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Research Article

Natural frequency analysis of laminated composite plates with lined cutouts using finite elements and machine learning

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Abstract

Laminated composite plates are extensively used in engineering applications because of their high strength-to-weight ratio and superior structural performance. However, stress concentration around cutout openings may reduce their durability and service life. Strengthening the cutout boundary through laminate lining offers an effective approach to improving structural integrity and vibration performance. This study investigates the fundamental natural frequencies of laminated composite plates with cutouts by comparing unlined and lined configurations, where the lining is achieved by increasing the number of laminate layers around the cutout edges. The lining factors are optimized using the Aqua Search meta-heuristic algorithm, while a topology optimization approach is employed to establish the convergence characteristics. The finite element formulation is based on a thin shell model without shear deformation using six-node linear strain triangular elements. Parametric analyses are performed under different loading conditions, including transverse and hygroscopic loading, and various boundary conditions, with particular emphasis on clamped laminated composite plates (CCCC). To validate the numerical results and evaluate predictive capability, five machine learning algorithms—Linear Regression (LR), Ridge Regression (RR), k-Nearest Neighbors (kNN), Random Forest (RF), and Gradient Boosting (GB)—are implemented in Google Colab®. Model performance is assessed using the coefficient of determination (R^2) and root mean square error (RMSE). Among the investigated models, Gradient Boosting demonstrates the highest prediction accuracy, achieving an R^2 value of 99.9% and an RMSE approaching zero. The non-dimensional natural frequencies predicted by the machine learning models are in excellent agreement with the finite element results, confirming the reliability of the proposed computational framework. The study demonstrates that cutout edge lining significantly enhances the vibration characteristics of laminated composite plates and that machine learning provides an efficient and accurate tool for predicting their dynamic behavior.

1. Introduction

The The use of laminated composites is inevitable in the field of Civil Engineering construction. The laminates are subjected to different types of loading such as mechanical loading, thermal loading, Hygroscopic loading, vibration loading and so on. The laminates are used with several different support conditions such as fixed (C), simply supported (S), hinged (H) and free (F) edges. The laminate has to perform in carrying the load as that of other construction material when it is used in a structure. The main focus of the present paper is to investigate the fundamental frequencies and mode shapes of laminates having cutouts with different boundary conditions. The laminate is discretized using six node linear strain triangular elements and the all the elements along the

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cutout edges are having different set of laminae compared to the rest of the laminate. This is to ensure the material will last longer and will not be subjected to wear and tear easily. This lining can be done in several ways such as increasing the number of layers along the the cutout edges, changing the moduli of elasticity of the material along the cutout edges. The present focus is only on increasing the number of layers and the modulus of elasticity in the elements along the cutout edges and investigate the fundamental frequencies and compare them with the fundamental frequencies of laminates without any change in the number of layers.

The increase in the number of layers for the elements along the cutout might pose a manufacturing constraint. The weaver has to increase the number of layers for the elements along the cutout edges. The manufacturer should be able to include this change during the manufacturing of laminates. The basic idea is to increase the life span of the laminate when used for a construction activity. The edge stiffening of the cutout can increase the resistance and improve its performance. This introduction will give a clear understanding of the proposed idea of lining of laminates by increasing the number of layers and modulus of elasticity for all elements along the edges of the cutout for different support conditions. The support conditions which are considered in this study are CCCC, SSSS, HHHH. Each type of structure as mentioned above is discussed in each of the following sub-sections. This section is outlined to efficiently describe information and basic concepts discussed in this study.

The recent papers on strengthening the open hole tensile strength have revealed that the structural performance can adversely get affected by failure at cutout or holes. Yu et.al., in 2022 [10] did his research work on using z-pins to prevent delamination damage. Experimental work on using z-pins has proved that the strain level at the notch can be reduced when compared with an unpinned specimen. There is cost factor involved to design and manufacture a z-pin and use it around the notch. There is also a chance where z-pins are not feasible to use such as wrapping a concrete member around an opening for increasing the strength of concrete member.

In this research work, we propose a new concept known as lining of laminates, where in the edges exposed in the cutout can be strengthened by increasing the number of layers along the cutout edges. The idea is similar to the edge of the cloth, wherein the edge of the cloth has a strength higher than the cloth and there will be no delamination damage along the exposed openings. The cloth edges are stronger and prevents the tearing along the edges. Similar to this, we can strengthen the edge of a laminate by lining the laminates along the cutout or open hole edges. The main advantage of this idea, it can be used anywhere and there are no practical constraints. The main limitation of this idea is that the manufacturer has to supply standard sizes and standard samples for testing in the laboratory. In the present study, we plot the curves to interpolate and determine the fundamental frequency for an optimal lining factor of material. The coding is done in ForTran® and MatLab® on an i7 processor laptop. The laptop has 4 cores and 8 logical processors with 8GB RAM, 1 TB hard drive and base speed of 2.59 GHz up to a top speed of 3.3 GHz. The characteristic curve clearly shows the convergence and which can be very useful to determine the lining factor for a desired frequency and vice versa. The lining is done along the edges of the cutout to increase the strength of laminate and which lowers frequency. The results are cross verified using AIML techniques. The input data is used to train machine learning algorithms. Five different machine learning methods have been used to validate our study and determine the performance metrics. Gradient boosting has been applied to determine the lining factors for other geometry ratios and predict the non-dimensional natural frequency of the laminate.

Based on prior published work, at this time of writing article experimental research has been done to determine the open hole tensile strength. The authors Gao et. al., in the literature have used z-pins arrangement to strengthen the area around the opening of a laminate. The limitation is that the thickness has to be considerable, and the use of z-pins needs equipment and skilled labor. This paper focuses on a low-cost alternative to strengthen the open holes tensile strength by providing more dense laminae around the cutout opening, this material with new material properties behaves similar to the edge of a cloth which has higher strength over the inner fibres of cloth. This idea to increase the laminae with suitable material can enhance the open hole strength of the laminate around the cutout opening.

1.1 Novelty of Work

The increased use of providing holes or openings in reinforced concrete members for pipes to pass through needs an additional provision to increase the strength at the section where material has been removed. The use of laminates to wrap around the openings is one such alternative to increase the load carrying capacity. The laminate is weaker around the cutout openings as shown in the Fig.1. There are investigations around the world to increase the open hole tensile strength of the laminate. This paper is focused on theoretical investigation of providing lining around the cutout area by increasing/changing the material of the fibres in the lining zone which is around the cutout opening. Lining zone is discussed in the subsequent sections in detail later.

1.2 Need for the Study

The laminates are widely in use in several areas of Civil Engineering. They are replacing the conventional materials due to their high strength and low weight. The laminates are also wrapped around the concrete members for increasing the strength of the member. They are provided around the openings in the concrete to reduce the stress carried by the concrete. The study on the behavior of the laminates is very important for a Civil Engineer. There are few issues with laminates being used as wraps around the openings in concrete. The laminate has to be teared to the same dimensions as the opening in the concrete. The cut edges are usually having threads which are set loose as a result the laminate cannot be glued the concrete member thoroughly and the strength of the member is not as it is expected to have. Similarly, these cutout openings have to be protected and we have to make sure that the laminate takes the load without failure at an early stage. Similarly, when a laminate having a cutout opening is used as a plate and when the load is applied on the laminate, the cutout edges are the first ones to tear and hence the entire laminate cannot be used to carry loads. In such cases, as well the cutout edges when they are strengthened, the laminate offers an increase in strength to carry loads. Hence, this present study is focused on lining of laminates used along with reinforced concrete members or as a transverse plate to carry loads. There is a definite need to study the laminate with the cutout edges being strengthened by increase the number of layers of laminae in the area close to the edges. When the edges are protected, then the material inside will start taking the load and the laminated plate can be used to carry the loads. The main focus of this research study is to apply the idea of lining of laminates along the edges of the cutout and determine the fundamental frequencies for different types of shells and plates with different support boundary conditions. We need to compare the fundamental frequencies of laminates with and without lining of laminates.

1.3 Aims and Objectives of the Study

Keeping in view of the requirement of the need for the study, the following aims and objectives have been set out.

- To find the non-dimensional natural frequency of the CCCC composite laminated shell with lining around the cutout opening
- To calculate a lining factor for a desired frequency of the CCCC laminated composite shell using aqua search meta-heuristic optimization algorithm
- To develop interpolation curves with different geometry ratios and determine non-dimensional natural frequency of the CCCC composite laminated plate with lining material along the cutout opening
- To apply AIML techniques and determine the non-dimensional frequency for a chosen a/b ratio, a/h ratio and lining factor and compare with the FEM result.

1.4 Scope of the study

- The study is within the scope of CCCC laminated composite plates with 0/90/0/90 lamina.
- The laminated composite plates are having eight layers of 0/90/0/90/0/90/0/90 for all the elements in the lining zone and along the cutout edges. The laminate is having four layers of 0/90/0/90 for the remaining portion of the plate.
- The non-dimensional fundamental frequency and mode shape with the maximum nodal displacement for the first mode of vibration is determined without increasing the moduli of

elasticity and also by increasing the moduli of elasticity for the elements along the edge of the cutout.

- The study is limited to linear elastic static analysis only.
- The Poisson ratio to remain constant and lining factor to remain constant for a given laminate.

1.5 Literature Review

The need of laminated composites in the field of Civil Engineering cannot be under emphasized. The use of laminated composites in the field of structural engineering has rapidly increased by replacing the structural outfits with laminated composite specimens due to their light weight, high strength to weight ratio, low corrosion, longevity, and low coefficient of thermal expansion [9]. Cutout edges of FRP composite with threads loosened and the bond is weak between concrete and composite laminate as shown in Fig.1. Laminated composites testing in the laboratory to determine the frequency as shown in Fig.2.

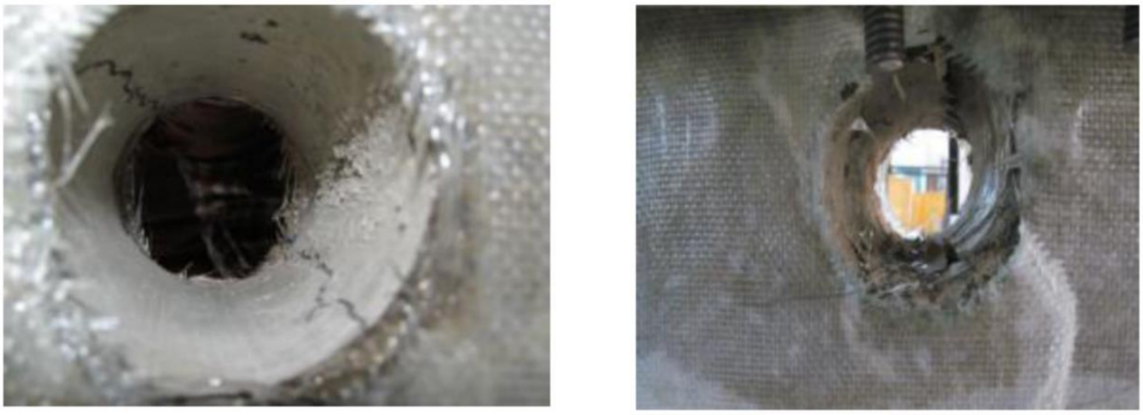
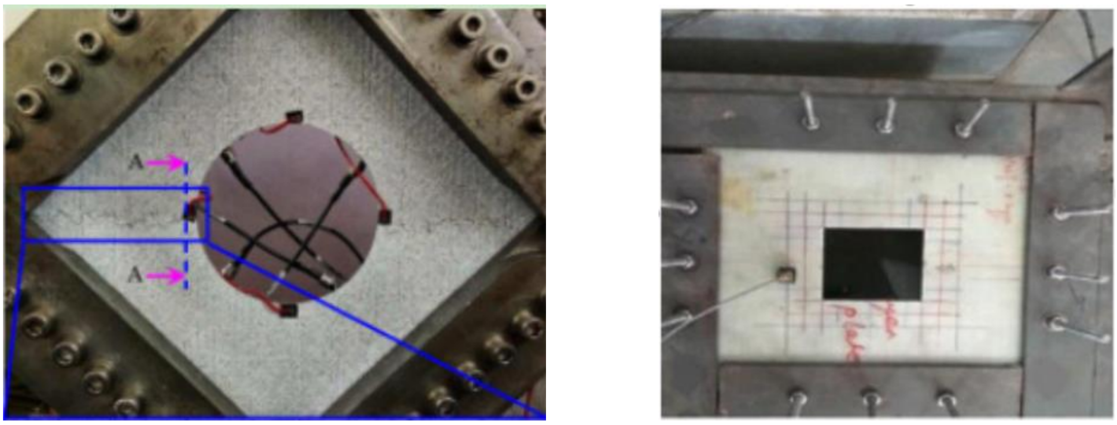


Fig. 1. Cutout edges of FRP composite with threads loosened and the bond is weak between concrete and composite laminate [9]



(a) Circular cutout [21] (b)rectangular cutout [22]

Fig. 2. Laminated composites testing in the laboratory

Cutouts are made to lighten the structure, to provide the access the different parts of the structure, changing resonant frequency, and for ventilation [6]. Sharmila Sahoo [1] did research on free vibration characteristics of stiffened composite hyperbolic paraboloid shell panel with cutout and presented the mode shapes. Eight node curved beam element is used to analyses cross ply and angle ply laminates. The study on natural frequencies can be used to develop design aids for structural engineers. This study provides the specific zones within which the loss of frequency is less than 5%-10% the frequency with a central cutout. Hachemi and Hamza-Cherif [5] present their research work on free vibration characteristics of laminated composite square plates having complicated cutouts using higher order shear deformation theory coupled with a curved quadrilateral p-element. The fundamental frequencies were studied for different characteristics of plate such as cutout shape, plate thickness, fiber orientation angle, and boundary conditions. Maharudra et.al. [2] focused on the buckling behavior of composite trapezoidal laminates panel with cutouts, subjected to nonuniform in-plane loads. The effect of various parameters such as cutout size, boundary condition, plywood orientation of panel is studied in greater detail. Ahmad et.al. (2013) [8] studied the effect of centrally located cutout and cracks emanating from the cutout on the free flexural behavior of the plate. The circular and the elliptical cutouts, crack length, thermal gradient with different boundary conditions have been studied in detail. The properties of functionally graded material are evaluated using Mori-Tanka scheme and the plate kinematics is based on the first order shear deformation theory. Sundarajan and Pratik [3] studied the effect of moisture concentration and thermal gradient on free vibration characteristics and buckling of laminated composites. The effect of plate geometry, cutout geometry, moisture concentration, thermal gradient and boundary condition is numerically studied. Tarun Kant and Swaminathan [18] in the year 2001 presented an analytical solution to find the natural frequency of the laminated composite. Sharmila Sahoo [1] did her research work on free vibration characteristics of stiffened hyperbolic laminated composite plates having cutouts in different shapes. Eight nodded curved shell elements with a three-node curved beam element is used to model the laminated composite shell panel with different shapes and sizes of cutouts. The effects of these parameters on fundamental frequencies are considered in detail. The design aids for laminated composites can be prepared for structural engineers. The practicing engineers can use these results when designing a cutout opening in the laminate.

Aydin Komur et.al., [4] did their research study on buckling analysis of a woven glass polyester laminated composite plate with a hole. An elliptical/circular hole is applied and tested for various plates both in shape and position of elliptical hole. They found that the increasing of hole positioned angle caused to reduce the buckling load. Mohammed Salih et.al., [7] did their study on free vibration analysis of perforated laminated composite plates. They worked on three support conditions namely SSSS, CCCC, SCSC. They effect of number of holes and hole area ratio, lamination angles and boundary conditions on vibration has been investigated in greater detail. The lamination angle has a clear effect on the change in the natural frequency of vibration.

The open hole tensile strength can be increased by using the lining of laminates to strengthen the edges, which is similar in blue color along the edge as shown in the image given here. This is quite suitable for the case of thin laminates. For example, the laminated composite wrap around the opening of a concrete beam is a thin laminate. The laminated composite. Oskana and Andrey [11] did their research work on prediction of stress strain state of a rectangular plate with a circular cutout. They used principal component analysis and convolutional neural networks. They estimated the mean square error loss function for both train data set and test data set up to 500 epochs. Input and hidden layers use hyperbolic tangent function as activation. Eight neurons are used to process float data and 76 neurons are used to process categorical data. Lakshmi Narayana et.al. [12] did their study on thermal buckling analysis of composite laminated plates having square or elliptical openings. They have used a 16-ply quasi-isotropic graphite epoxy symmetrically laminated composite plates. They found that for a rectangular laminated composite plate having a elliptical cutout opening, the magnitude of thermal buckling increased as the cutout orientation has increased from 0 to 90 degrees. Yassine El Kiri et.al., [13] developed an artificial neural network to predict the external corrosion, predict the burse pressures thus avoiding the need for complex finite element analysis anymore and any further. The results indicated that the parabolic shape is the most crucial parameter and is generally more detrimental to the structure. Agim Seranaj et al., [14] did present their research work to find the optimal size and position of openings in the reinforced concrete structure based on stress, displacement, and boundary cases. Several examples on the optimization of shells with advanced algorithms is done to validate their research work. Their research has reduced the weight of the structure by nearly 30% - 40% by providing openings in the structure. Vidhya Ramesh et.al., [15] developed a code using artificial neural networks to predict the compressive strength of a sustainable recycled concrete. Recycled aggregates concrete and treated textile waste water are used for eco-friendly concrete. Training and testing of model are done using experimental data compiled on eco-friendly concrete. The validation of the model is done by comparing the experimental results with the ANN predictions and these are found to be close.

Benbokhari Abdellatiff et al., [16] used artificial neural networks (ANN) to predict the seismic response of structures subjected to earthquake events. Three ANN models were used which are a hybrid of both supervised and unsupervised learning. Auto encoder ANN performed better than principal component analysis with ANN, PCA-ANN and ANN models. They are able to quickly and accurately predict the seismic responses of structures for unforeseen ground motion using only the building characteristics. Arun et.al. [17] used several machine learning algorithms, specifically Random Forest (RF), Gradient Boosting (GB) and Extreme gradient boosting (EGB) to predict the compressive strength is the main focus of their study. They judged their model's performance by calculating mean average error (MAE), root mean square error (RMSE) and R^2 values. The developed a powerful tool to predict the compressive strength in various scenarios. Gao et.al., [10] did their research study on designing and fabricating Z-pins to improve the open hole strength of a laminated composite. The cutouts cause damage to structural performance and their study of the behavior and response of the laminated composite having cutout openings is inevitable. Gao et.al., proposed fine Z-pins arrangement around the cutout opening to increase the strength of the opening when the CFRP laminated composite is subjected to axial loading as shown in Fig.3. The open hole in the laminate will definitely reduce the strength bearing capacity and the laminate will fail at the opening. They tested using fine z-pin made of T800S fibres having about 240 individual fibres with the volume fraction of 62% and the diameter of single filament is 5 μm. ASTM D3039 standard is used to conduct the tensile test of the specimens. The use of z-pins has increased the open hole strength by 9% and 54.1% reduction in the delamination damage area.

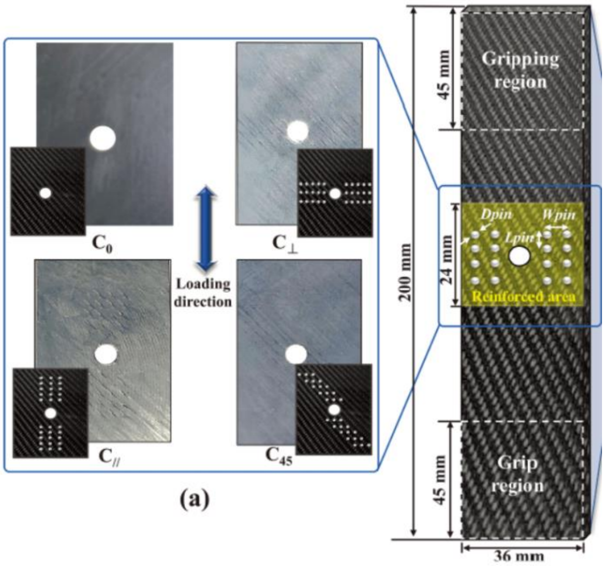


Fig. 3. Dimension of Z pinned open hole tensile (OHT) specimens prior to damage [10]

2. Methodology

The proposed way is to increase the stiffness and density of the elements along the length of the edge of cutout in order to reduce the displacement produced due to vibration. The laminated composite plate is having a dimension of 80 x 80 square and the dimension of cutout opening is 30 x 30 square at the center of the plate. The entire plate is meshed using first order six node linear strain triangular elements. The elements along the edge of the cutout are having eight layers of lamina 0/90/0/90/0/90/0/90 and the moduli of elasticity are multiplied by 20. The remaining portion of the laminated composite plate has four layers of lamina 0/90/0/90 as shown in the Fig.4. The dynamic analysis is performed. The stiffness matrix, mass matrix is calculated for each element and then assembled to obtain a global stiffness matrix and global mass matrix. The flowchart to perform the static analysis is as shown in the Fig.5. The coding in developed using VC++, ForTran® and Matlab® on Intel® Core™ i7-6700HQ CPU @ 2.60GHz.

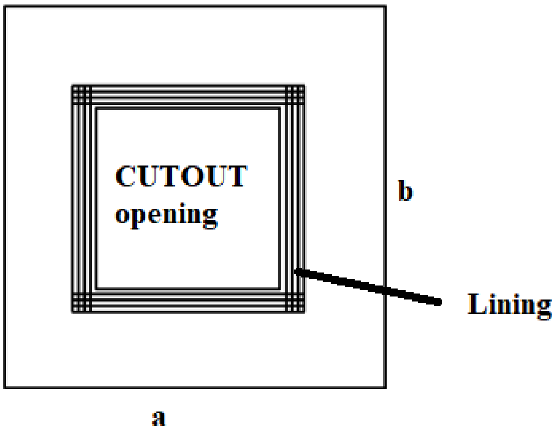


Fig. 4. Lining of laminate around the cutout edges

- Find the fundamental frequency, with usual notations;

$$\left|K - M\omega^2\right| = 0 \tag{1}$$

- Non-dimensional frequency;

$$\varpi = \omega\ a^2 \sqrt{\frac{\rho}{E_{YY}\ h^2}} \tag{2}$$

- The mode shape can be determined using eigen vector obtained by the relation

$$Kx = \omega^2 Mx \tag{3}$$

Determine the Optimal lining factor for a desired frequency with usual notations, $K = M\,\omega^2$ and $\varpi = \omega\ a^2\ (\rho/E_{22}h^2)^{1/2}$. Where; 'K' is the stiffness matrix and 'M' is the mass matrix, 'ω' is the natural frequency, 'ϖ' is the non-dimensional frequency (NDF), 'a' is the length of each side, 'h' is the thickness of the laminate. The variable is lining factor for each iteration and determine the natural frequency.

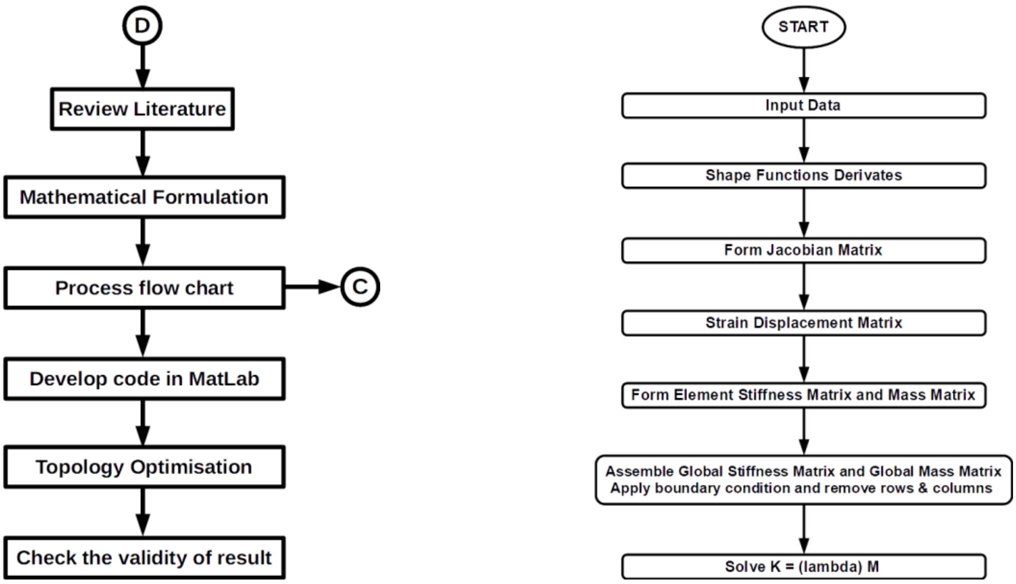


Fig. 5. Flow chart for dynamic analysis

Widely known well accepted topology optimization approach is used to determine the lining factor convergence curve using aqua search meta-heuristic algorithm. The procedure to perform can be found in author's earlier works [23]. Optimization is not the main content of this paper; the authors prefer to keep this to a minimum discussion.

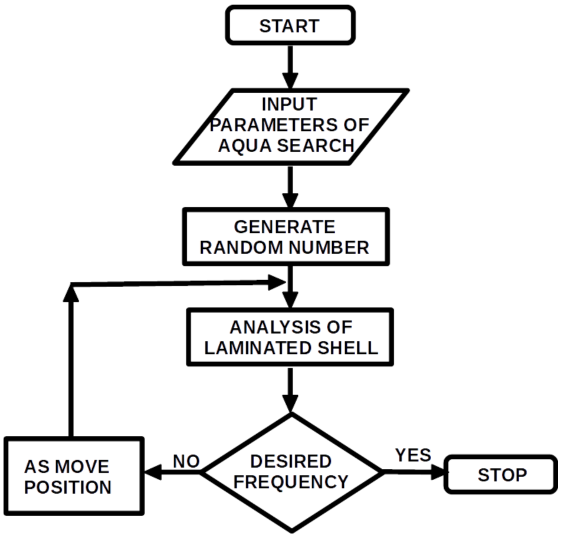


Fig. 6. Optimization process

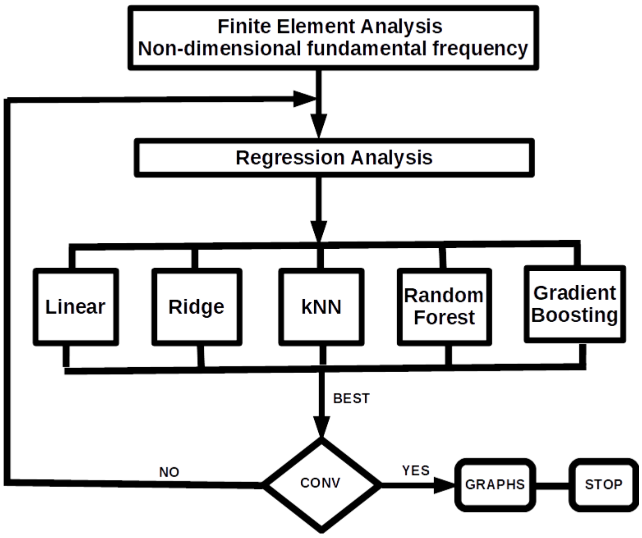


Fig. 7. Flow chart showing Artificial Intelligence and Machine Learning methods

In Fig.6 the random number is generated between from 0 to1. AS indicates aqua search algorithm to generate newer value of the lining factor. The diver moves towards another diver with the better value of objective function. The increase in the value of the lining factor results in the decrease in the natural frequency. In Fig.7 the flow chart shows the regression analysis using five different machine learning algorithms.

2.1 Theoretical Background

2.1.1 Aqua Search Meta-Heuristic Algorithm

Aqua search is a meta-heuristic algorithm, which is based on underwater sea divers searching in the deep sea. The sea is usually deep and is dark everywhere. The diver will have contact with the base station above using a wire or a wireless GPS device. The diver can use his instinct when choosing the search direction and this instinct helps in making decisions. The visibility of the water in the deep sea is very less and the diver usually has a light over his head which will have certain visibility. The visibility is different for different types of water sometimes clear sometimes not clear. The divers can exchange the information between them with an electronic device. The base station at the surface of the sea keeps track of every movement of the diver. The coordinates of the location and the direction of searching and the step length as well. The base station maintains interactions with everyone.

A weight is assigned to each water body depending on the type of the water body. Some water bodies are easy to search while some of them are not having clear water or have rocky terrain inside the deep sea. The design domain to search is the water body, the divers are the search particles and the search object is the point where all divers ideally should converge. Sometimes, all the divers may not converge a few of them might converge and others might not. There are cases when the divers are searching in several different directions and cannot reach the convergence point.

The cartesian distance can be calculated similar to the firefly algorithm. There are instructions sent from the base station to the divers. The diver can follow these instructions or use his instinct to make the search direction and take the search step length as well. This factor can is the factor1 which is a base station factor, value close to zero means the diver follows whatever the instructions he receives from the base station. If factor1 is taken as equal to one, then the diver goes by his instinct and his experience and he gives less weightage to the instructions received from the base station. When diver chooses to go by his experience and instinct, then there is another term which accounts for the instinct in the second term. This term is obtained by multiplying the factor1 with a gamma distribution, if needed we can use a random function as well.

The third factor is visibility in the deep sea. The water body is a deep sea, the water may not be clear and the visibility may be poor. The visibility of the human eye is clear up to a distance of 25 cm nearly. The visibility can be accounted by using an F-distribution. The value of degrees of freedom 'm' in the numerator can be taken as equal to 12. The value of the degrees of freedom 'n' in the denominator is taken as equal to 20. The value of sign is taken as positive when a randomly generated number is greater than 0.5. The value of the sign is taken as negative when the a randomly generated number is less than 0.5. This aqua search meta-heuristic algorithm has very few terms to model the equation of movement of the divers and search for the object in the underwater deep sea.

$$term1 = (1 - factor1) * (1 + RR)^{-t}$$

$$where\,(1 - factor1)\ is\ the\ base\ factor\ and\ varies\ from\ zero\ to\ one$$

$$\text{Cartesian distance, RR} = \sqrt{\sum_{k=1}^{d} \left(x_{i,k} - x_{j,k}\right)^2} \tag{4}$$

$$term2 = factor1 * rand(0,1)$$

$$term3 = Fd(t) * sign$$

The coordinates of the diver can be identified using an vector x. The size of the vector is the number of unknowns to be determined. The vector set of all x_i at any iteration can be useful to know the distribution of the material inside the design domain which is shown as a water body.

Fd(t) is the F-distribution where 't' is the iteration number. The degrees of freedom in the numerator is m may be taken as equal to 12 and the degrees of freedom in the denominator is 'n' may be taken as equal to 20. Sign is taken as positive unity when a randomly generated is greater than 0.50. Sign is taken as negative unity when a randomly generated is less than 0.50. The new position x_j for each diver can be determined by using the relation;

$$x_j = x_i + (term2 + term1) * (x_j - x_i) + term3 \tag{5}$$

The objection function at each diver is calculated and compared. They have electronic means to exchange the information. The minimum value of the objective function is the diver towards whom all the remaining divers will like to move. The direction and step length can be calculated using the relation given above. Aqua search deep sea search algorithm is very fast converging when compared with other meta-heuristic algorithms such as firefly algorithm.

2.1.2 Lining Zone

The layers of lamina are arranged in the lining area as shown in the Fig. 8 given below. The threads in the 90 directions can be closely spaced making the lining area denser. In the Lining area, the lamina is taken as 0/90/0/90/0/90/0/90 and the moduli of elasticity are multiplied by the lining factor. Lining factor is the ratio of the moduli of elasticity of the material in the lining zone to the modulus of elasticity of the material inside the material domain of the laminate. In the lining zone, the modulii of elasticity is taken as a multiple of lining factor.

- Exx lining zone = 130E3 * Lining factor
- Eyy lining zone = 8.5E3 * Lining factor
- Gxy lining zone = 6E3 * Lining factor
- Gxz lining zone = 6E3 * Lining factor
- Gyz lining zone = 3E3 * Lining factor

In other words, the theoretical assumption made here is given below here as follows:

- Modulus of elasticity of the element at the openings = Lining factor * Modulus of elasticity of the element.

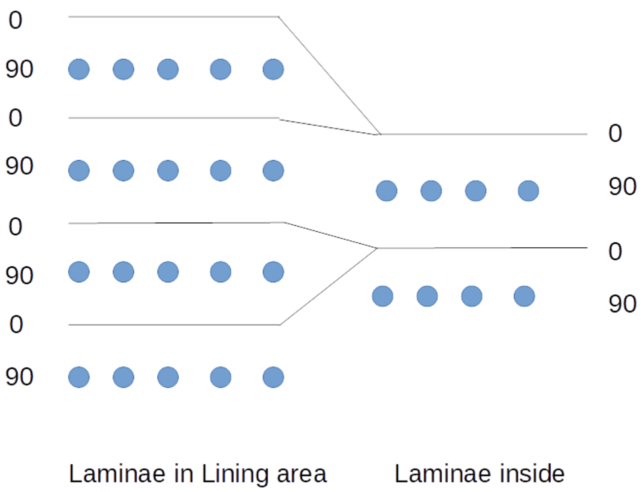


Fig. 8 Arrangement of 0/90/0/90/0/90/0/90 Lamina in the lining area

2.1.3 Six Node Triangle Diagram

Six node linear strain triangle elements and shape functions for 6N LST [20];

$$N_1 = \zeta(2\zeta - 1)$$
$$N2 = \xi(2\xi - 1)$$
$$N3 = \eta(2\eta - 1)$$
$$N4 = 4\zeta\xi$$
$$N5 = 4\xi\eta$$
$$N6 = 4\zeta\eta$$

$$\frac{\partial N_1}{\partial \xi} = 4\xi - 1 \qquad \frac{\partial N_5}{\partial \xi} = -4\eta \qquad \frac{\partial N_3}{\partial \eta} = 4\xi + 4\eta - 3$$

$$\frac{\partial N_2}{\partial \xi} = 0 \qquad \frac{\partial N_6}{\partial \xi} = 4 - 8\xi - 4\eta \qquad \frac{\partial N_4}{\partial \eta} = 4\xi$$

$$\frac{\partial N_3}{\partial \xi} = 4\xi + 4\eta - 3 \qquad \frac{\partial N_1}{\partial \eta} = 0 \qquad \frac{\partial N_5}{\partial \eta} = 4 - 4\xi - 8\eta$$

$$\frac{\partial N_4}{\partial \xi} = 4\eta \qquad \frac{\partial N_2}{\partial \eta} = 4\eta - 1 \qquad \frac{\partial N_6}{\partial \eta} = -4\xi$$

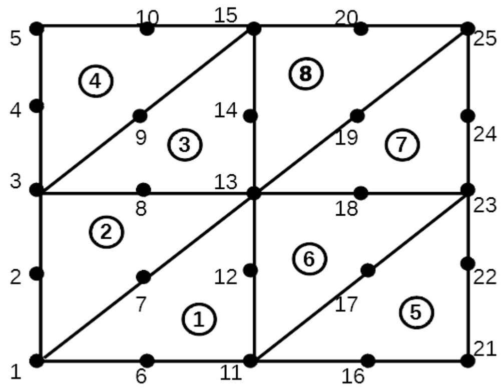
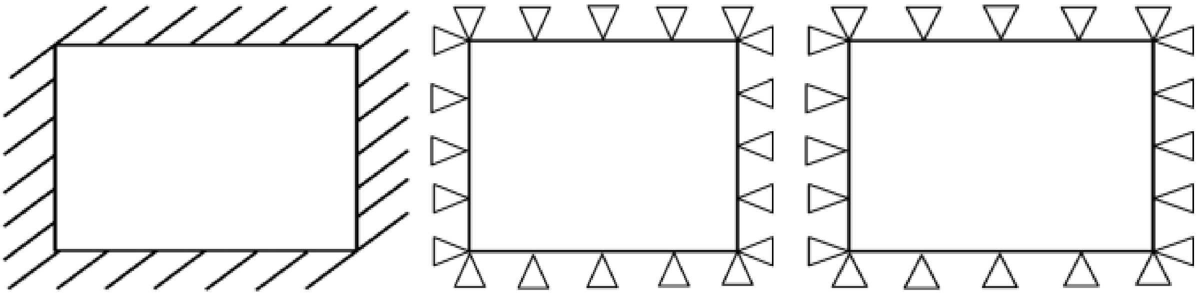


Fig. 9. Showing the finite element mesh and numbering of 6N LST element

The boundary conditions for CCCC numerically in the program are given as 11111 for each node along the boundary of composite laminate. The numerical representation of SSSS boundary condition is 10110, 01101 along x and y axis respectively. The numerical representation of HHHH boundary condition is 11110 and 11101 at each node along x and y axis respectively.



(a) (b) (c)

Fig. 10. Support conditions of the plate (a) CCCC, (b) SSSS, and (c) HHHH

2.1.4 Formulation of Laminated Plates

Let Ω be the domain in R^2 by the mid plane of the plate and u, v, w denote the displacement components in x,y,z directions respectively. Using the Kirchoff theory the displacements at any point can be expressed as [19];

$$
\begin{aligned}
u(x,y,z) &= u_o(x,y) - z\theta_x(x,y) \\
v(x,y,z) &= v_o(x,y) - z\theta_y(x,y) \\
w(x,y,z) &= w(x,y) \\
\text{where}, \ \theta_x &= \frac{\partial w}{\partial x} \quad \text{and} \quad \theta_y = \frac{\partial w}{\partial y}
\end{aligned}
\tag{6}
$$

In-plane strains through the following equation;

$$
\varepsilon = \left[\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}\right]^T = \varepsilon_o + z\kappa \tag{7}
$$

where, ε_o, κ are the in-plane deformations and curvatures of the middle surface respectively;

$$
\varepsilon_o = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} u_o \\ v_o \\ w \end{bmatrix} \quad \text{and} \quad \kappa = \begin{bmatrix} 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & 0 & -\frac{\partial^2}{\partial y^2} \\ 0 & 0 & -2\frac{\partial^2}{\partial x \partial y} \end{bmatrix} \begin{bmatrix} u_o \\ v_o \\ w \end{bmatrix} \tag{8}
$$

Strain displacement matrix is given in Eq. (7). In case of a cylindrical shell, the radius of curvature, R_{xx} is infinity and as curvature, C_{xx} is the reciprocal of the radius of curvature, R_{xx} which means that $C_{xx} = 0$. For the sake of our formulation, we consider $C_{yy} = -1/R_{yy}$. E_{11} and E_{22} are Young's Modulus of Elasticity parallel and perpendicular to the fibre orientation. G_{12} is the shear modulus and are υ_{12} *and* υ_{21} are Poisson' ratios. As shown in the Fig.11, the layers of lamina are given with the distance is measured from center of lamina.

$$b = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & 0 & -\frac{\partial^2}{\partial y^2} \\ 0 & 0 & -2\frac{\partial^2}{\partial x \partial y} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{N_i}{R_x} \\ 0 & 0 & \frac{N_i}{R_y} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & -N_i C_{xx} \\ 0 & \frac{\partial}{\partial y} & -N_i C_{yy} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial^2}{\partial x^2} \\ 0 & 0 & -\frac{\partial^2}{\partial y^2} \\ 0 & 0 & -2\frac{\partial^2}{\partial x \partial y} \end{bmatrix} \tag{9}$$

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\ \overline{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta+\cos^4\theta) \\ \overline{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta + (Q_{12}-Q_{22}+2Q_{66})\sin^3\theta\cos\theta \\ \overline{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\ \overline{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta + (Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta \\ \overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta+\cos^4\theta) \\ Q_{11} &= \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}}, \quad Q_{12} = \frac{\upsilon_{12}E_{11}}{1-\upsilon_{12}\upsilon_{21}} \\ Q_{22} &= \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}}, \quad Q_{44} = G_{13} \\ Q_{55} &= G_{23}, \quad Q_{66} = G_{12} \\ \upsilon_{21}E_{11} &= \upsilon_{12}E_{22} \end{aligned} \tag{10}$$

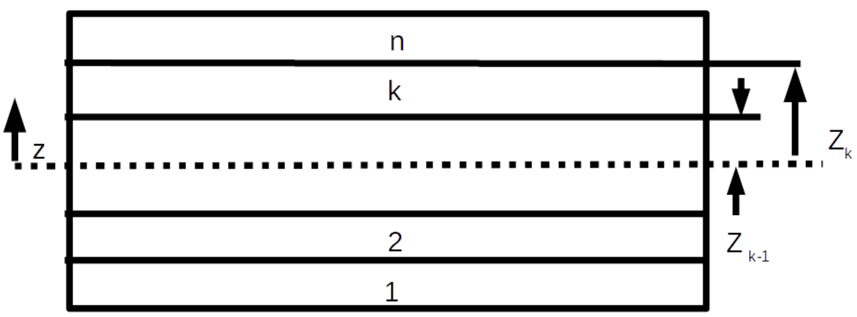


Fig. 11. Showing the layers of lamina

$$[E]=\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \tag{11}$$

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right) \tag{12}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right) \tag{13}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right) \qquad \text{i, j} = 1,2,6 \tag{14}$$

The value of z is measured from the center of the lamina as shown in the Fig. 11. The physical and the displacement field are expressed as;

$$x^h(\xi,\eta) = \sum_{A}^{nm} R_A(\xi,\eta) P_A \tag{15}$$

$$u^h(x(\xi,\eta)) = \sum_{A}^{nm} R_A(\xi,\eta) q_A \tag{16}$$

The strains can be expressed to following nodal displacements as;

$$[\varepsilon_o \quad \kappa]^T = \sum_{A}^{nm} \left[B_A^m \; B_A^b\right]^T q_A \tag{17}$$

$$B_A^m = \begin{bmatrix} R_{A,x} & 0 & 0 \\ 0 & R_{A,y} & 0 \\ R_{A,y} & R_{A,x} & 0 \end{bmatrix} \quad and \quad B_A^b = \begin{bmatrix} 0 & 0 & -R_{A,xx} \\ 0 & 0 & -R_{A,yy} \\ 0 & 0 & -2R_{A,xy} \end{bmatrix} \tag{18}$$

When the radii of curvature are equal to infinity in case of straight edge, and when the curvature is equal to zero, B_A^m and B_A^b are membrane and bending strain displacement matrices using the derivatives of the function.

Global stiffness matrix $$K = \int_{\Omega} \left\{ \begin{bmatrix} B^m \\ B^b \end{bmatrix}^T \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} B^m \\ B^b \end{bmatrix} \right\} d\Omega \tag{19}$$

Force vector is $$f = \int_{\Omega} pR d\Omega \tag{20}$$

For sinusoidal loading, $$p = p_0 \sin(\pi x / a) \sin(\pi y / b) \tag{21}$$

Where, q is the global displacement vector

2.1.4.1 Hygroscopic Analysis

The formulation of to determine the force vector is given here below.

$$\text{Strain Tensor} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = [T] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix} \tag{22}$$

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \frac{1}{2}\beta_{xy} \end{bmatrix} = [T^{-1}] \begin{bmatrix} \beta_1 \\ \beta_2 \\ \frac{1}{2}\beta_{12} \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\ \sin\theta\cos\theta & -\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \frac{1}{2}\beta_{12} \end{bmatrix} \tag{24}$$

Where, $\frac{1}{2}\alpha_{12} = \frac{1}{2}\beta_{12} = 0$

$$\text{Hygro strain} = \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} = \begin{bmatrix} \Delta C\beta_x \\ \Delta C\beta_y \\ \Delta C\beta_{xy} \end{bmatrix} \tag{25}$$

Total strain = Mechanical strain + Thermal Strain + Hygro strain;

$$\begin{bmatrix} \varepsilon_x^{Tot} \\ \varepsilon_y^{Tot} \\ \gamma_{xy}^{Tot} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^{Mech} \\ \varepsilon_y^{Mech} \\ \gamma_{xy}^{Mech} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Th} \\ \varepsilon_y^{Th} \\ \gamma_{xy}^{Th} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} \tag{26}$$

The total strain is given as;

$$[\varepsilon^{Tot}] = \begin{bmatrix} \varepsilon_x^{Tot} \\ \varepsilon_y^{Tot} \\ \gamma_{xy}^{Tot} \end{bmatrix} = \begin{bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Th} \\ \varepsilon_y^{Th} \\ \gamma_{xy}^{Th} \end{bmatrix} - \begin{bmatrix} \varepsilon_x^{Hy} \\ \varepsilon_y^{Hy} \\ \gamma_{xy}^{Hy} \end{bmatrix} \tag{27}$$

The stress is given as;

$$[\sigma] = [\overline{Q}][\varepsilon^{Tot}] = [\overline{Q}](\{\varepsilon^0\} + z\{k\} - \{\varepsilon^{Th}\} - \{\varepsilon^{Hy}\}) \tag{28}$$

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ k \end{bmatrix} = \begin{bmatrix} N^{Th} \\ M^{Th} \end{bmatrix} + \begin{bmatrix} N^{Hy} \\ M^{Hy} \end{bmatrix} = [F^N] \tag{29}$$

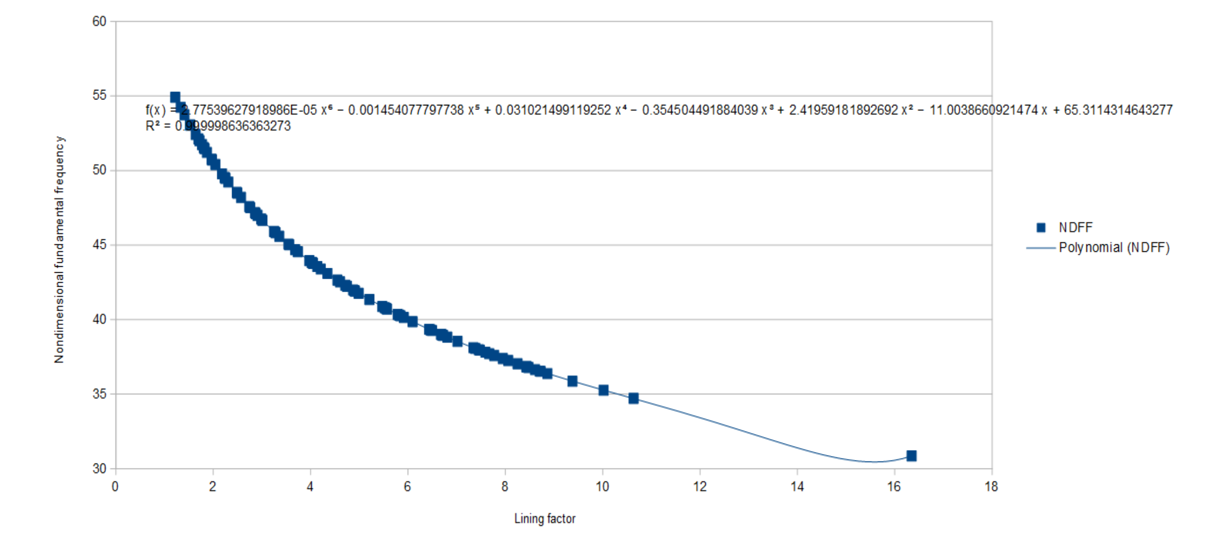
$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} \left(1, z, z^2\right) \overline{Q}_{ij}\, dz \quad i, j = 1, 2, 6 \tag{30}$$

The element level nodal load vector due to hygroscopic forces is given as $\iint [B]^T \{F^N\} dxdy$

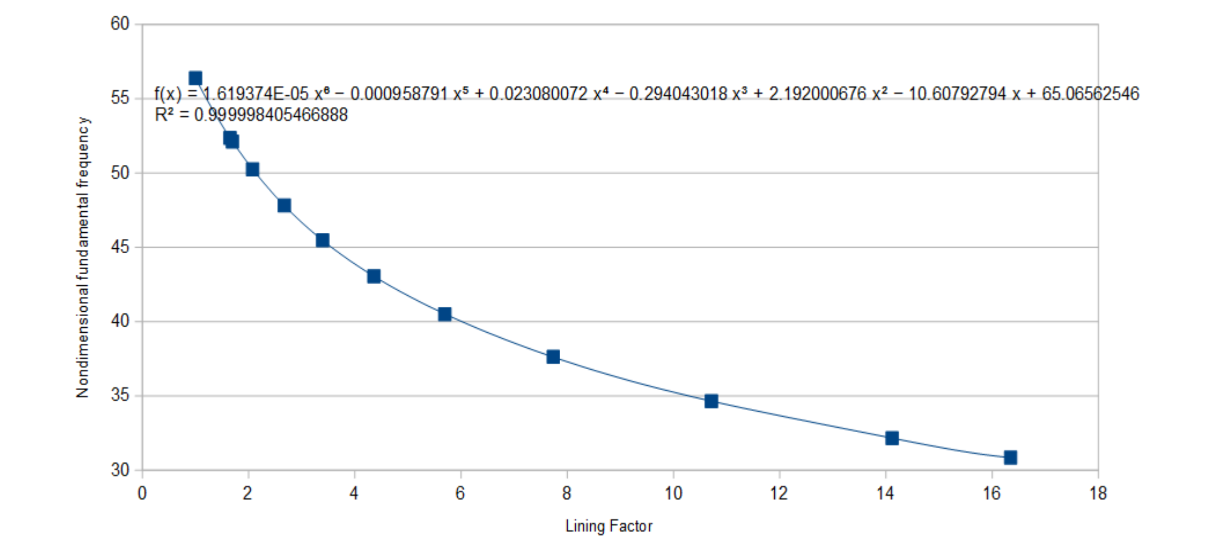
3. Aqua Search V/S Firefly Algorithm

3.1 Problem Statement to Compare Fire Fly Algorithm and Aqua Search Meta-Heuristic Algorithm

CCCC laminated composite conoidal shell is having a size of 80 x 80 in plan. The laminate is discretized using six node linear strain triangular elements. The total number of elements are 440 and total number of nodes are 960. A cutout of dimensions 30 x 30 is centrally placed. The elements surrounding the cutout having a width of 2.5. The number of laminas is four with the sequence of 0/90/0/90.



(a)



(b)

Fig. 12. (a) Fundamental frequency curve for CCCC laminated composite conoidal shell using firefly algorithm using VC++, (b) Fundamental frequency curve for CCCC laminated composite conoidal shell using aqua search algorithm using VC++

The material of the laminate in the lining zone for a width of 2.5 around the cutout is optimized for a desired frequency of the laminate. The lining factor is a multiple of the modulus of elasticity and is determined using Aqua Search meta heuristic algorithm. The objective function is to find the non-dimensional natural frequency of the laminate having the lining along the edge of the cutout. Aqua search (AS) meta-heuristics algorithm is used to determine the optimal lining factors to plot a non-dimensional frequency curve for a given set of geometry ratios. Interpolation curve to determine the non-dimensional frequency using Firefly algorithm is as shown in Fig. 12(a). The validation of aqua search meta-heuristic algorithm is done using a conoidal shell. Conoidal shell is considered as the benchmark structure to verify the results obtained from various algorithms used in the analysis of shell structures. Aqua search meta-heuristic algorithm gave better results with least number of search divers in the least possible time. Firefly algorithm took 5 search flies and 92 iterations and the program continued for nearly 21 hours. Aqua search took 2 search divers and 12 iterations and the program continued for nearly 144 minutes. The convergence criteria are met within a short period of time for aqua search meta-heuristic algorithm.

A characteristic curve showing the natural frequencies of the laminate is developed as shown in Fig.12b. A total of 92 iterations are performed for a duration of 21 hours for firefly algorithm and took 12 iterations for aqua search algorithm to arrive at the curve. The results obtained using AS algorithm are found to be in good agreement with the results obtained using analytical method as shown in Table 1.

Table 1. Comparison of frequency using aqua search meta-heuristic optimization, using polynomial relation and using analytical method

	Lining Factor	Frequency AS optimization	Frequency FFA	Frequency Analytical
	5.79111	40.3638034	40.3486	40.3438
Flies/Divers		2	5	
Iterations		12	92	
Time		144 minutes	21 hours	

4. Analysis

The results of aqua search meta-heuristic algorithm have led us to determine the optimal lining factors. The analysis is done for several cases with different types of loading and boundary conditions. Although the main focus of this paper is frequency analysis, we have checked this lining of laminates approach for every possible case on different types of shells as well.

4.1 Problem Statement

A square laminated composite plate is having a dimension of 80 x 80 square which has a cutout of dimension 30x30 square at the center of the plate. The modulus of elasticity of the material is given as E_{XX} = 130E3, E_{YY} = 8.50E3, G_{XY} = 6E3, G_{XZ} = 6E3, G_{YZ} = 3E3, Rho (ρ) = 100. The Poisson's ratio is taken as equal to 0.30. There are four layers of lamina which are considered here 0/90/0/90. The elements around the edge of the cutout are having eight layers of lamina 0/90/0/90/0/90/0/90 and the moduli of the elasticity of the material and the density of the material for these elements along the edge of the cutout are multiplied by 20. The plate is meshed using first order six node linear strain triangular elements. The number of elements is 440 and the number of nodes are 968. The number of nodes having support boundary condition are 128. The laminated composite plate is having three different types of supports.

4.1 Initial Investigation

4.1.1 Uniformly Distributed Load

4.1.1.1 CCCC Laminated Plate Carrying U.D.L.

Laminated plate carrying a uniformly distributed transverse loading acting vertically downward. The area near to the cutout edges undergoes less deformation as the suitable lining has been provided. The strain energy distribution shows that the support edges are carrying the maximum energy as shown in Fig.13 and the cutout edges are carrying the least. The cutout edges are safe and will not fail or tear apart early.

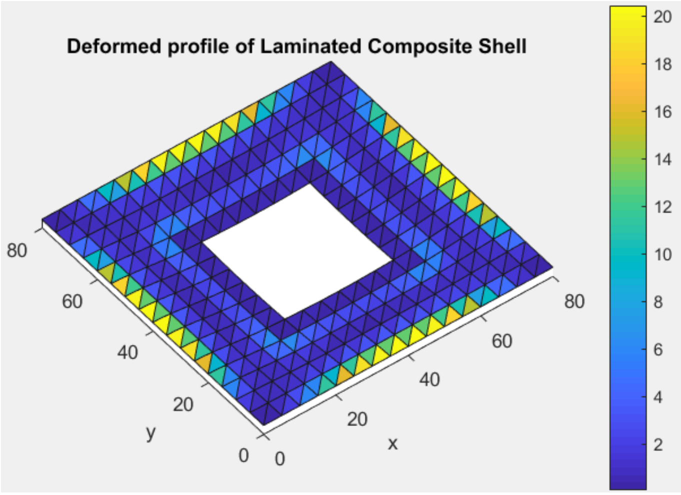


Fig. 13 Distribution of strain energy of a laminated composite plate with lining around the cutout edges and carrying uniformly distributed transverse loading

4.1.1.2 CCCC Laminated Composite Elliptical Shell Carrying U.D.L.

Fig.14 shows laminated composite elliptical shell subjected to uniformly distributed transverse loading, the first image on the left is without lining along the cutout edges, the corners are carrying high magnitude of strain energy and hence they are likely to fail first before the laminate starts to take any loading.

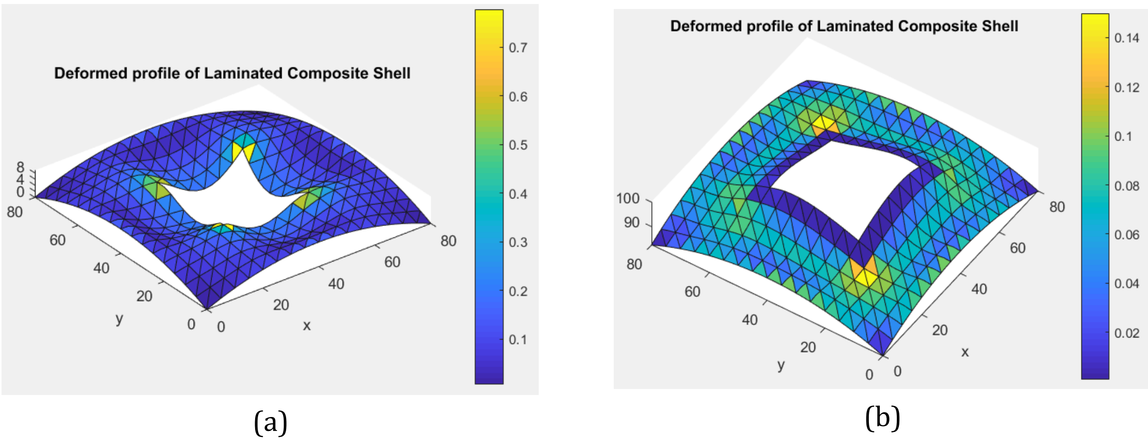
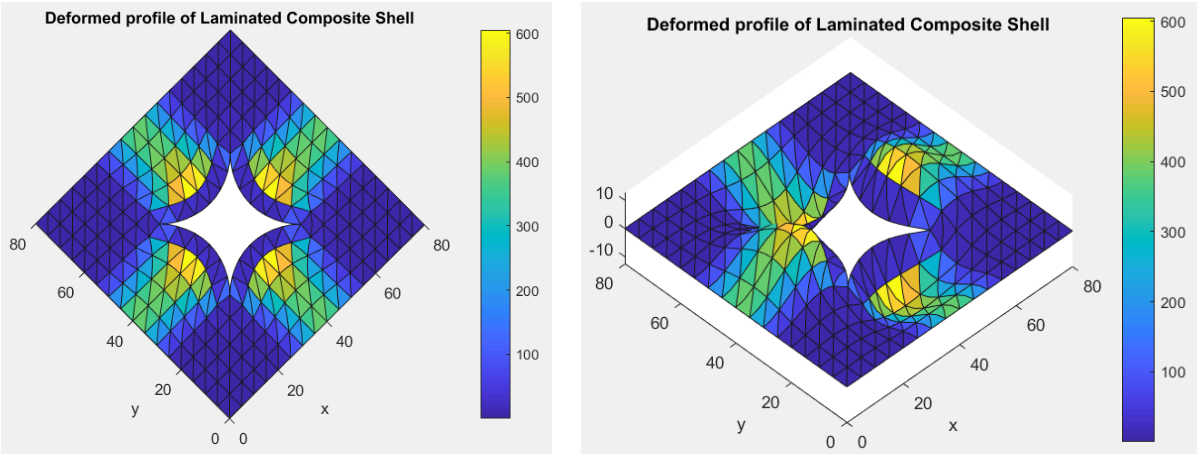


Fig. 14. CCCC elliptical shell carrying transverse loading with and without lining (a) CCCC elliptical shell without lining, (b) CCCC elliptical shell with lining

When the lining is provided along the cutout edges, the edges undergo less deformation and hence they are not carrying higher value of strain energy as shown in the deep blue color. The laminated shell is actually taking the load and undergoes deformation indicated by the strain energy in all the elements. This shows that the laminate is actually taking the load keeping the cutout edges as safe.

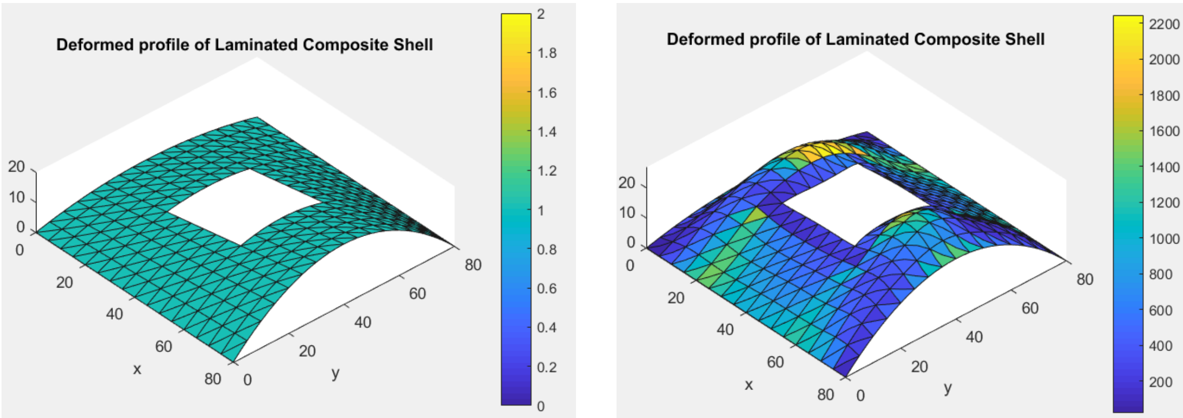
4.1.2 Hygroscopic Loading With 1.5% Moisture Content

Hygroscopic analysis of laminated composites with lining around the cutout opening is performed in this section. Different kinds of shells have been considered in this section namely, plate, conoidal shell, elliptical shell, cylindrical shell and hyperbolic shell with all the four edges as fixed. The loading in this case is 1.5% moisture content. The deformed profile of a CCCC plate with 1.5% moisture content is as shown in the Fig.15. The strain energy around the cutout edges is the least in deep blue color. The load is actually carried by the laminate and the edges do not fail at the beginning.



(a) (b)

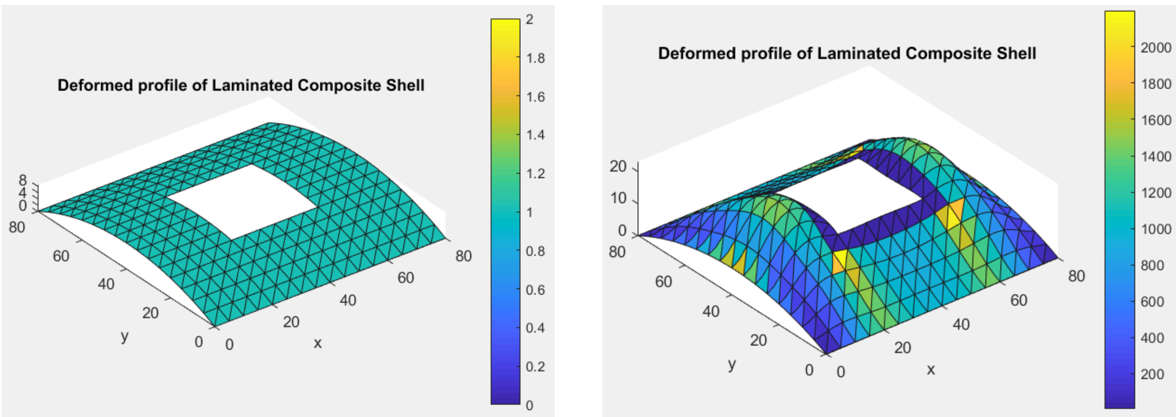
Fig. 15. Deformed profile of plate with 1.5% moisture content (a) Top view, (b) Isometric side view



(a) (b)

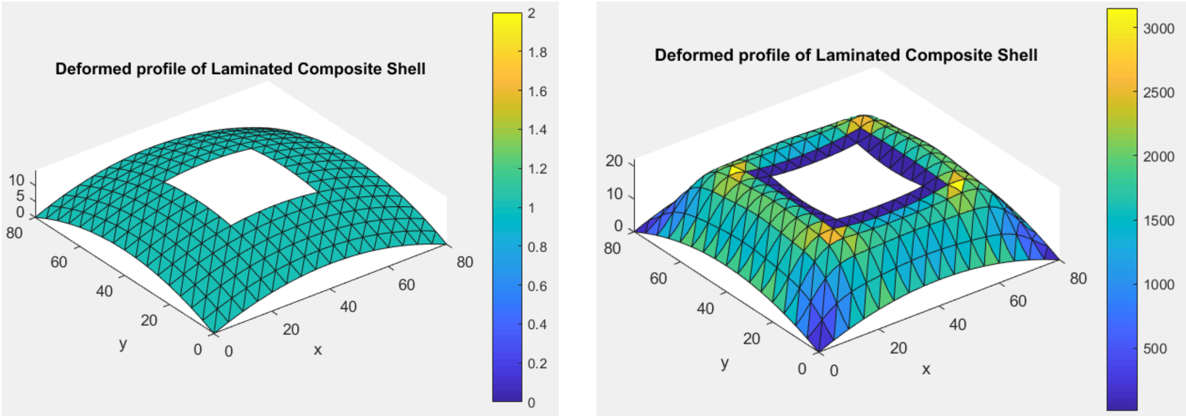
Fig. 16. Profile of a Conoidal shell carrying 1.5% moisture content (a) Undeformed profile of conoidal shell, (b) Deformed profile of conoidal shell

The undeformed profile of the conoidal shell is shown in the Fig.16 on left side. The shell has a cutout opening and is subjected to 1.5% moisture content as loading. The deformed profile of the conoidal shell is as shown on the right side. The cutout edges are in deep blue color shows at the edges the strain energy is less compared to the material inside. This means that the load is carried by the shell and the cutout edges are carrying least strain energy. Similar is the pattern observed with other kinds of shells namely cylindrical shell, elliptical shell and hyperbolic shell as shown in Fig.17, Fig.18 and Fig.19 given below.



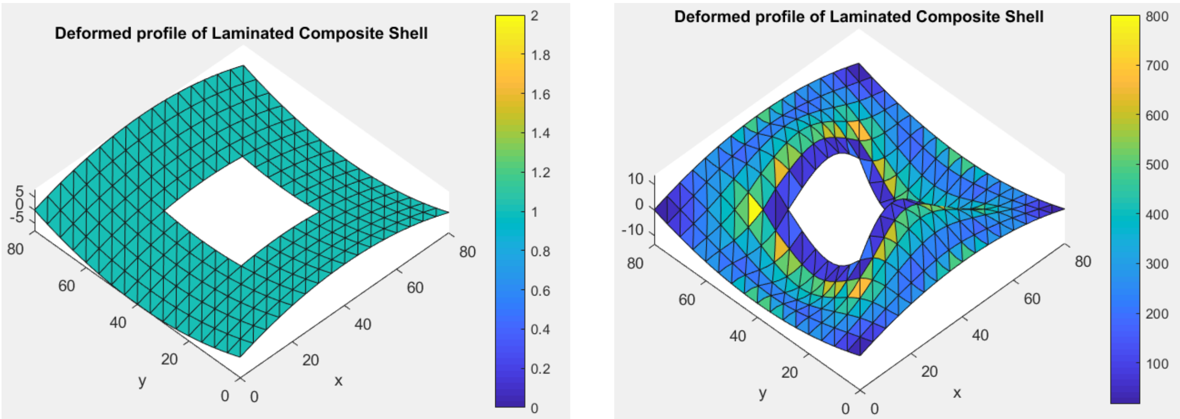
(a) (b)

Fig. 17. Deformed profile of a cylindrical shell carrying 1.5% moisture content (a) Undeformed profile of cylindrical shell, (b) Deformed profile of cylindrical shell



(a) (b)

Fig. 18. Deformed profile of a elliptical shell with (Rx = Ry) carrying 1.5% moisture content (a) Undeformed profile of cylindrical shell, (b) Deformed profile of cylindrical shell



(a) (b)

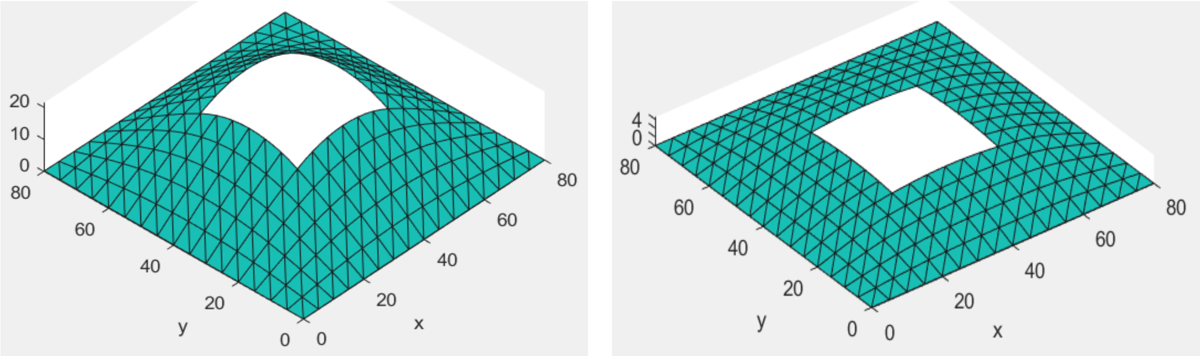
Fig. 19. Deformed profile of a hyperbolic shell (Rx = -Ry) carrying 1.5% moisture content (a) Undeformed profile of hyperbolic shell, (b) Deformed profile of hyperbolic shell

4.1.3 Vibration Analysis

First mode of vibration is taken for different kinds of laminated composite shell to measure the non-dimensional fundamental frequency (NDFF). The analysis is done for three different types of support conditions for the laminate, namely all edges of plate are simply supported, clamped, hinged. The results of fundamental frequency, first mode of vibration profile of laminate are shown below for each support condition for both laminated plate with edge stiffening lining and without lining. The results clearly show that the edge stiffening lining measures can reduce the fundamental frequency and also reduce the maximum displacement at node in the first mode of vibration. We consider 0/90/0/90/0/90/0/90 layers for elements along the cutout edge and the value of moduli of elasticity and the density are multiplied by lining factor of 20.

4.1.3.1 SSSS Laminated Composite Plate

The support condition for each node along X-axis is 10110 and the support condition for each node along Y-axis is 01101. The non-dimensional fundamental frequency is 12.3389. The mode shape is as shown in the Fig.18. By increasing the moduli of elasticity and density, the fundamental frequency changes to 8.8082. The maximum displacement in the eigen vector is 6.4392e-04 as shown in the Fig. 20.

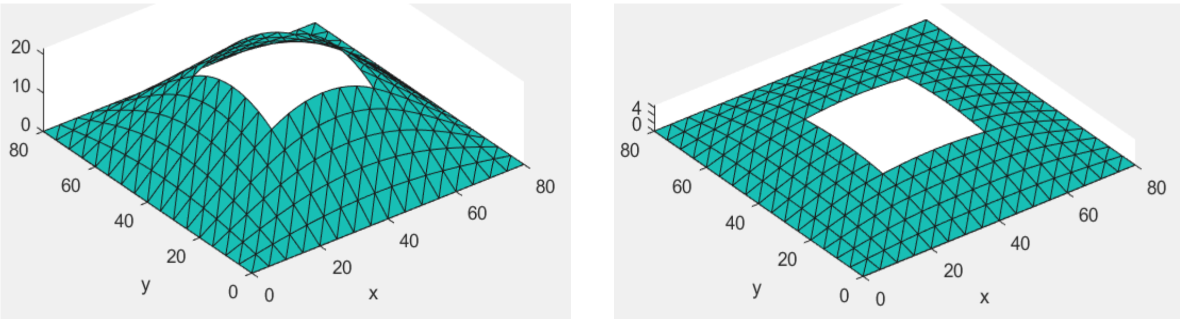


(a) (b)

Fig. 20. Fundamental mode shape of SSSS laminated plate having cutout (a) Without lining of SSSS plate NDFF = 12.3389, (b) With lining of SSSS plate NDFF = 8.8082

4.1.3.2 Hinged on All Four Edges (HHHH)Laminated Composite Plate

The support condition for each node along X-axis is 11110 and the support condition for each node along Y-axis is 11101. The non-dimensional fundamental frequency is 12.6368. The maximum absolute value of eigen vector is found to be 0.00214 as shown in the Fig.19. By increasing the moduli of elasticity and density, the fundamental frequency changes to 9.02285. The maximum displacement in the eigen vector is 6.4623e-04 as shown in Fig. 21.



(a) (b)

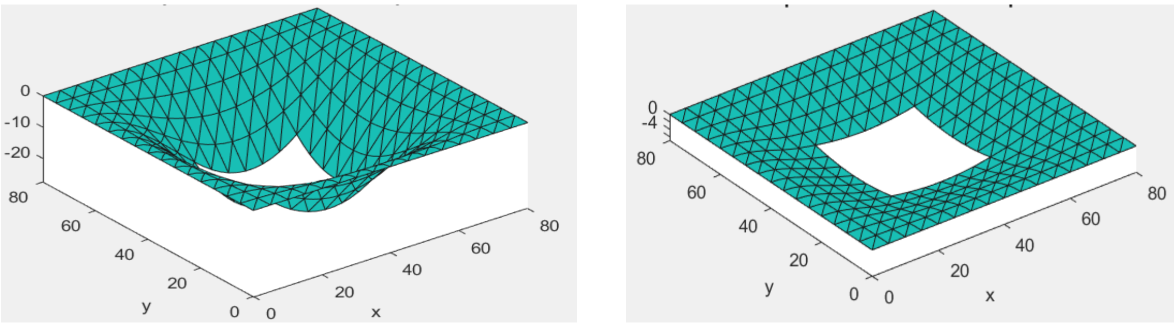
Fig. 21. Fundamental mode shape of HHHH laminated plate having cutout (a) Without lining of HHHH plate NDFF = 12.6368, (b) With lining of HHHH plate NDFF = 9.02285

4.1.3.3 Clamped / fixed on all four edges (CCCC) Laminated Composite Plate

The support condition for each node along X-axis is 11111 and the support condition for each node along Y-axis is 11111. The non-dimensional fundamental frequency is 36.4215. The maximum absolute value of the eigen vector is found to be equal to 0.002753. The fundamental mode shape as shown in the Fig.22 is drawn to a scale of 1 x 10^4. We can clearly observe the elements along the support boundary nodes are showing no displacement and no rotation as shown. By increasing the moduli of elasticity and density, the fundamental frequency changes to 16.45028. The maximum displacement in the eigen vector is 7.10201e-04. The CCCC support fixity has led to an increase in the fundamental frequency of the laminate higher than the frequency with SSSS and HHHH support condition as shown in Table 2.

Table 2. Showing the comparison of NDFF of laminated composite plate with different boundary conditions

Description	SSSS	HHHH	CCCC
Without Lining	12.3389	12.6368	36.4215
With Lining	8.8082	9.02285	16.45028



(a) (b)

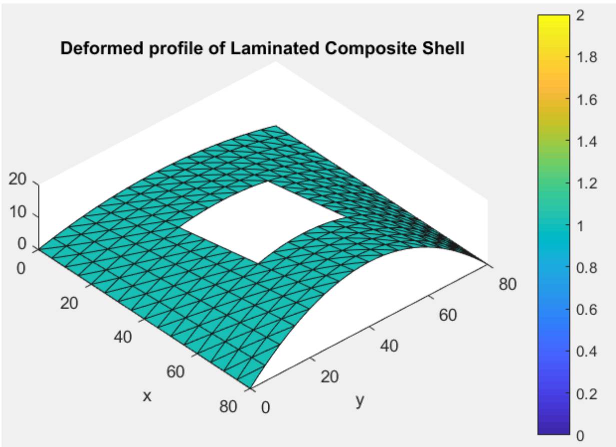
Fig.22 Fundamental mode shape of CCCC laminated plate having cutout (a) Without lining of CCCC plate NDFF = 36.4215, (b) With lining of CCCC plate NDFF = 16.45028

4.1.4 Vibration Analysis of Shells

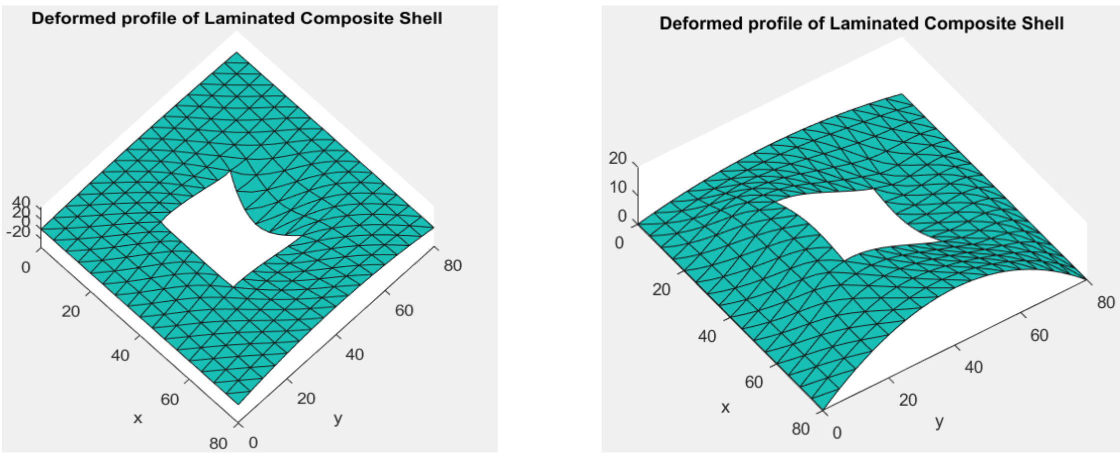
Vibration analysis is performed to determine the non-dimensional fundamental frequency (NDFF) of each kind of shell namely conoidal shell, elliptical shell, hyperbolic shell having fixed boundary conditions.

4.1.4.1 CCCC Laminated Composite Conoidal Shell

The first mode of vibration for a CCCC laminated conoidal shell is as shown in Fig. 23.



(a)



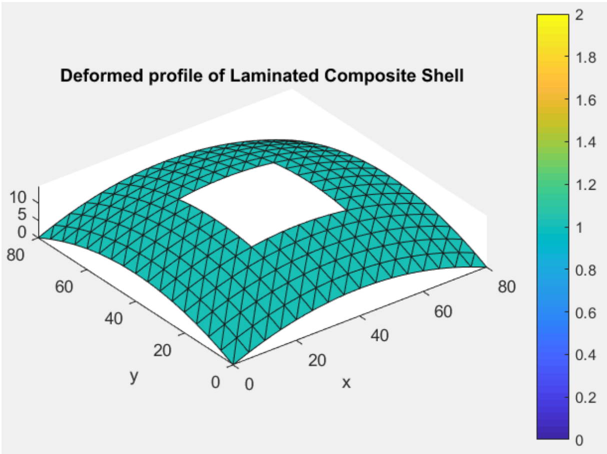
(b) (c)

Fig. 23. Conoidal shell first mode of vibration with and without lining around cutout edges (a) Undeformed profile of conoidal shell, (b) Without lining of conoidal shell NDFF = 88.277, (c) With cutout lining of conoidal shell NDFF = 45.42

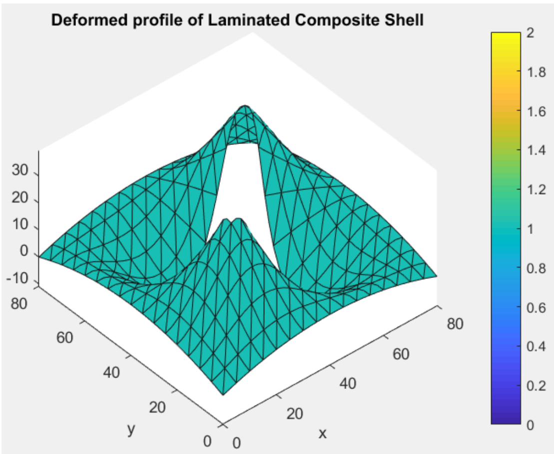
The first mode of vibration is as shown in the Fig.23 for the conoidal shell. The image on the left is the first mode of vibration without lining of laminates around the cutout openings. The non-dimensional fundamental frequency is 88.277. When the cutout edges are having the lining of the edges, the first fundamental frequency of vibration is lowered to 45.42. Lining helps to reduce the fundamental frequency and the edges of cutout are strengthened as well. The lining zone around cutout has higher modulus of elasticity.

4.1.4.2 CCCC Laminated Composite Elliptical Shell

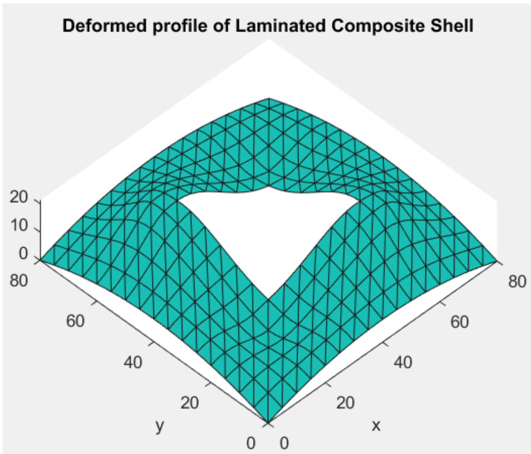
The first mode of vibration for a CCCC laminated elliptical shell is as shown in Fig.24. The elliptical shell with the radius Rx equal to radius Ry is analyzed and the fundamental frequency of vibration is determined. The non-dimensional fundamental frequency without lining is found to be 100.742 and with lining it is 60.458. Lining reduces the fundamental frequency of vibration and the edges are kept safe. The amplitude of vibration is lowered with lining the edges of the laminates.



(a)



(b)

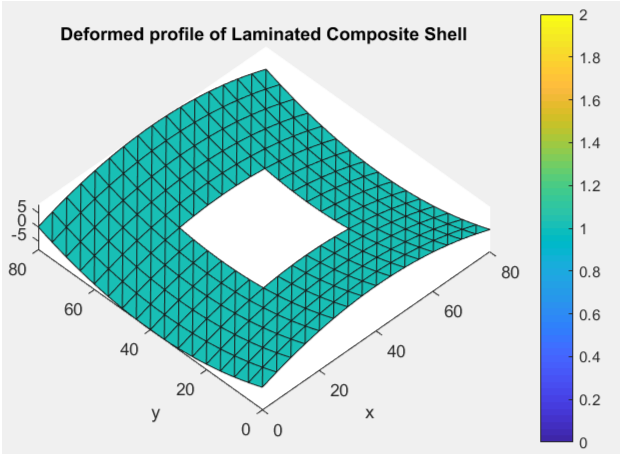


(c)

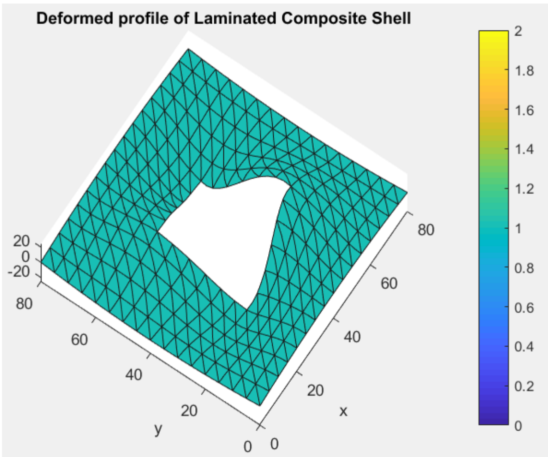
Fig. 24. Elliptical shell first mode of vibration with and without lining around cutout edges (a) Undeformed profile of elliptical shell, (b) Without lining of elliptical shell NDFF = 100.742, (c) With lining of elliptical shell NDFF = 60.458

4.1.4.3 CCCC Laminated Composite Hyperbolic Shell

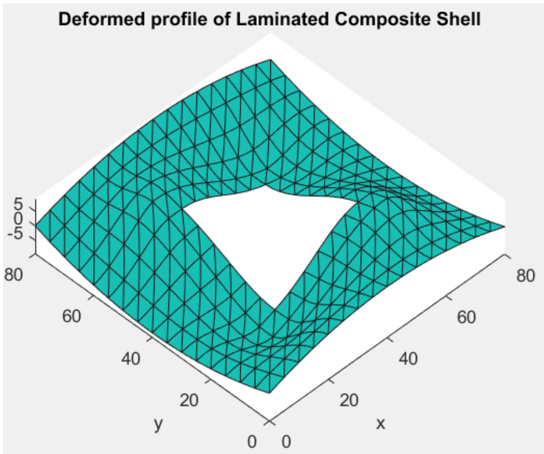
The first mode of vibration for a CCCC laminated hyperbolic shell is as shown in Fig. 25. Similar response of the laminate is observed in case of hyperbolic shells having fixed boundary condition. The non-dimensional frequency without lining the cutout edges is equal to 117.654 and reduces to 61.957 with lining the cutout edges. The amplitude of vibration is lower when the cutout edges are having lining. This section has done some extensive research on several different types of shell carrying different types of loading and boundary conditions. Every case, the advantages of lining of a laminate cannot be under emphasized. The strength of the laminate has increased, the maximum displacement has reduced in case of transverse loading, lowered the strain energy of elements in the cutout edges, the fundamental frequency of vibration has reduced, the maximum amplitude of vibration has reduced, and the edges are strengthened as well. In the next section, we determine the non-dimensional fundamental frequency of a CCCC laminated composite plate with 0/90/0/90 lamina and 0/90/0/90/0/90/0/90 lamina in the lining area at the cutout edges. NDFF is calculated for several different geometry ratios by varying the length of the edges. The thickness of the laminated plate is kept as equal to one.



(a)



(b)



(c)

Fig. 25. Hyperbolic shell first mode of vibration with and without lining around cutout edges (a) Undeformed profile of hyperbolic shell, (b) Without lining of hyperbolic shell NDFF = 117.654, (c) With lining of hyperbolic shell NDFF = 61.957

4.2 Analysis of CCCC Laminated Composite Plates with Lined Cutouts at Different Geometry Ratios

In the previous section, different cases of laminated composite plates and shells for different type of loading and support conditions have been analyzed. In this section, we present the non-dimensional fundamental frequency of a 0/90/0/90 CCCC laminated composite plate having lining around the cutout edges with a 0/90/0/90/0/90/0/90 lamina. The transition is smooth between the edge and the inner material of the composite laminate.

4.2.1 Problem Statement

A square laminated composite plate is having a dimension of 80 x 80 square which has a cutout of dimension 30 x 30 square at the center of the plate. The modulus of elasticity of the material is given as E_{XX} = 130E3, E_{YY} = 8.50E3, G_{XY} = 6E3, G_{XZ} = 6E3, G_{YZ} = 3E3, Rho (ρ) = 100. The Poisson's ratio is taken as equal to 0.30. There are four layers of lamina which are considered here 0/90/0/90. The elements around the edge of the cutout are having eight layers of lamina 0/90/0/90/0/90/0/90 and the moduli of the elasticity of the material and the density of the material for these elements along the edge of the cutout are multiplied by lining factor. The plate is meshed using first order six node linear strain triangular elements. The number of elements is 440 and the number of nodes is 968. The number of nodes having support boundary condition are 128. The laminated composite plate is having fixed or clamped support on all sides.

Several geometry ratios which are considered here are the ratio of the edge1 to edge2 which are 0.25, 0.50, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0. There are different values of the edge1 which are considered 50, 80, 100, 120, 160, 200. The thickness of the laminate is uniform throughout the laminate and is equal to unity. The optimal lining factors given by aqua search meta-heuristic algorithm are 16.3462, 1.69558, 1, 1.64888, 2.07497, 2.67246, 3.39439, 4.35938, 5.69433, 7.73602, 10.714, 14.1174. The ratio of the edge1 to the thickness of the laminate is taken as numerically equal to the value of the edge1 in plan of the laminated composite shell. The values of lining factor obtained from aqua search meta-heuristic algorithm have been used and the non-dimensional fundamental frequency are determined. The shell is discretized using six node linear strain triangular elements. Finite element analysis is done to determine the stiffness matrix (K) and mass matrix (M) and then the fundamental frequencies are determined as eigen values of the stiffness matrix and mass matrix. The eigen values are then transformed into non-dimensional fundamental frequencies. The first eigen frequency is usually the highest value and the first mode of vibration can be determined. The first mode shape for several different shells have been presented in the earlier section. A scatter plot of these NDFF for the case of composite laminated plate for different values of the lining factors have been presented. The scatter plot is drawn in Google Colabs. The coding is done using python and machine learning classes are imported and the scatter plot is drawn as shown in Fig. 26. In the next section, we have applied five machine learning methods namely linear regression, ridge regression, k nearest neighbor, random forest, and gradient boosting methods.

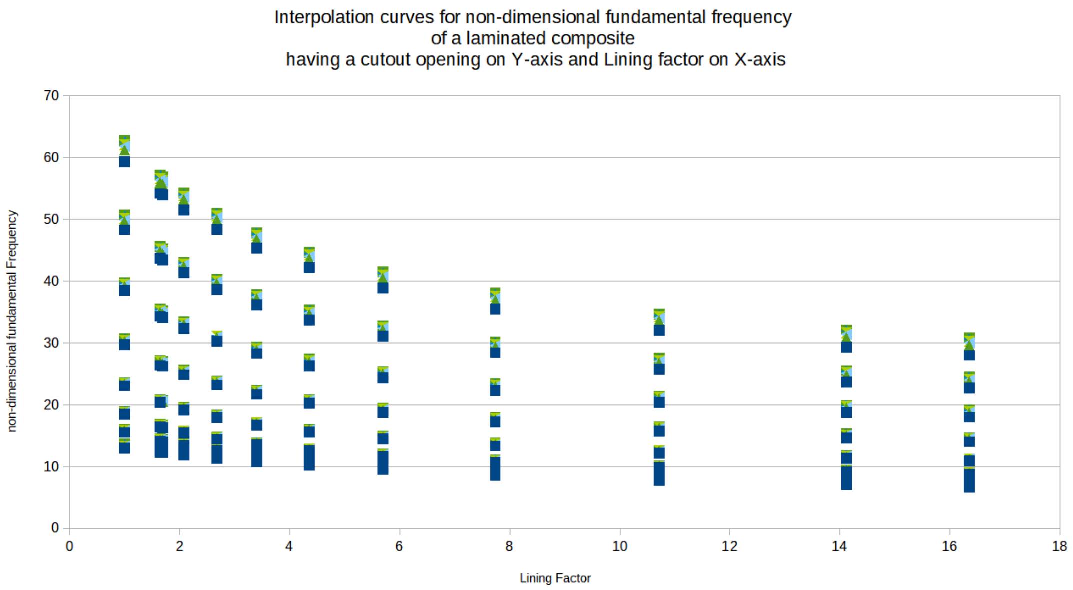


Fig. 26 Scatter plot of non-dimensional fundamental frequency characteristic for CCCC 0/90/0/90 laminated composite plate having cutout and 0/90/0/90/0/90/0/90 lining along the cutout edges using 6N LST FEM code for different aspect ratios (a/b = 0.25, 0.50, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0)

4.3 Apply AIML Techniques

In the previous section, a scatter plot is used to show the variation of non-dimensional fundamental frequency for different geometry ratios and lining factors. In this section, machine learning methods are applied to determine the NDFF and evaluate the performance metrics and choose the best method. Five different methods have been applied using artificial intelligence and machine learning (AIML) methods namely

- Linear Regression (LR)
- Ridge Regression (RR)
- kNearest Neighbor (kNN)
- Random Forest (RF)
- Gradient Boosting (GB)

Table 3. Comparison of performance metrics of different AIML algorithms

Method	MAE	RMSE	R2 Score
Linear Regression (LR)	6.8898	8.4996	0.5887
Ridge Regression (RR)	6.8898	8.4996	0.5887
kNearest Neighbour (kNN)	1.0637	1.946	0.9784
Random Forest (RF)	0.2545	0.431	0.9989
Gradient Boosting (GB)	0.168	0.2336	0.9997

Note: MAE - Mean Absolute Error, RMSE - Root Mean Square Error

Five machine learning methods namely linear regression, ridge regression, k nearest neighbor, random forest, and gradient boosting methods to interpolate and determine the value of non-dimensional fundamental frequencies for several geometry aspect ratios of edge1 to edge2 equal to 0.25, 0.50, 0.75, 1.0, 1.25, 1.50, 1.75, 2.0 and the value of edge1 are taken as equal to 60,70,90,100,110,130,140,150,170,180,190. The ratio of edge1 to thickness of laminate is taken as equal to unity for all geometry ratios. The coding has been done in python in Google Colabs® and the results are compared. The performance metrics chosen are mean absolute error, root mean square error and the R^2 values. Table 3 given below shows the comparison of these performance metrics. Clearly the best method is gradient boosting which is giving least error and higher R^2 value.

The graph shows the mean absolute error (MAE) for each method and compares with the error in x-axis and model in y-axis. Linear regression and ridge regression are having highest error and gradient boosting is the best method with the least value of MAE. Gradient boosting is better than other methods with least mean absolute error as shown in Fig. 27. The R^2 accuracy score graph clearly shows that gradient boosting is having a highest value of 99.9% and Linear regression has the least accuracy. Gradient boosting is the best method among all five methods as shown in the Fig. 28.

The graph in Fig.29 shows the comparative measure of predicted value with actual value. The scatter in green color is aligns along the best fit curve with Gradient boosting. The scatter in red color is linear regression which is the worst fit method. The error distribution in gradient boosting is normal distribution curve and as shown in the Fig. 30. The error distribution is shown in columns with maximum count at the center and least at the extremes. A random check has been done to verify the validity of the methods. A combination of geometry ratios is chosen at random whose non-dimensional fundamental frequency is determined by using finite element analysis. For a/b = 1.5, and a/h = 200 and lining factor = 2.67246. The graph is drawn for the given set of ratios for different lining factors and the polynomial equal is fit to these data points as shown in Fig.31. This polynomial equation is having a highest degree of seven. This polynomial equation is specific for this geometry ratio chosen and is also used to find the value of non-dimensional fundamental frequency for the given set of geometry ratios and lining factor. For the same set of geometry ratios, the non-dimensional fundamental frequency is determined by applying AIML Gradient boosting method and determine the non-dimensional fundamental frequency for 0/90/0/90 CCCC laminated composite plate having cutout and lining along the opening as shown in Table 4 given below.

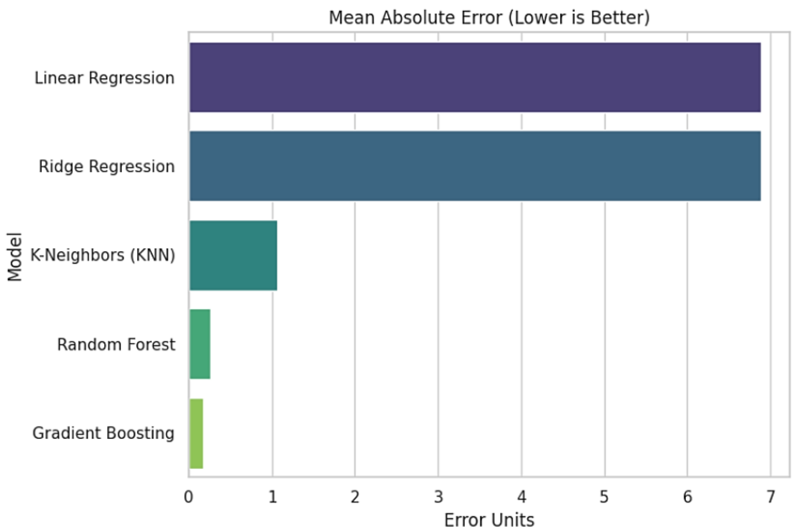


Fig. 27. Mean absolute error for five different machine learning methods

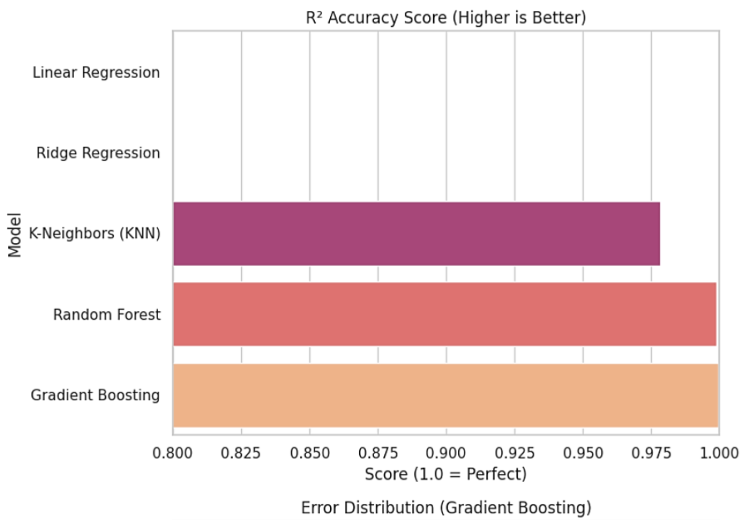


Fig. 28. Accuracy score for five different machine learning methods

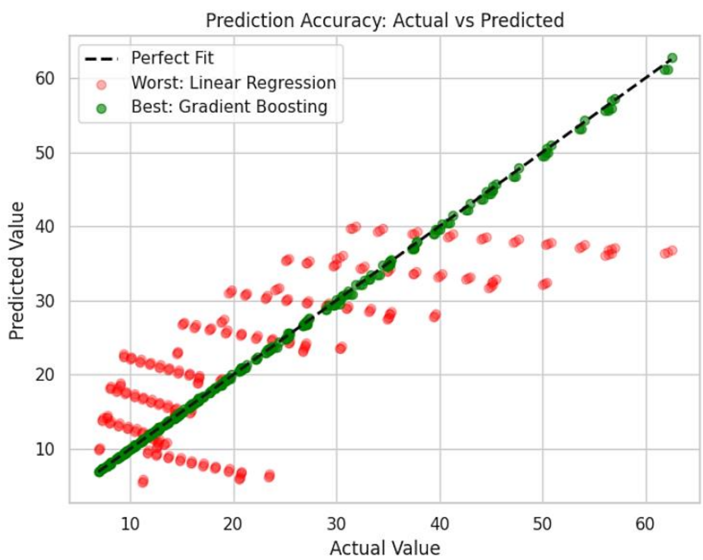


Fig. 29. Comparison of accuracy for best method and worst method

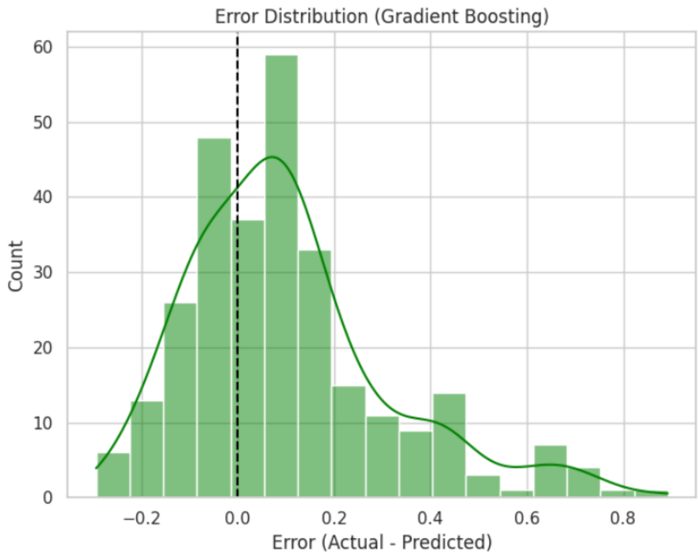


Fig. 30. Error distribution using gradient boosting

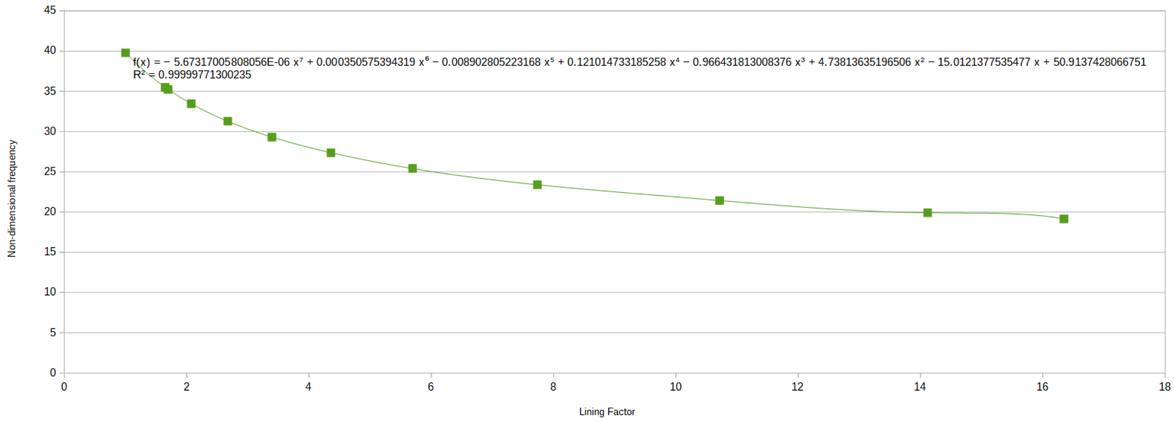


Fig. 31. CCCC laminated composite plate having lining at the cutout edges and analysed using FEM analysis for the ratios of edge a = 200, a/b =1.5, a/h = 1

Table 4. Comparison of NDFF using FEM, Gradient boosting, polynomial equation

a = 200 b = 133.33	a/b	Lining Factor	Using polynomial equation	Using AIML GB techniques	Using FEM
	1.5	2.67246	31.269649	31.25283	31.2864
% Error			-0.05354 %	-0.10729 %	

5. Conclusions

This research study in this paper is focused on lining of the laminated plate along the edge of the cutout. The analysis is done for different boundary conditions, different loading conditions, different lining factors to check the validity for different cases. However, the main analysis is done for only CCCC laminated composite plates. The optimal lining factors determined are using Aqua search algorithms. The study is focused mainly on non-dimensional fundamental frequencies of CCCC laminated composite plate having cutout although validation is done for different types of support and loading conditions. Several different type of loading conditions are checked and for different kinds of shells. In case of plates, three different support conditions were considered, namely SSSS, HHHH, CCCC. The fundamental frequency of vibration and the maximum displacement at node in the first mode of vibration were measured in each case. We observed with increase in the fixity the fundamental frequency of vibration has increased and the clamped plate has the maximum value of fundamental frequency. This is due to the increase in the stiffness of element having higher number of layers. As the stiffness increases the fundamental frequency increases and as the stiffness reduces the fundamental frequency reduces. Similarly, as the stiffness increase the displacement reduces, and as the stiffness reduces the displacement increases. As the mass increases the fundamental frequency reduces. With lining of laminates along the edges of the cutout, the frequency has reduced and the displacement at node in the fundamental frequency of vibration has been reduced as well. This proves that the edges along the cutout can be strengthened by lining and the longevity of the laminate might increase as well.

Gao et.al., paper on z-pins is an experimental proof of research currently going on in the area of laminated composites with a cutout opening. Z-pins are difficult alternative when the thickness of laminate is very thin. The proposed method of lining of laminate is easier and cost effective to apply and to increase the strength of edges in the opening.

The results obtained from optimization are in good agreement with the analytical results. The natural frequency of a laminated composite shell with lining around the cutout edges has been optimized for optimal lining factor. A check has been made for a geometry ratio of 1.5 and lining factor of 2.67246, the non-dimensional fundamental frequency using finite element analysis is found to be 31.2864. The results of finite element analysis using six node linear strain triangle are in close agreement with those obtained using artificial intelligence and machine learning algorithms which is 31.25283. The data points obtained from the aqua search meta-heuristic algorithm are curve fitted using a seventh-degree polynomial curve. The polynomial curve is used to determine the non-dimensional fundamental frequency and is in close agreement with the values obtained from finite element analysis. Five different machine learning models are used namely, Linear Regression (LR), Ridge Regression (RR), kNearest Neighbors (kNN), Random Forest (RF), Gradient Boosting (GB). Out of these five methods, Gradient Boosting (GB) is the best method and the Linear Regression (LR) model is the worst model. Three measures have been used to evaluate the performance of these models namely Mean Absolute Error (MAE), Root Mean Square Error (RMSE), R^2 value. The least error for Gradient Boosting (GB) is found to be nearly 0.1%. The complete data of non-dimensional fundamental frequency for different lining factors and geometry aspect ratio is calculated using both finite element analysis and machine learning gradient boosting methods.

5.1 Limitations of The Results from This Study

The lining factors considered in this study are below 17 and the values of NDFF can be checked. For the case of lining factors above this value, the analysis has to be done. The study is limited to thin laminated composites where shear deformation is not included. In case of thick laminated composites, shear deformation has to be included and the analysis has to be carried out again with increased number of degrees of freedom at each node. The aspect ratio considered in this study is less than or equal to 2. However, in case of higher values of aspect ratio the analysis has to be done separately. Newer AIML methods will emerge over the period of time and five different methods applied in this study may become obsolete in future.

5.2 Practical Industrial Applications

As discussed at the beginning of the paper in Fig.1 and Fig.2 where fibres are loose and which can be strengthened by applying the lining of laminates technique along the edges of the cutout. When transverse loading is applied to laminated composites where the edges are exposed and likely to fail first before carrying the expected load, lining the cutout edges can increase the performance of the laminate. Alyaa [24] in their research work have discussed the application of laminated composite to increase the strength of reinforced concrete member having openings.

5.3 Future Study

Non-linear reliability-based topology optimization of laminated composite shell structures having cutout opening and with the cutout edges are having lining and subjected to dynamic loading.

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