

The Distortion Energy Theory shear equation might not be sufficient with newly-developed drill pipes that have highly advanced material properties….

2.1. Governing Equations

In The hypotheses adopted by the mathematical model in this research are: Axial symmetry (Axisymmetry): It was assumed that the temperature distribution is symmetrical around the axis of the cylinder. This allows the equation to be simplified to one radial (r) and time (t). Heat transfer in the axial (θ) and perpendicular (z) directions has been neglected, reducing computational complexity and making the problem numerically solvable. Assume that the coefficient of thermal conductivity (k_r), density (ρ), and heat capacity (c_p) do not depend on temperature. This converts the equation into linear form and facilitates the solution. The effect of moisture in the heat equation has been neglected, simplifying the system to a pure thermal equation. It was assumed that each layer in a multilayer cylinder has homogeneous physical properties within its boundaries (not differing with the radial location within the layer). The general Fourier equation of heat in cylindrical coordinates was employed at the beginning of this analysis [17]:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_\theta \frac{\partial T}{\partial \theta}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right) \tag{1}$$

Assuming axial symmetry and heat transfer only in the radial direction, the equation is simplified to:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) \tag{2}$$

Where ρ is the density kg/m^3, c_p is the specific heat capacity $J/kg\ K$, T is the temperature t is time s, r is the radial coordinate, m, and k_r is the thermal conductivity in the radial direction W/mK. Chebyshev-Gauss-Lobatto points were used in the radial direction to ensure the stability of the solution:

$$r_i = \frac{a+b}{2} + \frac{b-a}{2} cos\left(\frac{(i-1)\pi}{N-1}\right), i = 1,2, \dots, N \tag{3}$$

Where a , b are the inner and outer radius of the cylinder, the second derivative of the degree n at the point r_i is rounded as follows:

$$\left.\frac{\partial^n T}{\partial r^n}\right|_{r_i} = \sum_{j=1}^{N} A_{ij}^{(n)} T(r_j) \tag{4}$$

Where $A_{ij}^{(n)}$are the weight coefficients derived from Lagrange polynomials. An even or adaptive time distribution was used to rapid changes in temperature. The first derivative of time is rounded by:

$$\left.\frac{\partial T}{\partial t}\right|_{t_k} = \sum_{m=1}^{M} B_{km}^{(1)} T(t_m) \tag{5}$$

Where $B_{km}^{(1)}$ are the temporal weight coefficients, after applying the DQM to the thermal equation, the system is written in matrix form:

$$M.T = F \tag{6}$$

Where M is the coefficient matrix (includes conduction and heat coefficients). T: temperature vector at all points and time moments. F: boundary conditions vector and heat sources.

2.2 Numerical Analysis

MATLAB was used as the main software to solve the recent equations numerically. While ANSYS was employed as a finite element tool to validate the MATLAB code, which is used for DQM. The model was sketched with the design modular tool in Workbench ANSYS 2023 R2 as half of a multilayered cylinder to reduce analysis time, and since it is symmetric, as shown in Fig. 1. Where the inner radius is 0.1m and the outer radius is 0.5 m., with fixed layer thickness. The mesh type employed in this analysis is shown in Fig. 2.

The Transient thermal solver was used to simulate this analysis with fine meshing (i.e., element size of 10 mm). The minimum mesh quality reached 91 % with the HEX20 element type. The analysis time taken was 300 s, while the boundary conditions are 200 ºC on the inner edge and 300 ºC at the outer edge, as shown in Fig. 3.

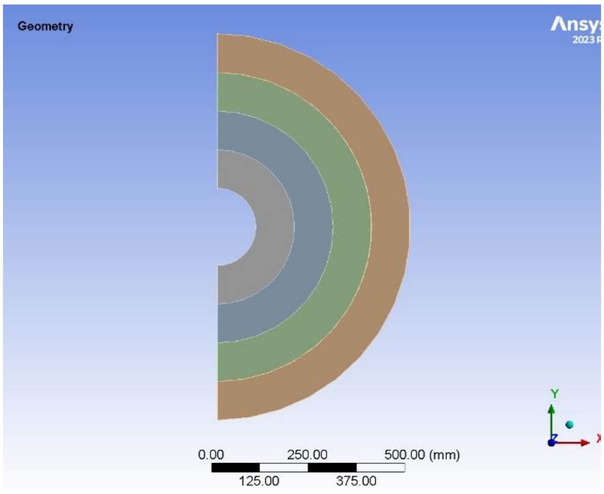

Fig. 1. ANSYS model

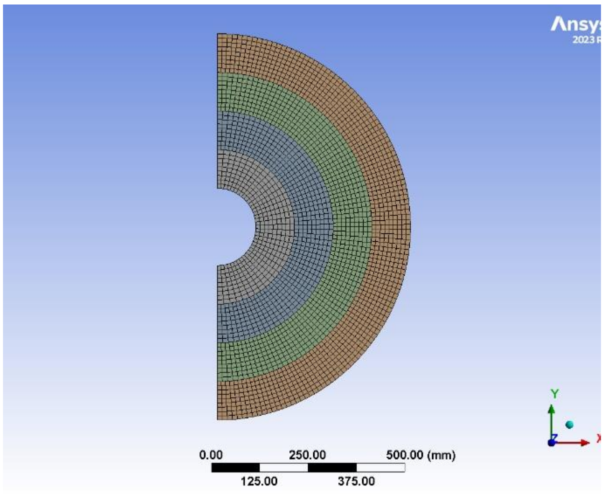

Fig. 2. The meshing of the model

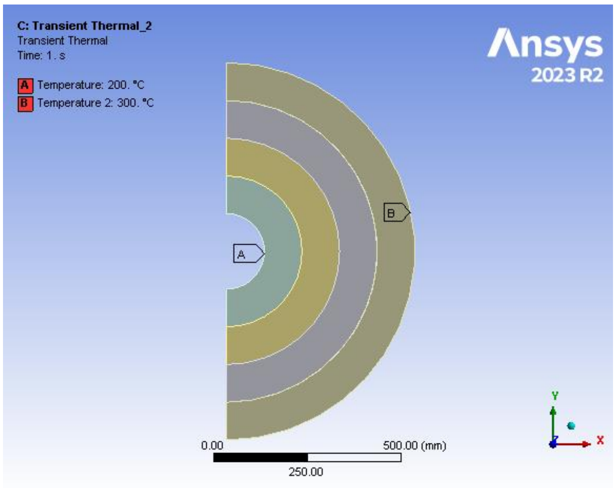

Fig. 3. The applied boundary conditions to the model

To ensure solution stability and accuracy, a grid independence test was performed. The spatial domain was discretized using non-uniform Chebyshev-Gauss-Lobatto points to cluster nodes near the boundaries where gradients are expected to be high. A mesh size of N=11 points per layer was found to be sufficient for convergence, as further refinement yielded negligible changes in temperature results (< 0.1%). The time integration was performed with a variable time step to capture the rapid initial transient response.

3. Results and Discussion

MATLAB software was used for the purpose of solving equations for the DQM and applying the boundary conditions represented by the inner radius of the cylinder of 0.1 meters and the outer diameter of 0.5 meters, and the temperature was applied in the inner layer at 200ºC and at the outer layer at 300ºC. Fig. 4 shows the distribution of temperature along the radius of the cylinder at several times during heat transfer – it can be seen that the edge conditions at the inner and outer surfaces, as well as the continuity of heat at the interlayer interfaces were achieved quite accurately in the numerical solution. In the first moments, the heat curve is low throughout the thickness (e.g. at about t = 0), then the temperature gradually increases over time in all internal positions with the cylinder boundary remaining at its specified values (200 at the inner surface and 300 at the outside). Over time, temperatures within each layer rise until they eventually reach a semi-stable state. Fig. 4 clearly shows that as time progresses from 0 to 100 and then 300 to 1000 seconds, thermal curves rise in each radius, indicating that the internal material gradually warms up and approaches stability.

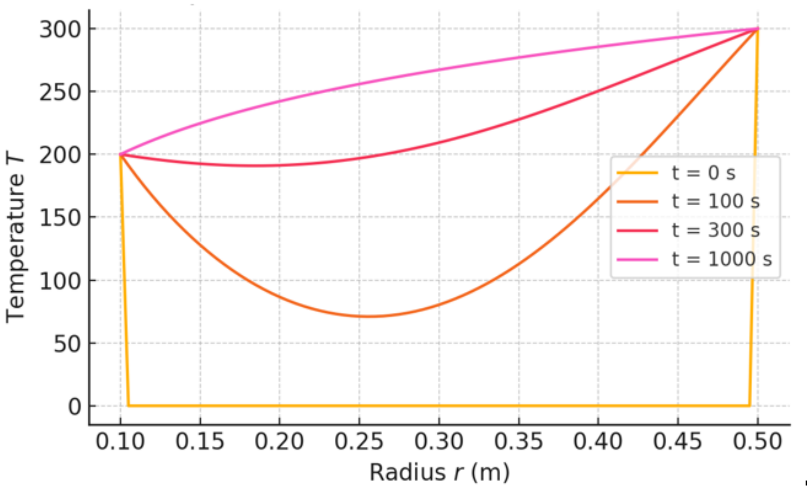

Fig. 4. Temperature distribution along the cylinder radius with different periods

Fig. 5 illustrates the temperature change over time in the middle of each of the four layers at radii (r=0.15, 0.25, 0.35, 0.45). The temperature at each of these points starts at a low value (close to zero in our reference state where we consider the inner material to be initially cold), then rises rapidly at first and gradually slows down as it approaches thermal stability. It can be seen about t≈800–1000 seconds, there are almost no temperature changes over time in all layers, which means reaching a steady state (constant temperature relative to time) that corresponds to physical expectations where heat balances across thickness and gradients become constant. Fig. 5 shows that the layer closest to the outer surface (at r=0.45 m) reaches a higher temperature (about 280–290) and precedes the inner layers in approaching their final value due to their proximity to the hot limit (300 ºC), while the layer closest to the cold inner surface (at r = 0.15 m) is the least hot (settles near 210 ºC) due to its proximity to the cold limit (200 ºC). In general, all layers converge to constant temperatures over time as seen in flat curves after a long time. It was noticed that the temperature rises rapidly at first and then the curves reach an almost constant value after a long time (thermal stability).

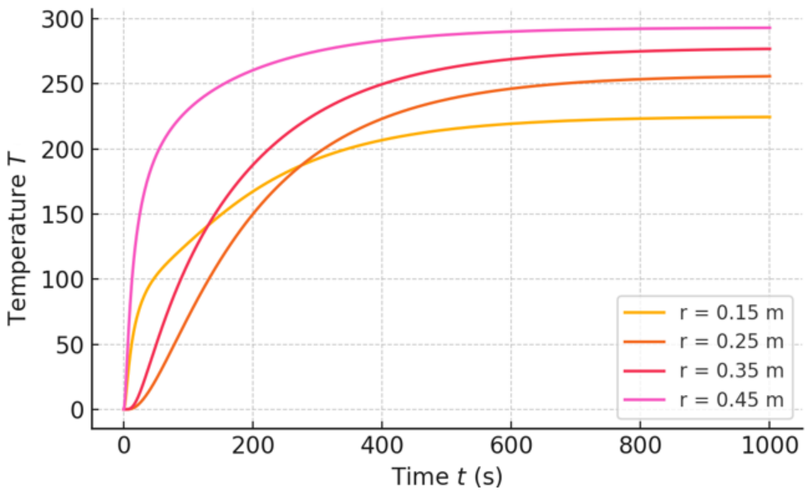

Fig. 5. Temperature distribution across time for each layer

After understanding the basic behavior, the effect of changing heat capacity, thermal conductivity, and layer thickness was studied using the same model. Here are the main findings: The base state (all layers have the same heat capacity) was compared with two cases, one in which the heat capacity of the inner layer is higher than the other, and the other where the heat capacity of the outer layer is higher as shown in Fig. 6. The diagram shows that increasing the heat capacity of the inner layer leads to a decrease in temperature near the inner radius compared to the baseline state, as the larger capacity inner layer absorbs more heat energy before it heats up, keeping it relatively colder. Similarly, increasing the heat capacity of the outer layer reduces the temperature near the outer surface because the outer layer stores more heat without significantly increasing its temperature. It can be seen from the orange curve (high internal capacity) in Fig. 6 that it is the basic yellow curve at the inner (about 0.1–0.2 m), and the pink curve (high external capacity) comes below the base at the outer (about 0.4–0.5 m). These results are consistent with the theoretical expectation that increasing the heat capacity of any layer delays its warming, making it instantaneously below the reference state in that region.

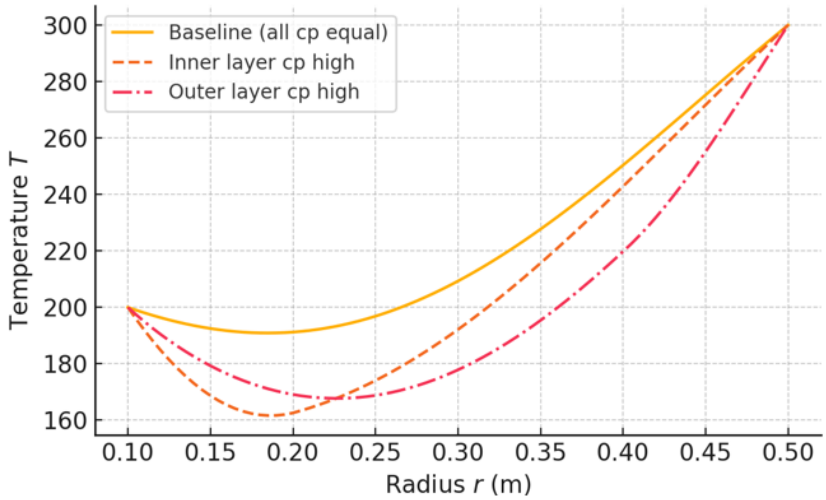

Fig. 6. The effect of heat capacity

Two cases were studied with regard to the thermal conductivity coefficient as in Fig. 7: increasing the conductivity coefficient of the inner layer (from 400 to 1200 W/m·K) with the rest of the layers remaining at the original value of 400, then increasing the conductivity coefficient of the outer layer (to 1200) while remaining 400. The diagram shows that raising the conductivity coefficient of the inner layer leads to a rise in temperature near the inner side, as the high-conductivity inner layer transfers heat more quickly than the adjacent layers, gaining more heat than the hot outer surface through the middle layers, raising its temperature from the base state. Similarly, an increase in the conductivity coefficient in the outer layer raises the temperature near the outer surface because the outer layer itself heats up faster when in contact with the hot surface and transfers heat more efficiently inside. In Fig. 7, the dashed orange curve (higher conductivity of the interior) lies above the basic yellow curve at areas near r=0.1–0.2 m, and the pink dashed curve–dot (higher conductivity of the outer layer) is above the yellow at r=0.4–0.5 m.

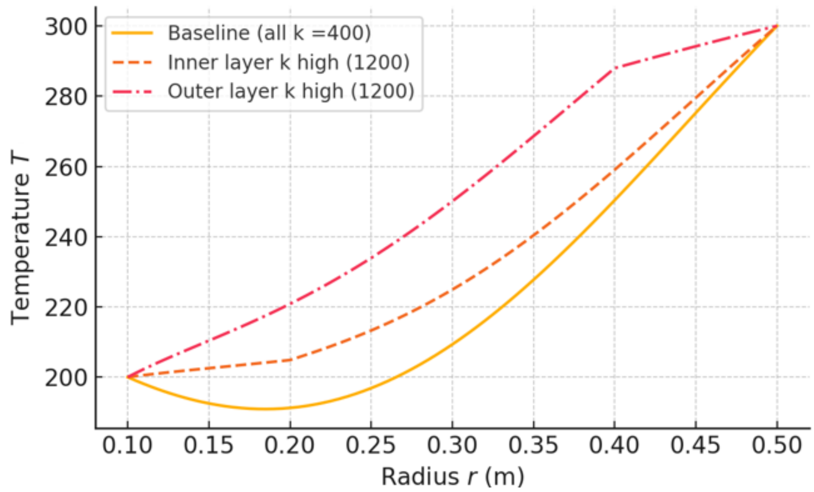

Fig. 7. The effect of thermal conductivity

A case was adopted in which the conductivity coefficient of the outer layer is high (1200) and the rest of the layers are 400 (as in Fig. 8), then the thermal distribution was compared when changing the thickness of the outer layer while keeping the total number of layers four. The basic case of this comparison is when the thicknesses of the layers are equal (0.1 m each from 0.1 to 0.5 m). The second case is when the outer layer is thicker (e.g. 0.2 m) and the three inner layers share the remaining thickness (0.2 m spread over three thinner layers). The third case is reversible: the outer layer is thinner (0.05 m) and the inner layers are thicker to compensate for the rest. Fig. 8 shows that when the outer layer of high conductivity is thicker, the temperature rises at all radii compared to the base state while reducing the thickness of the outer layer with high conductivity leads to a decrease in temperature at all radii.

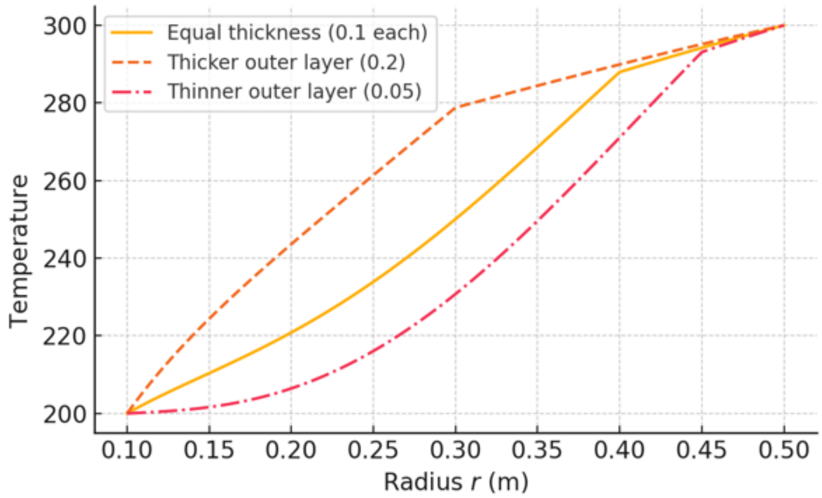

Fig. 8. The effect of different outer layer thicknesses

The reason is that the thicker the high-conductivity outer layer, the higher the overall thickness ratio with higher conductivity, so the heat moved more efficiently deeper inward, raising the temperature of all layers. When the high-conductivity outer layer is too thin, the bulk of the thickness is lower, slowing heat diffusion and lowering internal temperatures. In Fig. 8 the dashed orange curve (thick outer layer) is above the yellow curve (equal thicknesses) across the range, while the pink dashed curve-dot (thin outer layer) is below the yellow across most of the range, confirming the above conclusion.

Similarly, the case of the inner layer with a high conductivity coefficient (1200 W/m·K) and the rest of the layers was adopted as 400 W/m·K, and then the thermal distribution was compared when the thickness of this inner layer increased or decreased. Fig. 9 shows that increasing the thickness of the high-conductivity inner layer leads to a decrease in temperature across the radius (below the base state at all positions, while reducing the thickness of the outer layer (in the same case as the inner layer with high conductivity) leads to a rise in temperature at all positions. The explanation here is slightly different: when the high-conductivity inner layer is thick, it quickly transfers heat to the cold inner surface and dissipates it, keeping most of the thickness cooler (So the dashed orange curve under the yellow reference curve in Fig. 9). When the inner layer of high conductivity is of normal thickness but the outer layer becomes thinner (i.e. the part with low conductivity has become smaller), a larger part of the total thickness is of high conductivity (the inner layer itself + the layers with the highest conductivity expands outward), and the effect of external heating is transmitted more efficiently across most of the thickness, rising temperature everywhere (pink dashed curve-dot over yellow). It should be noted that the two cases of Fig. 9 show opposite but independent effects: increasing the thickness of the high-conductivity layer on the cold side cools the inside, while reducing the thickness of the layer on the hot side (or its equivalent by increasing the thickness of the inner layers at the expense of the exterior) heats the inside – which is exactly what the two comparisons achieve in the figure.

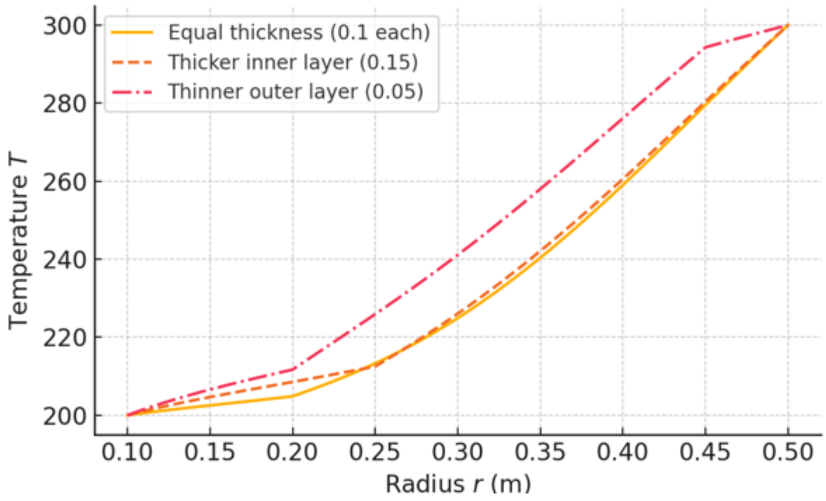

Fig. 9. The effect of inner layer thickness

The problem at hand was reconstructed using ANSYS software to simulate transient heat transfer in a multilayer cylinder under the same conditions as the original study that adopted the differential squaring method. These conditions were represented by a hollow cylinder with an inner radius of 0.1 m and an outer radius of 0.5 m, radially divided into four layers of equal thickness (0.1 m per layer). A constant temperature of 200 °C was applied to the inner limit, and to the outer limit of 300 °C uniform physical properties were adopted in all layers (density ρ=8000, kg/m^3., thermal conductivity k=400 W/m.K), with a uniform value of specific heat capacity, c_p. for all layers (c-p.=1000 J/kg. K). The simulation was run up to a time of t=100 seconds, and the temperature distribution across the radius was monitored at this time. To achieve spatial accuracy, a finely divided radial grid (each layer contains a sufficient number of elements) was used to ensure convergence of results, and emphasis was placed on calculating temperatures at selected locations (mid-layer at r = 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, and 0.5 m). The comparison between the ANSYS results and DQM is presented in Fig. 10 and

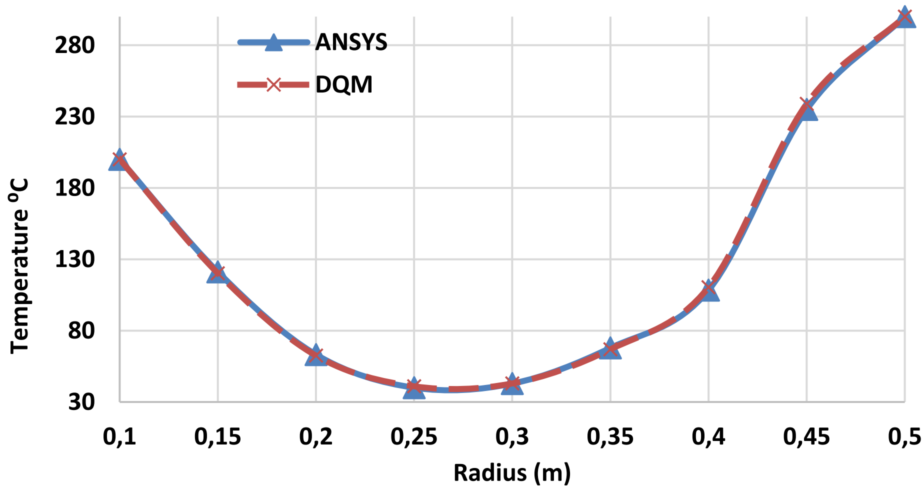

Fig. 10. Results comparison between ANSYS and DQM

The ANSYS temperature results are shown in Fig. 11. Table 1 shows the temperature values calculated for different radius values, by both the DQM approach and using ANSIS. Temperature values are found to be very similar in both approaches, with differences not exceeding multiple degrees, where the maximum error percentage reached 2.4 %. Figure 11 shows the temperature results generated by ANSYS simulation of a multilayer cylinder, with thermal distribution across the radius at 100 seconds.

Table 1. The Comparison between the results of ANSYS and DQM.

Radius (m)	T °C (ANSYS)	T °C (DQM)	Error (%)
0.1	200	200	0.00%
0.15	121.18	120.2	0.81%
0.2	63.229	62.5	1.15%
0.25	40.014	41	2.40%
0.3	42.889	43	0.26%
0.35	67.866	66.9	1.42%
0.4	108.34	110.4	1.87%
0.45	234.94	238.87	1.65%
0.5	300	300	0.00%

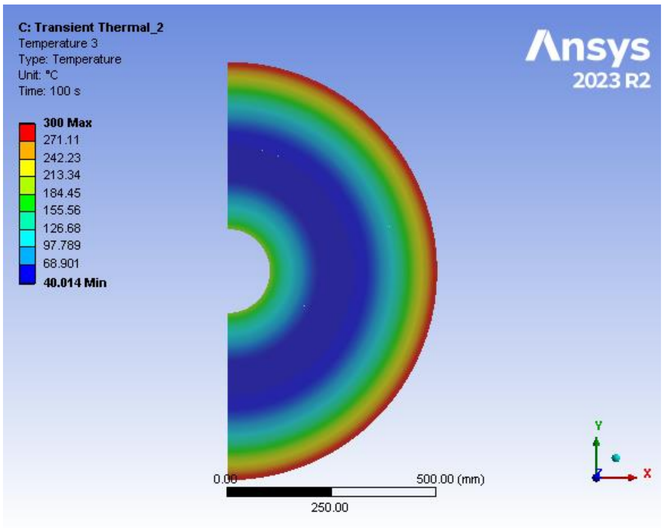

Fig. 11. ANSYS temperature results

4. Conclusions

This study successfully developed and validated a novel numerical approach for analyzing transient heat transfer in multilayered cylindrical structures using the Differential Quadrature Method (DQM). The investigation represents the first comprehensive application of DQM to solve the transient Fourier heat conduction equation in multilayered cylinders with distinct material properties, addressing a significant gap in the numerical heat transfer literature. The DQM implementation demonstrated exceptional accuracy and computational efficiency compared to conventional numerical approaches. The validation against ANSYS finite element simulations confirmed the reliability of the proposed model, achieving a high degree of agreement with maximum relative errors restricted to 0–2.4% across all radial positions. This validates the DQM as a powerful alternative to traditional Finite Difference or Finite Element methods, offering high precision with significantly fewer grid points.

Parametric studies provided profound insights into the thermal behavior of composite cylinders. It was observed that:

- Heat Capacity: Increasing the heat capacity of specific layers acts as a thermal buffer, effectively delaying the local temperature rise. Specifically, a higher capacity in the inner layer maintains lower temperatures near the core for longer durations.
- Thermal Conductivity: Variations in thermal conductivity create preferential heat flow paths. A high-conductivity outer layer accelerates heat ingress from the external boundary, raising the overall temperature profile of the cylinder more rapidly than modifications to inner layers.

- Layer Thickness: Geometric configuration plays a crucial role; increasing the thickness of insulating layers (low conductivity) significantly dampens the transient thermal response.
- Transient Behavior: All configurations demonstrated expected thermal evolution from initial conditions to steady-state, validating the physical consistency of the numerical model.

The novel contribution of this work lies in the algorithmic formulation that seamlessly handles interface conditions in multilayered media without the need for complex mesh generation techniques required by FEM. Future research will extend this DQM framework to handle non-linear heat conduction problems where material properties are temperature-dependent, as well as non-Fourier heat conduction models for micro-scale applications. The presented method provides engineers with a rapid and accurate tool for the preliminary design and optimization of thermal insulation systems in piping and aerospace structures.

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