

## Performance assessment of RC beams under out-of-plane parts effect retrofitted with steel plates

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| Article Info                                                                                                                                                                                                                                 | Abstract                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| <p><b>Article History:</b></p> <p>Received 21 Jul 2026</p> <p>Accepted 07 Sep 2026</p> <p><b>Keywords:</b></p> <p>Out-of plane parts beams;</p> <p>Non-straight sections;</p> <p>FE analysis;</p> <p>Ductility;</p> <p>Energy absorption</p> | <p>This research investigates the failure mechanism and inelastic deformation of out-of-plane parts beams wrapped externally with steel thin plates. The retrofitting with thin plates is associated with the geometric discontinuity zone making this zone highly vulnerable to crack initiation, stiffness degradation, stress concentration, and extreme damage. The theoretical and experimental programs were matched by simulating three experimental models developed in a previous study to ensure the finite element analysis procedure could achieve accurate results for this modeling. The results obtained by validation, the maximum load and maximum displacement, as well as the relationship between load and displacement, showed a good agreement. The parametric study involved examining thirteen beams divided into three groups. The first group included examining four under out-of-plane parts beams with a varying number of connected segments. The second and third groups included eight models, partially and fully reinforced by thin steel plates. The results showed that the geometric discontinuity altered the mechanism of force transfer from pure flexural stress to a coupled stress compound. This effect is initiated at the connection region where the shear, twisting, and bending developed which initiated cracks at this region and governed the failure. The strength and stiffness decreased when the number of the connected segment increased. The decreased in the load was decreased up to two time due to the increase in the number of discontinuity points. Retrofitting the members with thin plates partially at the connection zone reduced the stress concentration and redistributed the stress far away from the discontinuity zone. Fully wrapping the beams with steel plates enhanced the load resistance by two times in addition to changing the failure mode from brittle shear-flexural to ductile flexural mode.</p> |

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### 1. Introduction

The shape and geometric form of a concrete structural element play a fundamental role in determining its structural behavior, alongside architectural requirements that may impose specific shapes or aesthetic details that contribute to the building's architectural character. In many structures, particularly mid-rise buildings or those with modern designs, designers may need to change the straight path of columns or lintels to a non-linear path for purely architectural purposes, or to address conflicts with other structural or mechanical systems such as service pipes or air ducts [1]. This change in the path of members can also be an engineering option to reduce the number of columns and achieve architectural flexibility in building spaces, as demonstrated in numerous practical applications, such as the BMW welt building in Germany, National Taichung Theater in Thailand, Sky House project in Tokyo, Centre Pompidou-Metz in France, and some complex buildings in China. Changing the path of a concrete beam from its straight line leads to a clear change in its structural behavior under the influence of applied loads. The response is not

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limited to resistance to bending or shear alone, but rather introduces additional components, including torsion, buckling, and other combined effects. The variance in the response for this type of member shows complex behavior dissimilar to those presented in conventional straight beams which causes a complex failure mechanism. Due to the importance of these phenomenon, many studies investigated the RC beams under variable load configurations such as bending, shear, and torsional loads [2].

Owenati [3] investigated the effect of the load configuration on the RC beams with varied ratio of steel reinforcement (longitudinal and transverse reinforcement). The results showed that cracking pattern and failure mode were affected by changing the configuration of the applied load. Ali and Anis [4] analyzed the load configuration effect on the secondary RC beams when applied two types of loading. The first load was pure torsional moment while the second one is combined with static vertical load. Results of the load-deflection relationship showed that ductility decreased significantly when subjected to combined loads, while the magnitude of the torsional angle increased at the maximum load. In contrast, Kaminski and Pavlak [5] presented a study that combined numerical analysis and laboratory tests on rectangular and angular beams under two types of loading: pure torsional moment and combined loads of torsion, bending, and shear. The results showed that the load capacity and stiffness decreased when the element was subjected to a combination of bending and torsion compared to when the element was subjected to pure torsion. The recommendations of ACI 445.1R-12 [6] also supported these conclusions, indicating that the presence of bending moments reduces torsional ductility and changes the direction of radial stress angles and crack patterns from oblique under pure torsion to nearly vertical under bending. On the other hand, El-Sayed et al.'s study [7] focused on the effect of cable inclination angle and different reinforcement ratios. The results showed that increasing the inclination angle had no significant effect on ultimate load resistance or cracking resistance, but it did significantly affect overall stiffness and deflection. Meanwhile, increasing the longitudinal reinforcement ratio improved diagonal cracking resistance and ultimate strength. Kay and Li [8] also examined the effect of corner column loss in reinforced concrete frames. Their experimental and analytical results indicated a gradual collapse and the formation of ductile joints at the ends of beams at corner joints, especially with moderate transverse reinforcement. Rahal and Collins's study [9] also showed that end-reinforced beams subjected to both bending and shear, and with the addition of torsion, exhibit a significant decrease in their strength, necessitating consideration of torsional reinforcement in the design. Meanwhile, Talaitapa and Mostafinejad [10] studied beams under pure shear and combined shear forces with torsion, and found that the beam under shear alone recorded the highest load-bearing capacity, while the beam under shear and high torsion recorded the lowest capacity, while pure torsion yielded intermediate results. In another study, Amolo and Ezeagu [11] analyzed normal-strength beams under combined bending, flexural, and shear forces. Their results confirmed that failure results from the combined effect of these forces, and that increasing the proportions of longitudinal and transverse reinforcement increases the beam's resistance. Finally, Ersoy and Ferguson [12] studied L-shaped concrete beams with reduced longitudinal and transverse reinforcement ratios under torsion. Experimental and numerical analysis results showed that reducing longitudinal reinforcement leads to earlier failure compared to reducing transverse reinforcement, highlighting the critical effect of longitudinal reinforcement on torsional resistance. Muhammed and Adel A. Al-Azzawi [13] conducted an experimental study on the behavior of RC beams with longitudinal out-of-plane sections, which the results showed that the presence of the out-of-plane section resulted in a reduction in the maximum load of approximately 80%. In 2022, Hasan et al. [14] conducted an FE analysis study to analyze the behavior of RC beams with out-of-plane sections. The results showed a clear decrease in load-bearing capacity with increasing number of sections; the decrease was 35% for single-segment beams, 33% for two-segment beams, and 68% for three-segment beams. Although most previous studies focused on analyzing reinforced concrete beams under combined loads of bending, shear, and torsion, with the aim of identifying their effects on ultimate strength, these studies have not adequately addressed the complex behavior of beams with out-of-plane members (BWOPP beams). This type of beam is characterized by fractures or changes in its longitudinal path, imposing complex behavior influenced by the number of beam's segments and their connection locations.

Previous investigations of non-straight reinforced concrete beams have addressed folded, kinked, and skew geometries, in which the member axis remains within a single plane and the geometric discontinuity generates secondary in-plane forces. Beams with out-of-plane parts differ fundamentally, since each connection displaces part of the axis outside the loading plane and introduces an eccentricity that accumulates torsional demand along the member. The behavior of such beams has been reported only for a limited number of experimental specimens, and neither the influence of the number and location of the out-of-plane segments nor the effectiveness of external steel jacketing in restoring the lost capacity has been quantified. Accordingly, the present study examines the failure mechanism and inelastic deformation of RC beams with out-of-plane parts under combined flexure, shear, and torsion, and identifies the optimum strengthening configuration for this geometric defect. Based on thirteen simulated FE models, the study establishes and provides design-oriented recommendations regarding the plating scheme required to compensate for out-of-plane connectivity.

## **2. FE Analysis**

To analyze the structural behavior of reinforced concrete beams with out-of-plane sections (BWOPP beams), the FE method was adopted as the primary tool for representing numerical models within ANSYS APDL. In this study, concrete was modeled using appropriate solid elements in ANSYS. These elements allow for the representation of cracking and the nonlinear behavior of concrete when it reaches compressive or tensile strength. Reinforcement steel was introduced either as link elements connected to the concrete using the Embedded Region feature, or by defining an integrated model within the concrete elements themselves when needed to reduce analysis time [15, 16]. Nonlinear analysis was implemented to capture the actual behavior of the beams. The Large Deflection option was used in the solution settings to ensure that large deformations that may occur due to combined loads are accounted for. The contact properties between the different beam components were also precisely defined. A suitable mesh was adopted to balance the accuracy of the results with the efficiency of the calculations. Regarding the loading and support conditions, the beam ends were fixed in various ways to simulate the support conditions assumed in experimental or practical models. Loads were applied gradually to ensure complete tracking of the load-deformation path until failure [17]. Regarding the modeling, first step is the choice of the element types which the type of the element controls the behavior of the modeled beam. The concrete, steel rebar, and steel plates were chosen to simulate the real behavior of the simulated materials. For the concrete material, SOLID65 was used to simulate the behavior of the concrete material because this element has the same behavior of concrete under stresses, as it is specifically designed to take into account the nonlinear behavior of concrete. Concrete is brittle in tension and strong in compression, and therefore cannot be modeled as a simple linear elastic element. What distinguishes this element is its ability to simulate two fundamental phenomena in concrete: Cracking in tension and crushing in compression [18]. That is, it can identify the moment when concrete begins to crack in the tension zone, as well as the point at which collapse occurs in the compression zone. These features are not present in other solid elements, such as SOLID45 or SOLID185, which only model elastic or elastic-plastic behavior without capturing cracking and crushing. SOLID65 also allows for embedded reinforcement by defining the proportions and orientations of the reinforcement bars within the element itself, facilitating the representation of the complex behavior of reinforced concrete without the need to draw each bar separately. This feature reduces modeling complexity and shortens analysis time while maintaining reasonable accuracy. The LINK180 element is ideal for representing steel reinforcement because it is a three-dimensional bar element capable of accurately transmitting axial tensile and compressive forces, which is consistent with the fundamental role of reinforcement in reinforced concrete. This element is characterized by its ability to represent the nonlinear behavior of steel through realistic stress-strain curves, allowing it to capture the yield and ductility phases of the structural member [19]. The SHELL41 element is used to model thin steel plates. Because it relies on four-degree-of-freedom nodes, the model can accurately handle bending and shear deformations while reducing numerical analysis time [19].

### 3. Material Modeling

#### 3.1 Concrete, Steel Section, and Rebar

To simulate the BWOPP beams, the material should be defined and modeled by using the suitable element and define the behavior of the material with use of stress strain curve under uniaxial test. The stress strain curve of the concrete and steel can be defined according to the equation imported from previous studies. For the concrete, stress strain curve of quasi-brittle material such as concrete was defined by Hognestad model [20] which stated that the concrete demonstrates linear behavior at the first stages of loading in compression and tension reaching to the yield point which after this point the concrete suffers from developing cracks causing stiffness degradation and strength loss until the ultimate load. Then, the curve softens reaching to the mechanism point as revealed in (Fig. 1). The simulation of the stress strain curve for the concrete was defined in the FE software. The mechanical properties of the concrete such as the compressive strength, tensile strength, modulus of the elasticity in addition to the Poisson's ratio were defined to simulate the real behavior of the concrete. The tensile strength was taken of 12% of the compressive strength while the modulus of the elasticity and Poisson's ratio were 27,806 MPa and 0.2 respectively. For the steel reinforcement, steel plates, and bearing plates, the stress strain curve was modeled under the assumption of the isotropic homogeneous elastic perfect plastic material according to the ductile theory of materials offered by Von Mises. The modulus of the elasticity and Poisson's ratio were 200,000 MPa and 0.3 respectively. The different yield strength is the governing for the steel reinforcement, steel plates, and bearing plates which were 420 MPa, 350 MPa, and 600 MPa respectively.

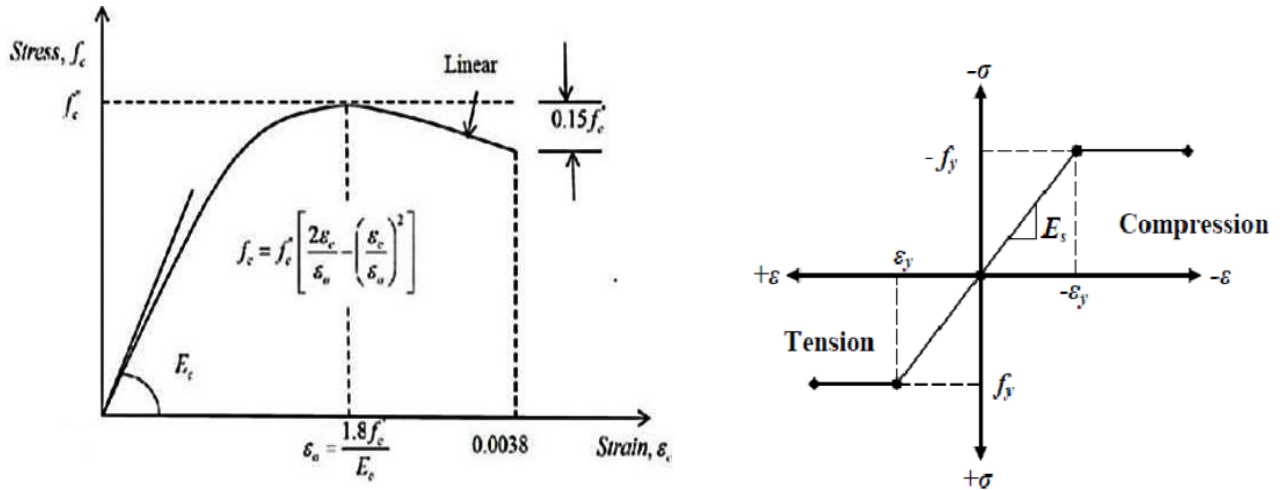


Fig. 1. Beam with out-of-plane parts applications [10]

#### 3.2 Meshing and Convergence Criteria

The meshing section is one of the most critical steps when preparing numerical modeling for reinforced concrete beams with out-of-plane sections (BWOPP beams) using ANSYS APDL. This is because the quality of the mesh determines the accuracy of the numerical solution and the extent to which the results approximate the actual behavior of the structure. A carefully designed finite element mesh was constructed to accurately represent the structural behavior of the studied beams while maintaining the efficiency of the numerical analysis. The mesh density affects directly on the behavior and accuracy of the RC beam which denser mesh led to higher accuracy and real behavior of the test concrete. But the chose of the mesh is a sensitive manner which the Increasing the mesh density causes higher accuracy but this increase is to a certain element which more increase causing more analysis time and more accumulative error due to the matrix multiplication. In this study, the mesh density was variable because the model size was large. Increasing the mesh density to achieve the required accuracy would lengthen the analysis time and cause a non-convergence problem, where the analysis would stop due to the accumulation of numerical errors within the computational matrices. So, a variable mesh density was chosen along the model. Higher

mesh density was used in the most sensitive areas, such as loading zones and regions of directional changes, BOWPP beams. These joints are more sensitive due to changes in load paths in these zones, in addition to the denser mesh at support zones. The mesh density was reduced in intermediate zones that were not subject to dimensional or directional changes or the presence of loads. Also, the fineness of the mesh directly affects the computational convergence of the analysis. A mesh density was chosen to achieve numerical convergence. Additionally, convergence is affected by certain input values in the analysis. A tolerance of at least 0.005 was selected when applying forces, and values of open and closed crack factors of at least 0.2 were used. Finer mesh was selected at the connection zone, loading and supports regions. Maximum element size was 50 mm was selected with aspect ratio not exceeded 1:2 to ensure the accuracy and convergence of the results. A mesh sensitivity analysis was conducted on the validation model using four mesh densities comprising 2,862, 3,971, 5,128, and 6,791 elements. The predicted ultimate load and mid-span deflection were compared with the experimental values of 369.0 kN and 61.21 mm, and the computational time was recorded, as summarized in Table 1 and illustrated in (Fig. 2).

Table 1. Sensitivity analysis results for simulated models

| Mesh | Element size (mm) | No. of elements | Pu (kN) | Deflection at Pu (mm) | CPU time (min) | Change in Pu (%) | Change in deflection (%) |
|------|-------------------|-----------------|---------|-----------------------|----------------|------------------|--------------------------|
| M1   | 50                | 2,862           | 453.18  | 72.04                 | 8.0            | 22.81%           | 17.69%                   |
| M2   | 35                | 3,971           | 403.29  | 70.36                 | 13.0           | 9.29%            | 14.95%                   |
| M3   | 25                | 5,128           | 384.73  | 69.14                 | 22.0           | 4.26%            | 12.96%                   |
| M4   | 20                | 6,791           | 396.55  | 69.55                 | 38.0           | 7.47%            | 13.63%                   |

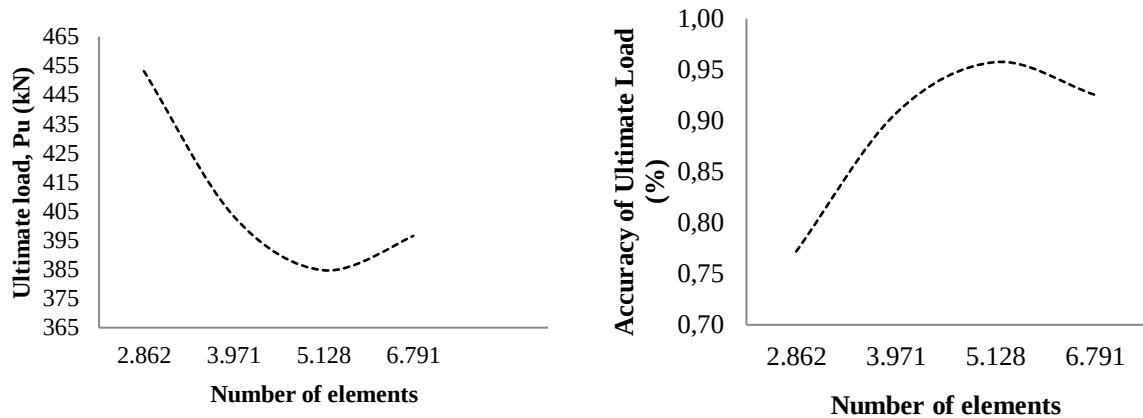


Fig. 2. Sensitivity analysis of simulated models

Increasing the number of elements from 2,862 to 5,128 reduced the deviation in ultimate load from 22.81% to 4.26% and in deflection from 17.69% to 12.96%, with the computational time increasing from 8 to 22 minutes. A further increase to 6,791 elements reversed this trend, raising the deviations to 7.47% and 13.63% respectively at a cost of 38 minutes. This non-monotonic behavior is characteristic of the smeared cracking formulation of the SOLID65 element, in which the cracking band width is governed by the study in the absence of fracture-energy regularization; an excessive number of elements intensifies damage localization and alters the load step at which numerical divergence, taken as the ultimate load, occurs.

A mesh of 5,128 elements was therefore adopted for all models and maintained across all configurations, so that the observed differences reflect the investigated variables rather than the discretization. The residual deviation in deflection, close to 13% for all meshes, is not governed by the mesh density but by the stiffness idealizations adopted, in particular the assumption of perfect bond and the selected shear transfer coefficients.

### 3.3 Modeling Input and Solution Controls

Concrete was modelled using the SOLID65 element, which incorporates a five-parameter failure surface accounting for the pressure-sensitive brittle response of concrete under multiaxial stress states. The concrete was assigned an elastic modulus of [27,800] MPa and a Poisson's ratio of [0.2]. The shear transfer coefficient for open cracks was taken as [0.3] and for closed cracks as [0.9]; values below 0.2 were found to cause severe convergence difficulties and were therefore avoided. The uniaxial tensile cracking stress was set to [4.2] MPa. The crushing capability was [35 MPa]. Compressive behavior beyond the elastic limit was represented by a multilinear isotropic hardening curve reaching a peak stress of [35] MPa at a strain of [0.0033]. Reinforcing bars were modelled with LINK180 and the steel plates with [SHELL41] as shown in (Fig. 3), both assigned a bilinear isotropic hardening law with a yield strength of [410] MPa, an elastic modulus of [200,000] MPa, a tangent modulus of [260] MPa, and a Poisson's ratio of [0.3]. Von Mises equivalent stress was adopted as the post-processing criterion for all steel components, since the change in beam path at the out-of-plane connections produces multiaxial stress states that no single stress component can represent. In the concrete domain, von Mises contours are reported only qualitatively to illustrate load-path redirection at the connection zones, while failure itself is governed by the concrete failure surface described above. Perfect bond was assumed at both the reinforcement-concrete and plate-concrete interfaces. This assumption is supported by the adequate embedment length and mechanical interlock of the tested specimens, and by the limited interfacial slip reported for comparable composite configurations. Both the reinforcement-concrete and the plate-concrete interfaces were modelled as fully bonded through nodal connectivity. For deformed reinforcing bars, mechanical interlock limits relative slip to a magnitude that has a negligible effect on the global response prior to yielding, and the assumption is therefore appropriate for a study concerned with strength, stiffness, and failure mode rather than with bar slip or crack width.

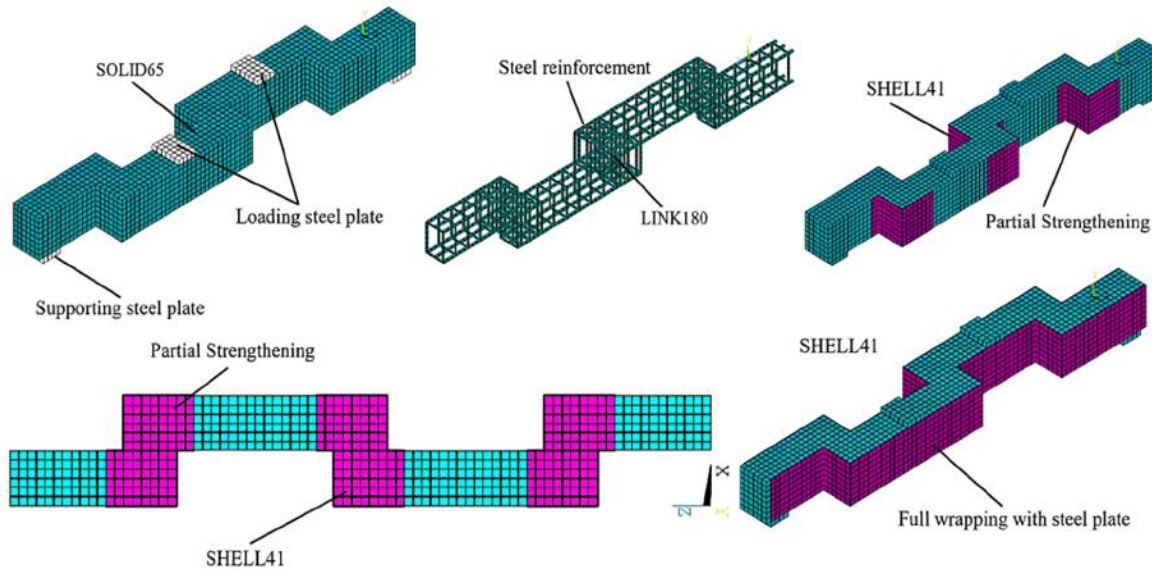


Fig. 3. FE model mesh of BWOPP beams

The assumption for the plate-concrete interface is validated by the computed response: failure in all configurations was governed by concrete. The nonlinear solution was conducted with full Newton-Raphson method by using automatic time stepping, which allowed the reduction of substep size in crack neighborhood and near ultimate load and the increase during near-linear stage. The convergence criteria were evaluated on the force with tolerance [0.005] and on the displacement criterion with a tolerance of [0.05]; each substep allowed a maximum number of [200] equilibrium iterations. Line search was used to regularize convergence in the post-cracking region (more pronounced for the BWOPP configurations due to flexural-shear-torsional interaction). The analysis was stopped at the load step corresponding to the solution divergence, which is consistent with standard SOLID65 modelling practice; this step was accepted as the ultimate load.

### 3.4 Loading and Support Conditions

The models were restrained with fixed ends supports as revealed in (Fig. 4). The models were restrained the vertical and horizontal displacement in addition to restrained rotation to simulate the real behavior and permit the stresses of shear, flexural, and torsion to develop. The load was applied on the steel plates which the load value divided on the number of nodes to distribute the load over area and avoid the stress concentration at the node attaching to loading zones

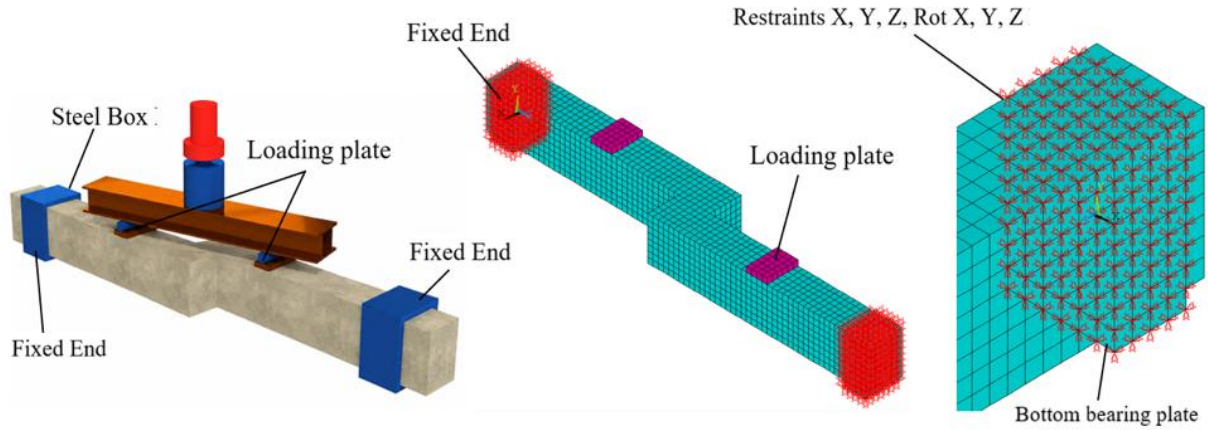


Fig. 4. Loading and Support conditions of BWOPP beams

### 4. Verification of Finite Element Modeling

The verification is considered the first step to ensure the accuracy of the followed procedure of modelling and obtaining results. In this research three experimental beams presented by Mohsin et al [23] was modeled and analyzed which varied with the concrete type. The experimental study included three models were NSC-10P (normal-strength concrete with one out-of-plane portion), HSC-10P (high-strength concrete with one out-of-plane portion), and HSC-10PT (high-strength concrete with the addition of longitudinal and transverse reinforcement to resist torsion). The study recorded experimental values for both ultimate load and maximum deflection, in addition to load-deflection curves as revealed in (Fig.5.) and Table 2. In this research, numerical models were prepared that were completely identical in terms of geometric dimensions, reinforcement ratios, material properties, and type of applied loads, using ANSYS APDL.

Table 2. Validation outcomes of FE simulation and experimental specimens

| 8,5      | Exp. Pcr | ANS. Pcr     | Exp. Pu           | ANS. Pu          | Exp Max. disp.         | ANS Max. disp.         |
|----------|----------|--------------|-------------------|------------------|------------------------|------------------------|
| NSC-10P  | 26       | 24.31        | 369.2             | 384.73           | 61.21                  | 69.14                  |
| HSC-10P  | 48       | 44.94        | 385.99            | 377.18           | 51.22                  | 43.36                  |
| HSC-10PT | 48       | 50.54        | 460.49            | 483.4            | 49.62                  | 42.84                  |
|          | MAPE Pu% | MAPE. Disp.% | Exp. Twist. angle | ANS. Twis. angle | Exp. Energy Absorption | ANS. Energy Absorption |
| NSC-10P  | 4.0366   | 11.4695      | 2.58              | 2.32             | 14775.76               | 14289.8                |
| HSC-10P  | 2.3358   | 18.1273      | 2.44              | 2.18             | 13068.91               | 12897.45               |
| HSC-10PT | 4.7393   | 15.8263      | 2.65              | 2.39             | 16155.78               | 15916.18               |

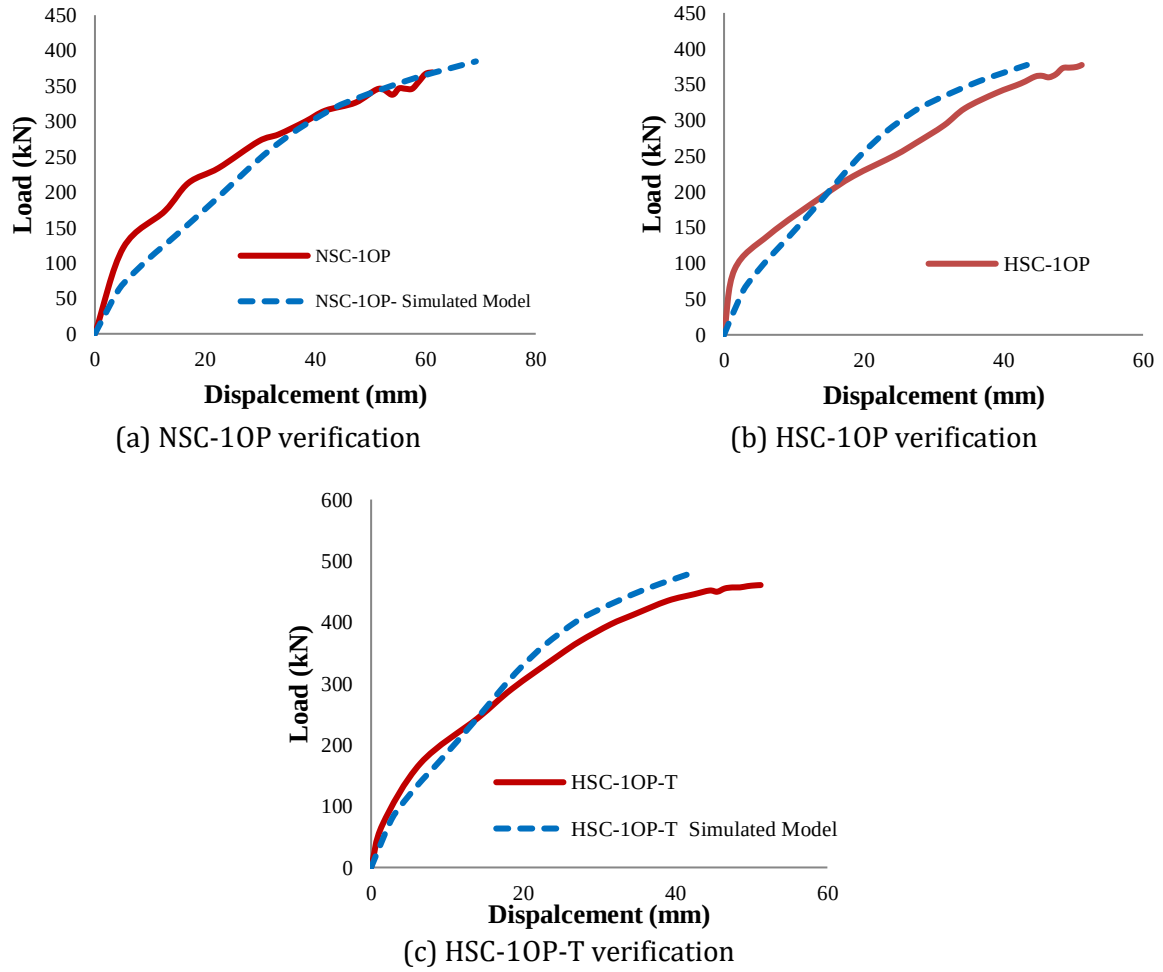


Fig. 5. Load-displacement relationship Comparison between experimental and FE results

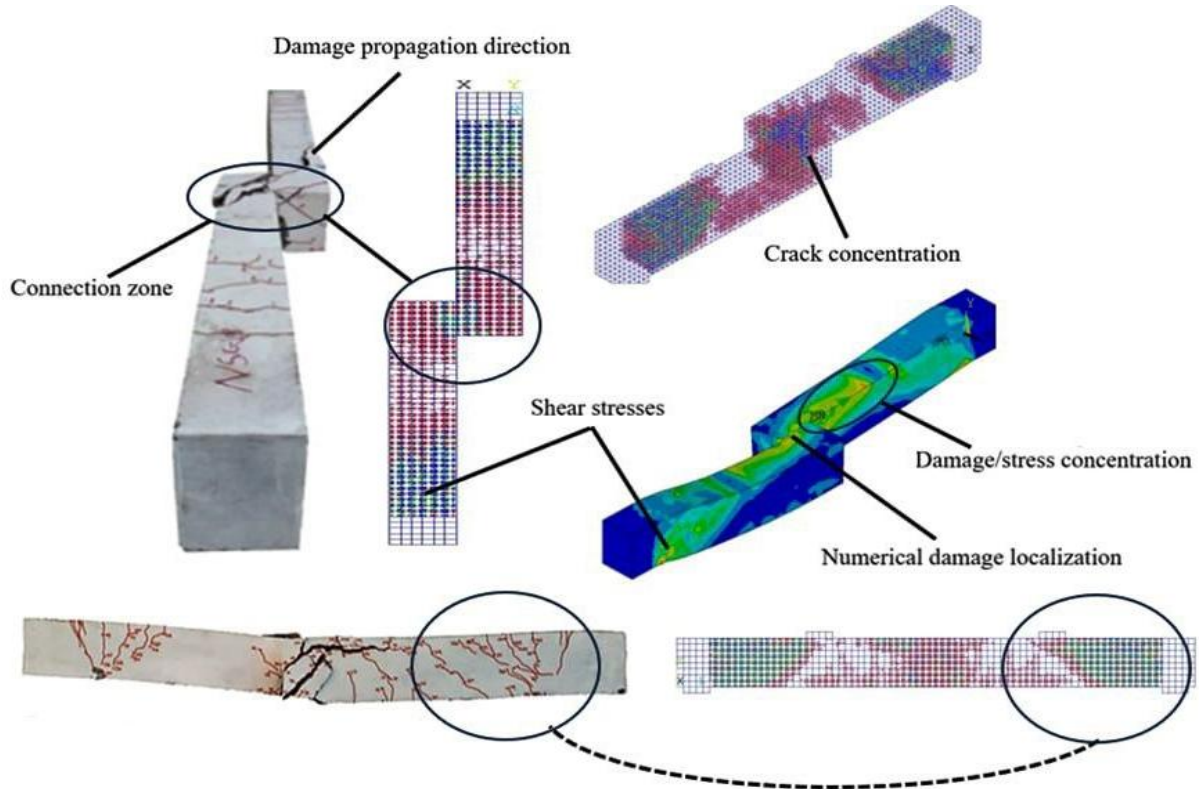


Fig. 6. NSC-10P Experimental and Simulated failure mechanism

The same support and loading conditions as those used in the experimental study were applied to ensure direct comparability of the results. The comparison between the experimental and thermotical results can be explained more clearly with use of statistical calculations such as MAPE calculations for the ultimate load and maximum displacement which the ultimate load capacity showed that the error ranging between 2.3 to 4.7% but displacement demonstrate higher error ranging from 11-18% approximately. This higher variance is influenced by the boundary conditions, material ideal behavior, in addition to the analysis factors and analysis method. These ratios are considered acceptable in numerical studies of reinforced concrete, especially since concrete materials are inherently nonlinear and semi-brittle, making perfect agreement between numerical and experimental analyses difficult to achieve. For the energy absorption results, the results were comparable in range between 96.7% to 98.7% approximately.

For the failure mechanism comparison, the simulated experimental models showed the initiation of shear cracks at the supports which followed by stress concentrations and cracks initiation at the connections zone in the same way of the experimental beams testing. The simulated and experimental models show the same failure mechanism which was not conventional flexural failure which started from initiation of combined stresses at the connection zone which extended inclined and longitudinal along the beam. The variance in some cracks amount in the experimental beams due to non-homogeneity of the concrete which make the concrete crushed at these points as revealed in (Fig. 6).

## 5. Analytical Study

The specimens (total of thirteen models) were made of RC sections measuring 150×200mm with a total length of 2000mm, with 12mm longitudinal reinforcement and 8mm transverse reinforcement, according to the experimental previous study [12]. The models were designed according to experimental previous study [12], the first mode was the reference beam while twelve of them parameterized according to number of segments and strengthening method. The tested specimens were made of RC sections and reinforced with 12mm longitudinal reinforcement and 8mm transverse reinforcement, according to design of the experimental previous study [12]. The beams were evenly spaced at 81mm intervals to control cracks.

Table 3. Details of specimens

| ID        | f <sub>c</sub> (MPa) | E <sub>c</sub> (GPa) | Beam Dim.    | Plane parts Number                             |
|-----------|----------------------|----------------------|--------------|------------------------------------------------|
| RC1       | 35                   | 27.80                | 150×200×2000 | NSC beam with a straight segment               |
| BWOPP1    | 35                   | 27.80                | 150×200×2000 | NSC beam divided into two connected segments   |
| BWOPP2    | 35                   | 27.80                | 150×200×2000 | NSC beam divided into three connected segments |
| BWOPP3    | 35                   | 27.80                | 150×200×2000 | NSC beam divided into four connected segments  |
| BWOPP4    | 35                   | 27.80                | 150×200×2000 | NSC beam divided into five connected segments  |
| BWOPP1-PS | 35                   | 27.80                | 150×200×2000 | Partially plated two-segment NSC beam          |
| BWOPP2-PS | 35                   | 27.80                | 150×200×2000 | Partially plated three-segment NSC beam        |
| BWOPP3-PS | 35                   | 27.80                | 150×200×2000 | Partially plated four-segment NSC beam         |
| BWOPP4-PS | 35                   | 27.80                | 150×200×2000 | Partially plated five-segment NSC beam         |
| BWOPP1-S  | 35                   | 27.80                | 150×200×2000 | Fully plated two-segment NSC beam              |
| BWOPP2-S  | 35                   | 27.80                | 150×200×2000 | Fully plated three-segment NSC beam            |
| BWOPP3-S  | 35                   | 27.80                | 150×200×2000 | Fully plated four-segment NSC beam             |
| BWOPP4-S  | 35                   | 27.80                | 150×200×2000 | Fully plated five-segment NSC beam             |

The compressive strength of the normal strength concrete was 35 MPa, and the yield strength of the longitudinal steel was 413 MPa as demonstrated in (Fig. 7) and Table 3. The models were divided into three series for three variables respectively which the first variable was the number of beam path segments. The models included a reference straight beam (RC1), one-joint beams (BWOPP1), two-joint beams (BWOPP2), three-joint beams (BWOPP3), and four-joint beams (BWOPP4). This change in the number of the joint connection was performed because the in more segments connected developed a combination of shear, flexural and torsional stresses so the presence of these segments will provide a combined behavior of stresses, while the second and third included strengthening of the models with partial and full wrapping by steel plate of 3 mm thickness. A 25 mm thick steel plate were used under the loading for the models to avoid the stress concentration, achieving smoother stress transmission across the section, and reducing the possibility of brittle failure at the connection points. This composite solution (Steel–Concrete Composite Action) enabled a dual evaluation of the feasibility of combining reinforced concrete with external steel as a means of addressing the shortcomings of beams with out-of-plane sections. Specimens were loaded under two-point loading and under strict end-to-end restraints to simulate a combination of bending, shear, and torsion. ANSYS APDL was used for numerical modeling, with load distribution at nodes, to ensure accurate representation and match the numerical behavior with the experimental response. This procedure is expected to enable accurate comparisons between encased and un-encased beams, and between straight beams and those with out-of-plane components, providing a rich database for future development of theoretical models.

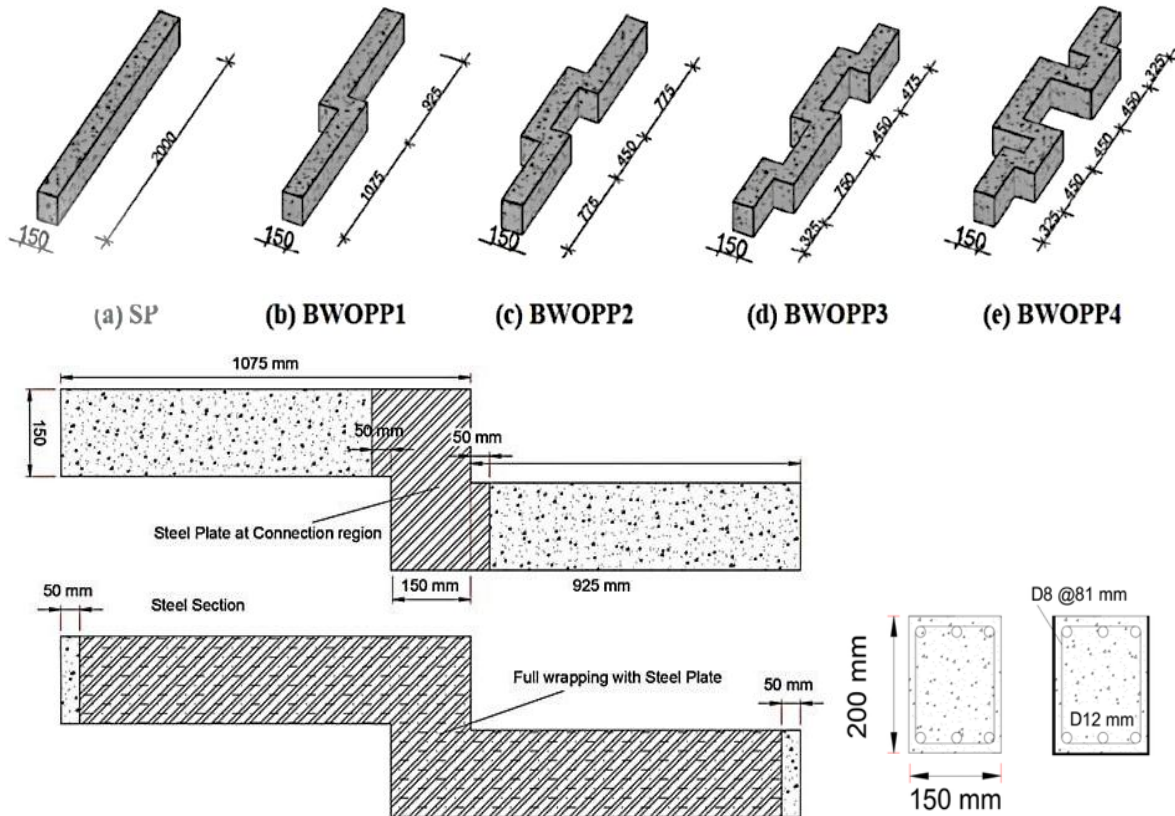


Fig. 7. Beams details

## 6. Results and Discussion

### 6.1 Load-Displacement Relationships

Table 4 and (Fig. 8) illustrated the results of the parametric models. According to the obtained results, the BWOPP beams behavior was dissimilar to those of straight beams which the cracking strength, ultimate load, in addition to the developed stresses were affected significantly. The control beam (straight beam) exhibited higher strength against the cracking and ultimate load which this ability reflects the high stiffness of the RC beams.

Table 4. Results of BWOPP beams

| Par.                     | ID        | Yield load (kN) | Failure load (kN) | $\delta$ corr. to Pu (mm) | Twist. Angle (Deg.) | DI    | Tn      | Failure Type  |
|--------------------------|-----------|-----------------|-------------------|---------------------------|---------------------|-------|---------|---------------|
| Connected segment number | RC1       | 182.13          | 601.13            | 24.13                     | 0.000               | 3.411 | 1054.57 | Flexure       |
|                          | BWoPP1    | 78.05           | 384.26            | 69.12                     | 2.328               | 3.711 | 919.90  | Shear-Torsion |
|                          | BWoPP2    | 59.22           | 297.26            | 58.37                     | 2.600               | 4.145 | 761.58  | Shear-Torsion |
|                          | BWoPP3    | 32.15           | 250.09            | 46.35                     | 2.942               | 4.689 | 618.02  | Shear-Torsion |
|                          | BWoPP4    | 21.85           | 174.66            | 19.61                     | 3.245               | 5.172 | 528.59  | Shear-Torsion |
| Partial strengthenin     | BWoPP1-PS | 156.31          | 610.74            | 36.98                     | 2.038               | 6.936 | 1552.79 | Shear         |
|                          | BWoPP2-PS | 132.11          | 488.43            | 25.04                     | 2.136               | 7.269 | 1201.78 | Shear         |
|                          | BWoPP3-PS | 107.43          | 364.15            | 23.58                     | 2.074               | 7.147 | 1184.02 | Shear         |
|                          | BWoPP4-PS | 84.06           | 299.07            | 17.91                     | 2.012               | 6.848 | 987.63  | Shear         |
| Full strengthenin        | BWoPP1-S  | 226.34          | 685.86            | 31.10                     | 1.757               | 9.578 | 1982.91 | Shear         |
|                          | BWoPP2-S  | 177.29          | 578.31            | 23.79                     | 1.842               | 9.407 | 1561.11 | Shear         |
|                          | BWoPP3-S  | 138.37          | 467.61            | 27.02                     | 1.834               | 8.884 | 1553.83 | Shear         |
|                          | BWoPP4-S  | 101.96          | 382.51            | 21.28                     | 1.735               | 8.204 | 1369.97 | Shear         |

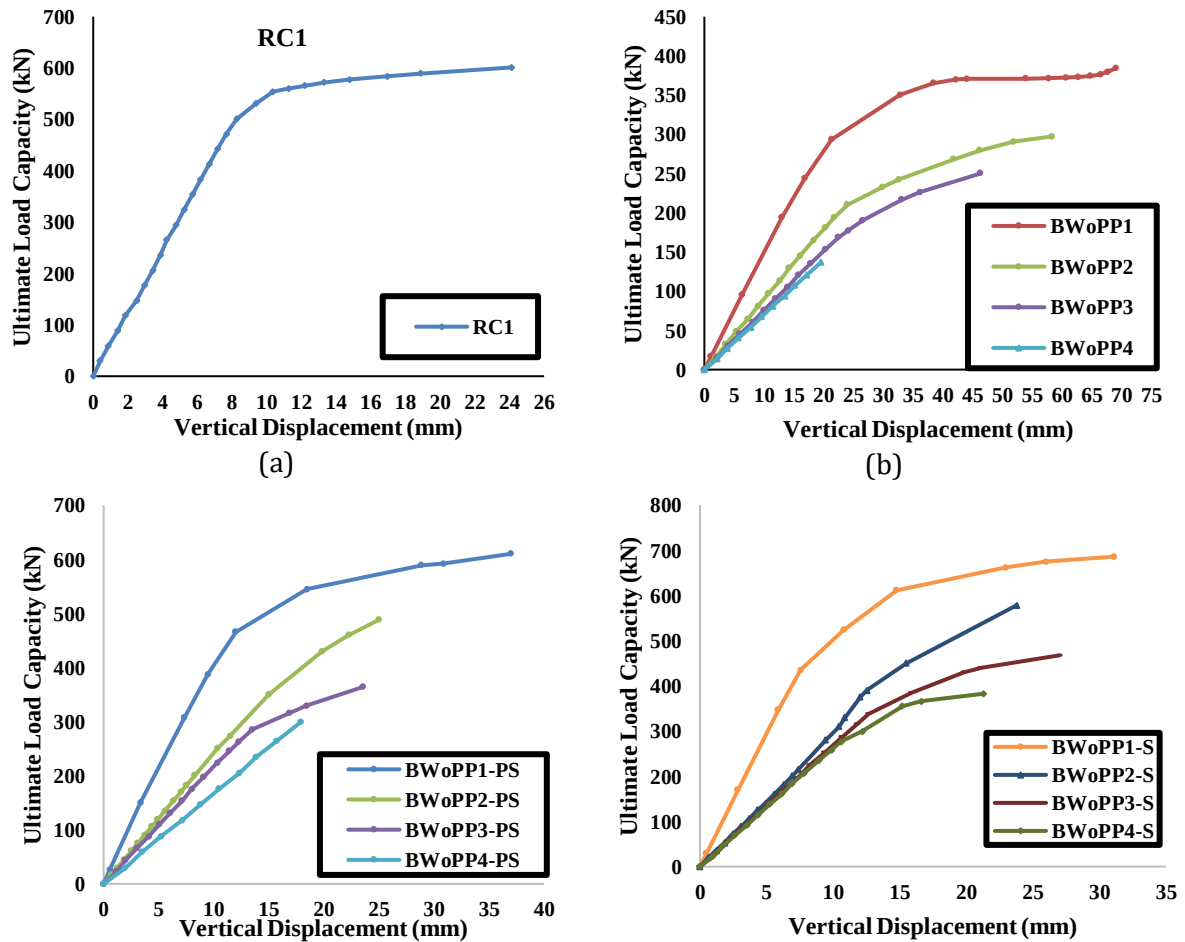


Fig. 8. Load-deflection curves retrofitted a non-retrofitted beams with out-of-plane parts effect

This ability was reduced when the concrete beam segmented into many parts (such as BWOPP beams) which the change in the direction causes reduction in the stiffness and reduced strength against the cracking and ultimate load. In addition, the ductility of the members decreased notably due to the exposure of the beam to combined of stress due to the change in the member direction and segmentation the member into several parts. Owainait [3] proved this statement which stated that the combination of stresses reduces stiffness and increase in the initiation of cracks. This is consistent with current observations that the presence of out-of-plane components accelerates the propagation of diagonal cracks and hastens failure. On the other hand, the results of the current study highlighted the role of steel jacketing using steel box sections in improving the performance of beams. This reinforcement contributed to enhancing shear and torsional resistance, reducing the large deformations that appeared in unreinforced models, and providing more stable post-cracking behavior. This trend is consistent with what Rahal and Collins [9] and Amulu & Ezeagu [11] reported, which states that increasing the proportions of longitudinal and transverse reinforcement or incorporating high-strength steel increases the overall capacity to resist combined loads and enhances the structure's resilience against premature collapse. It is worth noting that comparing the results with those of Elsayed et al. [7] In their study on the effect of the reinforcement pitch angle, they demonstrate that adding external steel elements is more effective in counteracting torsional failure than increasing the pitch ratio of the reinforcing bars alone. The current study also demonstrates that the behavior of beams equipped with steel cladding differs qualitatively from the classical behavior of reinforced concrete, generating a composite behavior that mimics composite materials (composite Action), achieving a better balance between strength and ductility.

## **6.2 Comparison Between Models with and Without Retrofitting**

In the case of the BWoPP1 series, a distinct divergence in the load–displacement response is observed between the un-strengthened beam (BWoPP1), the partially strengthened configuration (BWoPP1-PS), and the fully strengthened specimen (BWoPP1-S). The un-strengthened beam exhibited a rapid increase in load at small displacements, followed by an early reduction in stiffness, as reflected in the flattening of the curve. The behavior of these beams illustrates that the early cracks initiation was due to the shear-torsional stresses especially at the connection zone. The existence of the geometric discontinuity causes changing the path of stresses which developed a twisting stress. For this reason, the cracks extended longitudinally and inclined along the beam. It should be noted that the connection joint was not restrained which this feature permit for developing combined of stresses. The presence of the steel plates in these joints causes restraining the joint zone and redistribute the stresses along the span which reflects on enhancing the strength and stiffness of the beam.

The comparison between the jacketed and non-jacketed models showed that existence of thin plates at the connection zone enhanced the behavior which these plates reduced the negative effect of the discontinuity points but this enhancement was to certain limit which these enhancements decrease when the number of joint increased. In the BWoPP2 models, the presence of two discontinuity zones enhanced degradation of stiffness in the un-retrofitted models, while partial retrofitting improved stiffness and raised yield strength to a higher displacement. With full retrofitting, force transference became more uniform, and the two discontinuity zones no longer acted as independent sources of failure. BWoPP3 exhibited more nonlinear behavior due to multiple discontinuities, but partial laminations delayed cracking and increased the ultimate load, while full reinforcement achieved a more pronounced improvement in ductility. For BWoPP4, the unreinforced model was the weakest due to numerous discontinuity zones, but partial encapsulation mitigated the rate of damage development and improved the overall threshold response. The local steel plates at the four joints delay initial cracking, improve stiffness, and raise the ultimate load capacity compared with the un-strengthened beam. Still, the multiple unconfined spans between the strengthened joints remain prone to stress concentration, so the limited ductility behavior and a clear softening trend once cracking initiate. The fully strengthened BWoPP4-S achieves the most dramatic performance improvement relative to its un-strengthened counterpart. The curve demonstrates a significantly higher elastic stiffness, elevated ultimate load, and a broader deformation range prior to failure as revealed in (Fig. 9 d).

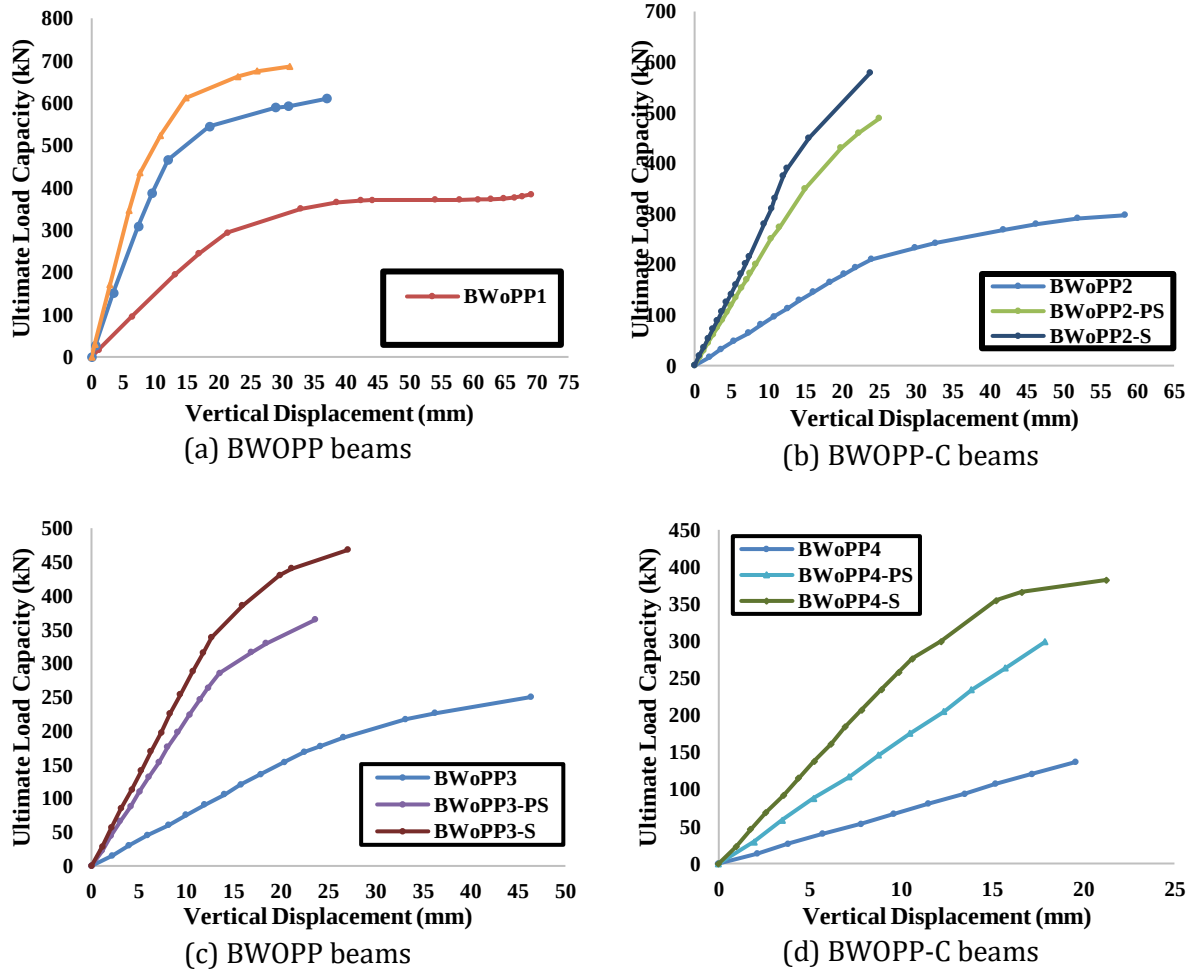


Fig. 9. Load-displacement relationship of BWOPP beams

### 6.3 Twisting Angle

The angle of twist is measured for the non-straight beams at the ends of the twisted segment. The average value of twisting angle was calculated from the displacement-induced rotation of the beam cross-section. The lateral and vertical displacements of two corresponding points located on opposite sides of the cross-section were extracted from the FE results. The rotation angle was determined from the relative displacement between the two points, and the values obtained from the two sides were averaged to represent the twisting angle of the beam. Regarding the acceptable serviceability limit for the twisting angle, there is no universally prescribed limiting value for the angle of twist of RC beams in the commonly used design codes. Serviceability under torsion is generally evaluated in relation to cracking, deformation, and the intended structural function rather than by a single universal twist-angle limit. The analysis results demonstrated that the geometric discontinuity causes torsional effect on the non-straight models which the beam with connected segment illustrates twisting angle of 2.344 Degrees, while increase the number of the connected segment enhanced the angle of twisting to 2.76, 3.1, and 3.78 degrees. The increase in the angle of twisting reach to 17.9% for the model with three connected segments. This percentage increased for the models (BWOPP3, BWOPP4) to 33% and 61% respectively as shown in figure below. Jacketing the connection joints with thin plates reduced the twisting angle to lower values which redistributed the shear torsional stresses along the beam. The angle of twisting for retrofitted models was reduced by 16.6%, 11.4%, 16.4%, and 13.5% when compared with the corresponding beams (BWOPP2, BWOPP3, BWOPP4) respectively as demonstrated in (Fig. 10 b).

The reduction in strengthening efficiency with increasing segment number can be attributed to the role of the external steel plates in confining the critical connection regions. The steel plates provide local confinement and help reduce the adverse effects associated with these connection zones,

thereby improving the torsional resistance of the strengthened beam. However, as the number of segments increases, the number of connection regions also increases, and consequently a larger portion of the strengthening system is occupied by these interfaces. This reduces the effective contribution of the steel plates to enhancing the overall torsional resistance. Therefore, although increasing the number of segments provides additional confinement to the connection regions, the overall strengthening efficiency gradually decreases due to the increased number of connection interfaces. The ANOVA results indicate a statistically significant effect of the investigated strengthening configuration on the response of the beams, as evidenced by the high F-value (77.17) and the very low p-value ( $2.0 \times 10^{-6}$ ). The coefficient of determination ( $R^2 = 0.92$ ) indicates that approximately 92% of the observed variation is explained by the adopted model, demonstrating a strong relationship between the investigated strengthening parameter and the measured response.

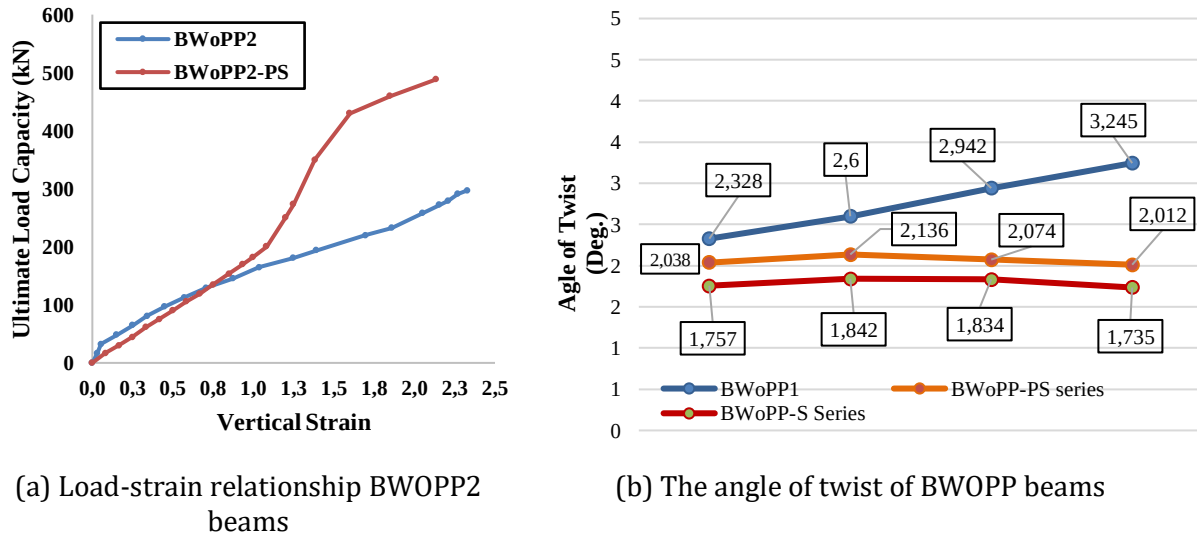


Fig. 10. Results of retrofitted and un-retrofitted of BWOPP beams

#### 6.4 Number of Segments Effect

Comparing the yield loads of the BWoPP1, BWoPP2, BWoPP3, and BWoPP4 models with the reference beam RC1 as revealed in (Fig. 11) clearly shows that the introduction of out-of-plane components results in a gradual and significant decrease in the yield load. The yield load for the straight beam RC1 was 182.13 kN, while it decreased to 78.05 kN in BWoPP1, a decrease of approximately 57%. It decreased to 59.22 kN in BWoPP2, a decrease of approximately 67%. It then decreased further in BWoPP3, reaching 32.15 kN, a decrease of approximately 82%. Finally, BWoPP4 recorded the lowest yield load of 21.85 kN, representing an 88% decrease compared to the reference beam. This clear sequence reveals that increasing the number of connected and out-of-plane parts exacerbates the deterioration of structural behavior at the yield stage. As the cracks accumulate at joints, the strains of the reinforcing steel increase under relatively low loads, resulting in early yielding. When examining the maximum load of the four models BWoPP1–BWoPP4 compared to the straight reference RC1 (601.13 kN), a clear monotonic decrease is evident with increasing number of out-of-plane parts. BWoPP1 achieved an ultimate capacity of 384.26 kN, representing 63.93% of that of RC1, a decrease of 216.87 kN (-36.08%). BWoPP2 decreased to 297.26 kN, representing 49.45% of that of the reference (a decrease of 303.87 kN, or -50.55%). The decrease continued in BWoPP3 to 250.09 kN, or 41.60% of RC1 (351.04 kN, -58.40%), while BWoPP4 recorded the lowest value of 174.66 kN, not exceeding 29.06% of the reference, with a difference of 426.47 kN (-70.94%). This numerical trend confirms that the increasing number of out-of-plane connections progressively weakens the ultimate strength of the beam. the presence of out-of-plane parts imposes successive eccentricities in the load path, generating effective bending moments in addition to the bending moments and shear forces. Critical regions particularly at joints reach failure limits before the beam reaches its flexural capacity. Localized crushing of concrete or premature capture of reinforcement occurs in high-torsion areas, limiting ultimate strength.

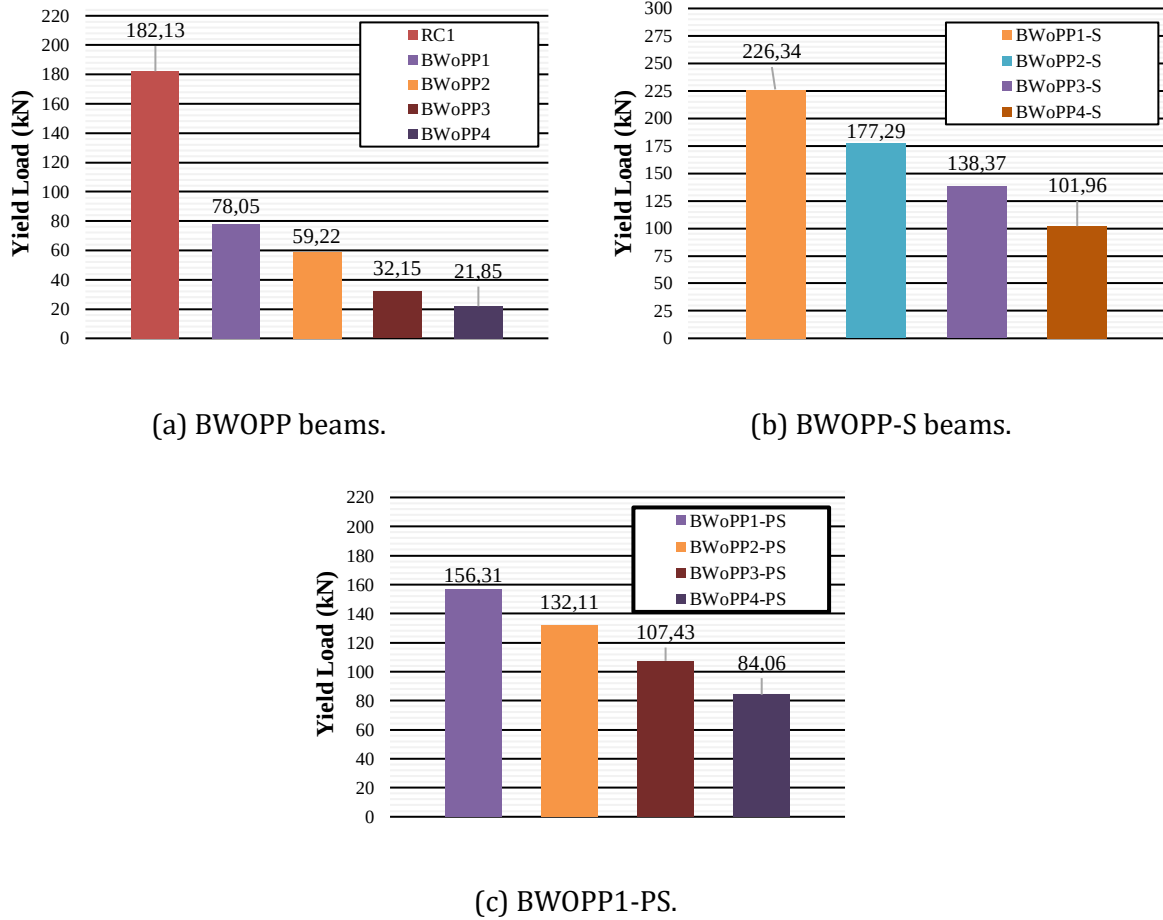


Fig. 11. Yield load of simulated models

### 6.5 Effect of Partial Strengthening

Comparing the BWoPP-PS specimens (wrapped with steel plate at the connection regions only) with their un-strengthened counterparts, a significant improvement in the yield load is evident due to local strengthening. The yield load of BWoPP1-PS increased to 156.31 kN from 78.05 kN in BWoPP1, an increase of +78.26 kN, or approximately  $\approx 100\%$  (the reinforced load is nearly twice that of the unreinforced one). In BWoPP2-PS, the yield load reached 132.11 kN, compared to 59.22 kN, an increase of +72.89 kN, or  $\approx 123\%$  (the reinforced load is  $\approx 2.23$  times that of the original one). The BWoPP3-PS achieved 107.43 kN versus 32.15 kN, an increase of +75.28 kN, equivalent to +234% (3.34 times the reinforced load), while the BWoPP4-PS achieved 84.06 kN versus 21.85 kN, an increase of +62.21 kN, or +285% (3.85 times the reinforced load). This sequence demonstrates that the effect of local wrapping intensifies as the number of out-of-plane parts increases, as the joint regions become the dominant force in yielding behavior. Scientifically, this significant increase in yielding is explained by the steel plate at the joint altering the force transmission mechanism and reducing the extreme values bending-shear-torsion interaction in non-straight beams. Wrapping creates a local confinement of concrete and increases the effective torsional stiffness and warping resistance, reducing stress concentration and controlling the opening of diagonal cracks at the onset of nonlinear loading. This confinement reduces interface slip and enhances the composite action between the outer steel and concrete, increasing the effective inner arm and delaying the yield condition of the longitudinal reinforcement. Wrapping also acts as a closed loop around the joint section, improving resistance to Saint-venant buckling and warping. Force paths are rebalanced so that the joint is not solely responsible for shear-torsion, and moments and shear are distributed over a longer portion of the section. As the number of sections increases, points of discontinuity and directional changes multiply, making joints more sensitive. Therefore, the effect of local strengthening is greater in BWoPP3 and BWoPP4, where yields are moved from very low

levels to levels approaching or exceeding two-to-four times the un-strengthened loads. When comparing the failure load between the models encased at the contact areas only (BWoPP-PS) and their un-retrofitted counterparts, the effect of the steel plate in raising the ultimate load capacity of the beams is clearly evident. The failure load for BWoPP1-PS increased to 610.74 kN compared to 384.26 kN for BWoPP1, an increase of 226.48 kN or approximately 59%. For BWoPP2-PS, the ultimate load reached 488.43 kN compared to 297.26 kN in the un-encased version, an increase of 191.17 kN or 64%. For BWoPP3-PS, the ultimate load reached 364.15 kN compared to 250.09 kN, an increase of 114.06 kN or approximately 46%. While the BWoPP4-PS recorded 299.07 kN compared to only 174.66 kN, an increase of 124.41 kN, or 71%, the highest among all models. In the reinforced models, the steel cladding slows crack propagation and increases the section's ability to resist combined forces resulting from bending, shear, and torsion, allowing the beam to withstand greater loads before failure. It is worth noting that the improvement rates were not equal across models, as the effect of reinforcement was more pronounced the more out-of-alignment parts there were. In BWoPP4-PS, which contains the largest number of joints, the greatest effect of reinforcement was demonstrated, with an increase of over 70%. This demonstrates that the joint areas are the most susceptible to failure in this type of beam, and that local reinforcement provides maximum effectiveness when these areas are multiple and complex.

### 6.6 Effect of Full Strengthening

When comparing the failure load between the models wrapped at the contact areas only (BWoPP-PS) and their un-retrofitted counterparts, the effect of the steel plate in enhancing the ultimate load capacity of the beams is clearly evident. The failure load for BWoPP1-PS increased to 610.74 kN compared to 384.26 kN for BWoPP1, an increase of 226.48 kN or approximately 59%. For BWoPP2-PS, the ultimate load reached 488.43 kN compared to 297.26 kN in the un-retrofitted version, an increase of 191.17 kN or 64%. For BWoPP3-PS, the ultimate load reached 364.15 kN compared to 250.09 kN, an increase of 114.06 kN or approximately 46%. While the BWoPP4-PS recorded 299.07 kN compared to only 174.66 kN, an increase of 124.41 kN, or 71%, the highest among all models. It is worth noting that the improvement rates were not equal across models, as the effect of reinforcement was more pronounced the more out-of-alignment parts there were. When comparing the behavior of beams with out-of-plane sections during the yielding phase as revealed in Figure 11, a clear pattern emerges: full steel reinforcement along the beam (BWoPP-S) increases the yield load to levels exceeding both local reinforcement at the contact points only (BWoPP-PS) and un-strengthened beams. This is demonstrated numerically in all models; for example, the yield in BWoPP1 increased from approximately 78 to 156 with local reinforcement and then to 226 with full reinforcement, and in BWoPP4 from approximately 22 to 84 and then to approximately 102 kN. In this order, the effectiveness becomes: full reinforcement > local reinforcement > no reinforcement, with the gain from local to full remaining, albeit decreasing proportionally as the number of sections increases due to the expansion of weak zones along the beam. Full steel cladding not only repairs a local weak point at the joint, but also reshapes the path of force transmission along the section: the effective moment of inertia in bending increases, the torsional stiffness increases, and the concrete confinement is strengthened, inhibiting and delaying crack expansion. When comparing the ultimate load between the different models, it is clear that strengthening the beams with full steel plates along the beam (BWoPP-S) provided significantly higher capacity compared to partial strengthening at the joints (BWoPP-PS) and unreinforced beams. For example, the failure load in the BWoPP1-S model reached 296.44 kN compared to 245.72 kN in its partially reinforced counterpart, an increase of  $\approx 50.7$  kN ( $\approx 20.6\%$ ). The unreinforced model, on the other hand, achieved only 186.24 kN, an increase of more than 59%. For BWoPP2-S, the ultimate load was 251.62 kN compared to 211.94 kN in the partially reinforced case, an increase of approximately  $\approx 18.7\%$ , while the unreinforced model achieved only 169.11 kN, approximately 48% less than the fully reinforced model. For BWoPP3-S, the ultimate load was 212.05 kN compared to 181.67 kN for the partially reinforced model ( $\approx 16.7\%$  increase), compared to 146.20 kN in the unreinforced case (an increase of more than 45%). Finally, for BWoPP4-S, the ultimate load was 174.23 kN, outperforming the partially reinforced model (152.08 kN) by  $\approx 14.5\%$ , while the unreinforced model did not exceed 125.76 kN, a difference of approximately 38% as revealed in (Fig. 12).

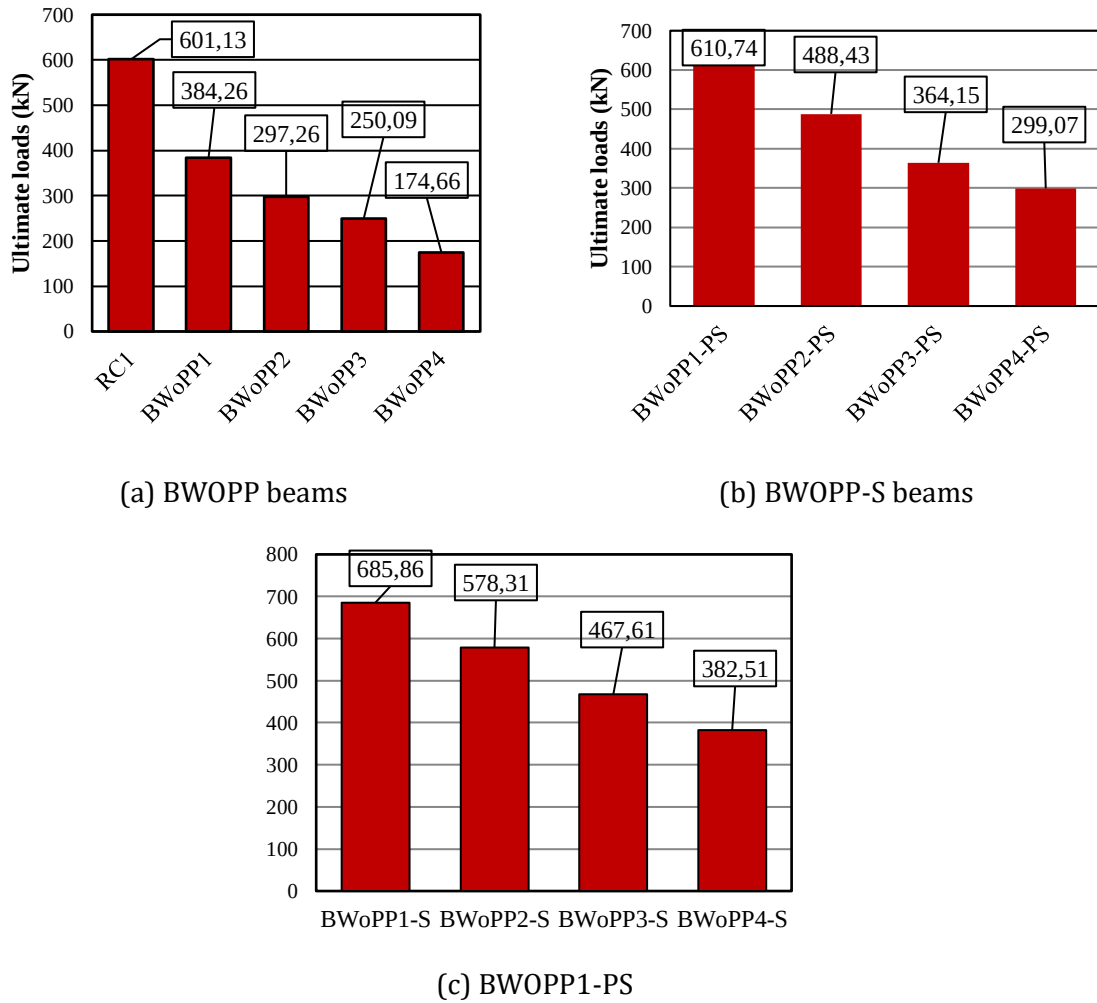


Fig. 12. Ultimate load of BWOPP beams

### 6.7 Displacement Ductility Index

According to the definition of ductility, Punmia et al. and Khamees et al. [25,26] stated that the ability of the materials to resist the load under increased amount of deformation. Also, Kim et al. and Maghsoudi et al. [27,28], provided a method for calculation of ductility index according to the ratio between the values of deflection at ultimate and yield points which named the ductility index for displacement. The ductility index (DI) across the three series is presented in (Fig. 13) shows a clear structural trend that directly reflects the failure mechanism and force transmission. In the first series (unreinforced beams with continuous sections), the DI increases gradually as the number of sections increases: from 3.711 in BWoPP1 to 4.145, 4.689, and then 5.172 in BWoPP4, i.e., relative increases of approximately 8.8%, 21.5%, 37.5%, and 51.6% compared to the straight reference beam RC1 (DI = 3.411). This increase, despite the failure mode being shear-torsion-dominated, is explained by the definition that the DI is measured as the ratio of ultimate displacement to yield displacement; As the geometric fractures increase, the yield point decreases rapidly due to stress concentration at the joints and the displacement of the neutral axis. However, the final displacement does not decrease as much due to nonlinear redistribution and continued cross-sectional buckling. The DI increases, even though the failure is more brittle. In the second series (partially strengthened at joints only), the DI jumps to much higher levels: 6.936, 7.269, 7.147, and 6.848, respectively. Compared to un-strengthened specimens of the same grade, the ductility increases by approximately 86.9%, 75.4%, 52.4%, and 32.4%, respectively. The third series (full-strengthened along the span) exhibits the highest ductility values ever: 9.578, 9.407, 8.884, and 8.204. Compared to partial strengthening, these values increase by 38.1%, 29.4%, 24.3%, and 19.8%. Compared to un-strengthened spans, the ductility increases to massive increases of 158.1%, 126.9%, 89.5%, and 58.6%, respectively, while exceeding 140-181%

compared to the RC1 reference. However, a slight, steady decrease in DI is observed within this series with increasing parts (from 9.578 to 8.204) because the frequency of fractures generates areas of disturbance and distributed warping that a single sheathing even a complete one cannot completely eliminate, thus reducing some of the gains.

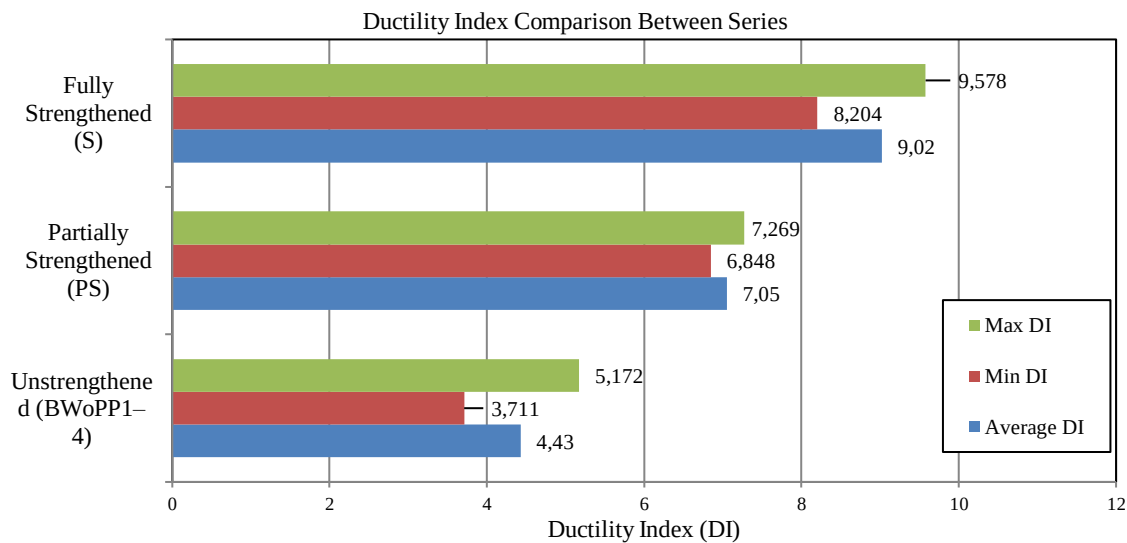


Fig. 13. Ductility index of the BWOPP beams

When the three series are placed side by side, the structural picture becomes clear: Unreinforced reinforcement achieves higher ductility than RC1 due to the yield displacement decreasing faster than the final displacement, but it remains brittle to failure; local reinforcement stabilizes the joint and nearly doubles ductility in the few-part models before its relative benefit declines as the weak zones spread; Full reinforcement creates a continuous composite effect that distributes forces and converts the failure mechanism into a soft bend, providing the highest DI and the most reliable operational behavior. As shown in Table 4, the statistical results of the ductility coefficient showed significant differences between the un-strengthened, partially strengthened, and fully strengthened specimens. The average ductility of the un-strengthened specimens was approximately 4.429 with a standard deviation of 0.637 and a relatively high coefficient of variation of 14.37%, indicating less consistent behavior and clear dispersion in performance. The partially strengthened specimens, on the other hand, showed a higher average of 7.050 with a very low standard deviation of 0.193 and a coefficient of variation not exceeding 2.73%, which reflects a consistent return of local strengthening at the contact areas and its direct effect in improving behavior without significant variation between specimens. The fully reinforced specimens achieved the highest ductility coefficient, with a mean of 9.018, a moderate standard deviation of 0.618, and a coefficient of variation of 6.85%. A one-tailed ANOVA test confirmed highly significant differences between the means at a high confidence level, with the calculated F value of 77.17 and a p-value of  $2.0 \times 10^{-6}$ . This means that more than 92% of the variance in ductility can be explained by the type of strengthening system used.

Table 5. Statistical analysis results of retrofitting technique

| Series    | Mean  | St. Deviation | Count | COV (%) | A Nova test     |                       |                    |
|-----------|-------|---------------|-------|---------|-----------------|-----------------------|--------------------|
|           |       |               |       |         | Un-strengthened | Partial Strengthening | Full Strengthening |
| Full Str. | 9.018 | 0.61791       | 4     | 6.852   | 3.711           | 6.936                 | 9.578              |
| Partial   | 7.051 | 0.19250       | 4     | 2.731   | 4.145           | 7.269                 | 9.407              |
| Un-Str.   | 4.429 | 0.63661       | 4     | 14.372  | 4.689           | 7.147                 | 8.884              |

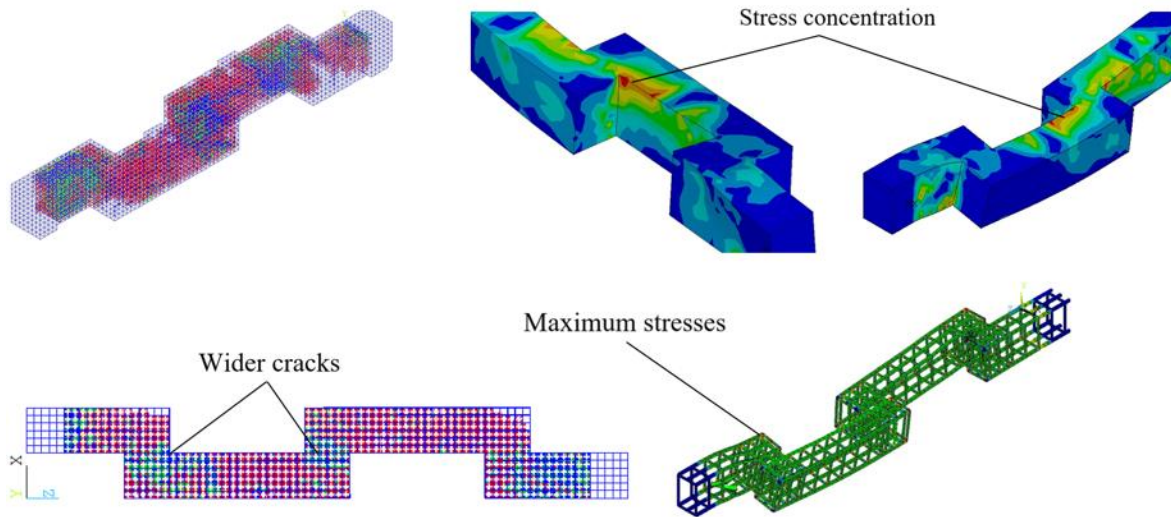
To further quantify the statistical significance and magnitude of the strengthening effect, 95% confidence intervals and effect-size measures were calculated. The mean ductility indices were 4.429, 7.050, and 9.018 for the unstrengthened, partially strengthened, and fully strengthened beams, respectively, with corresponding 95% confidence intervals of 3.416–5.442, 6.744–7.356, and 8.035–10.001. The effect size was very large ( $\eta^2 = 0.945$ ; Cohen's  $f = 4.141$ ), indicating that the strengthening configuration accounts for a substantial proportion of the observed variation in the ductility index. A post-hoc power analysis at  $\alpha = 0.05$  indicated an achieved statistical power approaching 1.00, reflecting the large observed effect.

Table 6. Mean ductility index and 95% confidence intervals for the investigated strengthening series

| Strengthening series   | Mean DI | SD    | 95% CI       |
|------------------------|---------|-------|--------------|
| Unstrengthened         | 4.429   | 0.637 | 3.416–5.442  |
| Partially strengthened | 7.050   | 0.193 | 6.744–7.356  |
| Fully strengthened     | 9.018   | 0.618 | 8.035–10.001 |

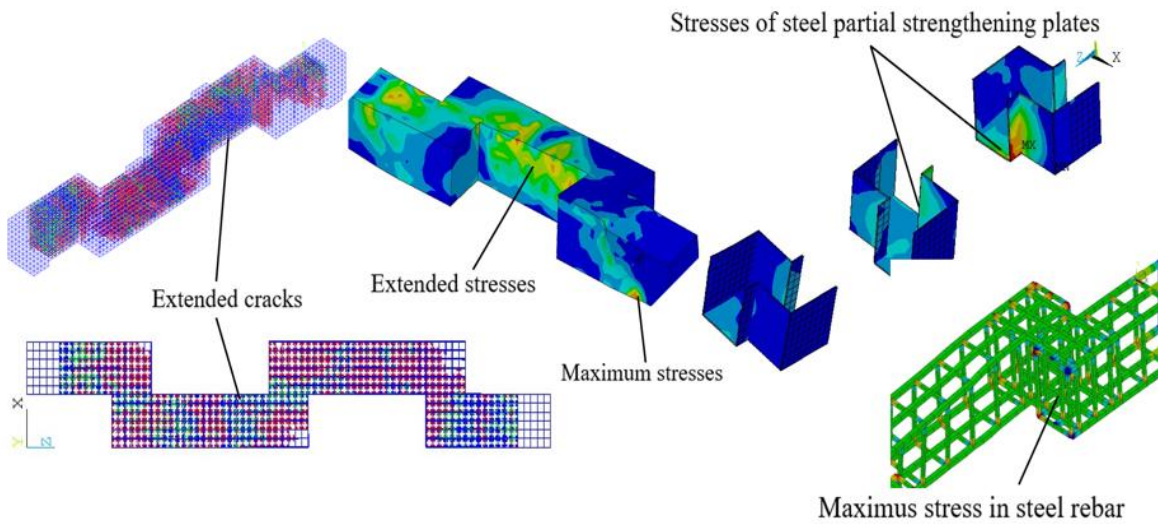
## 6.8 Energy Absorption

Table 7 and 8 exposed the results and statistical analysis of the energy absorption. The energy absorption in the various models clearly reflects the beam's ability to absorb and dissipate loads before failure. This is a key indicator of the system's strength and efficiency under dynamic or quasi-dynamic loads. In the first series, i.e., the unreinforced beams (BWoPP1–BWoPP4), the energy absorption values gradually decrease from 919.901 to 528.599 kN.mm, compared to the reference RC1, which had an energy of 1054.57 kN.mm. This sharp decline, reaching approximately 50% at BWoPP4, reflects the brittle nature of shear-torsion failure in these models. Cracking is concentrated at joints, and the collapse process accelerates due to the limited energy absorption capacity of the unconfined concrete after cracking. Here, it is noted that increasing the number of connected parts leads to a multiplication of weak areas, distributing deformations and expanding diagonal cracks more rapidly, reducing the area under the load-displacement curve and, consequently, decreasing the energy reserve of the element. In contrast, the second series (partial reinforcement at joints) shows a significant improvement in values, with energy absorption increasing from 1552.794 in BWoPP1-PS to approximately 987.631 in BWoPP4-PS. Compared to their un-strengthened counterparts, these models record increases of up to 69% in BWoPP1, 58% in BWoPP2, 91% in BWoPP3, and 86% in BWoPP4. The scientific reason for this increase is that the local metal coating provides effective confinement at the joints, preventing the premature propagation of diagonal cracks and prolonging the post-cracking phase. This increases the total area under the load-displacement curve, thereby improving energy absorption. The third series (full-length reinforcement along the beam) demonstrated the highest energy absorption values, reaching 1982.917, 1561.117, 1553.837, and 1369.973, respectively. Compared to the unreinforced beams, these values represent significant increases ranging from 115% to 158%, and also significantly outperformed the partially reinforced models. The scientific reason for this is that the continuous metal cladding transforms the behavior of the element into a homogeneous composite with high stiffness and a larger deformation range before failure. It also reduces the disparity between stress zones, allowing for gradual energy absorption until later stages of loading. As shown in Table 8, the statistical results for energy absorption indicate a clear discrepancy between the un-strengthened, partially reinforced, and fully reinforced models. The un-strengthened series showed relatively low average energy absorption values with significant variance, represented by a high coefficient of variation. This reflects the sensitivity of these models to changes in geometric and structural properties due to the absence of additional energy dissipation mechanisms. In contrast, the partially reinforced series showed a significant increase in average absorption with a relatively small standard deviation, reflecting the role of localized coatings at the contact points in enhancing energy dissipation capacity, although some variation remained due to the lack of comprehensive reinforcement along the span.

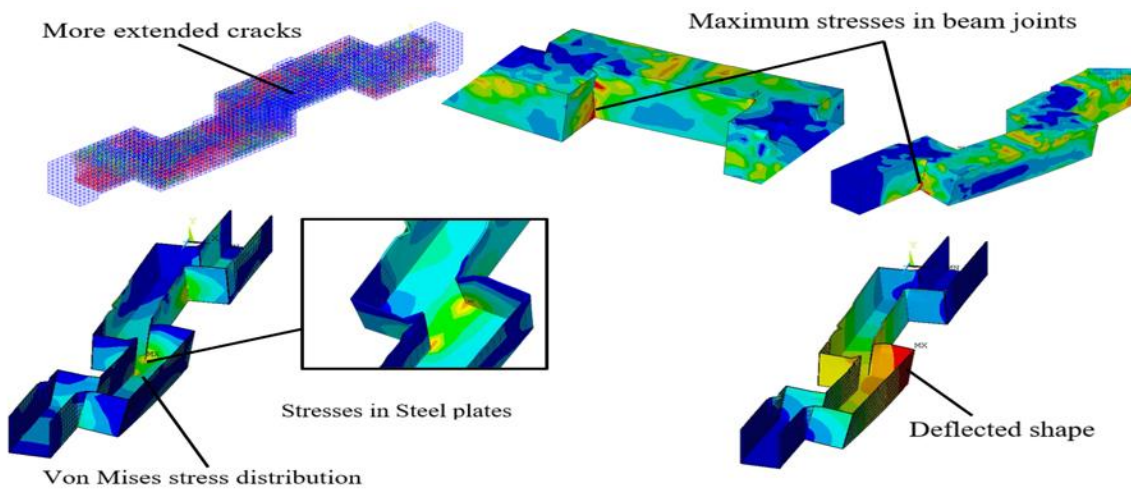


(a) BWOPP3

Fig. 14. Failure mechanism and stress distribution of BWOPP3 beams



(b) BWOPP3-PS



(c) BWOPP3-S

Fig. 15. Failure mechanism and stress distribution of BWOPP3 beams

The fully reinforced models recorded the highest average energy absorption, with a significant decrease in the coefficient of variation, confirming the effectiveness of the comprehensive strengthening mechanism in achieving greater structural stability and improving the ability to withstand repeated or combined loads. The results of the ANOVA test between the groups supported these observations, showing significant differences between the three series.

Table 7. Results of energy absorption

| Model  | Un-strengthened | Partial Strengthening | Full Strengthening | Partial vs. Un-strengthened (%) | Full vs. Un-strengthened (%) | Full vs. Partial (%) |
|--------|-----------------|-----------------------|--------------------|---------------------------------|------------------------------|----------------------|
| BWoPP1 | 919.901         | 1552.794              | 1982.917           | 68.8                            | 115.6                        | 27.7                 |
| BWoPP2 | 761.586         | 1201.783              | 1561.117           | 57.8                            | 105                          | 29.9                 |
| BWoPP3 | 618.027         | 1184.02               | 1553.837           | 91.6                            | 151.4                        | 31.2                 |
| BWoPP4 | 528.599         | 987.631               | 1369.973           | 86.8                            | 159.2                        | 38.7                 |

Table 8. Statistical analysis results of energy absorption

| Series       | Mean     | St. Deviation | Count | COV (%) | A Nova test     |                       |                    |
|--------------|----------|---------------|-------|---------|-----------------|-----------------------|--------------------|
|              |          |               |       |         | Un-strengthened | Partial Strengthening | Full Strengthening |
| Full Str.    | 1750.711 | 231.45        | 4     | 13.22   | 919.901         | 1552.794              | 1982.917           |
| Partial Str. | 1231.557 | 96.22         | 4     | 7.81    | 761.586         | 1201.783              | 1561.117           |
| Un-Str.      | 707.028  | 171.36        | 4     | 24.24   | 528.599         | 987.631               | 1369.973           |

## 7. Cracking Pattern and Stress Distribution

According to the obtained results, of the cracking pattern, failure mode, and stress distribution was notably affected by the change in geometry of the beam as shown in (Fig. 14). Stress contour legend: red indicates the highest stress levels, while blue indicates the lowest stress levels. For the non-straight beam, the stresses were concentrated at the connections zone (the zone of changing geometry). also increase the number of segments led to initiating of more stresses at the connection's zones. For the strengthening case, the existence of the wrapping steel plate affects the cracking pattern and stress distribution. Also, this variation included changing the strengthening from partial to full wrapping case. The changing in the direction causes changing in the path of the load which developed combined stresses. Shear stresses were initiated at the support zone while at the connection regions, combined shear-torsional stresses were developed. The torsion twisted the internal segments to the inside. The mid span segment tends to deflect in combination with twisting. The partial wrapping with steel plate showed that the existence of steel plate at connection region causes re-distribution of stresses and avoid shear stresses to concentrate in the concrete at the connection zone. In case of full wrapping, the steel plates redistributed that stresses along the entire beam which changed the failure mode from shear torsional failure to failure dominated mode. The presence of the steel plate wrapping enhanced the ductility of the concrete members which increased the deformation and distributed stresses along the entire beam larger than those of partial wrapping.

## 8. Conclusions

This study presents an analysis of the failure mechanism and deformation pattern for RC beam under the effect of out-of-plane parts. Because of the geometric discontinuity causes disturbances in load path and develop a complex combined of stresses at these joints, make a need to suggest and investigate a strengthening method to reduce the negative effect of the geometric discontinuity. The study proved the efficiency of reducing the defects of these joints which refers to the strengthening does not depend only on the wrapping by steel plate but also on the location of the thin plate. The conclusion can be summarized as following:

- The results showed that the straight reference beams (RC1) exhibited more stable structural behavior, with yield loads approximately 18–22% higher than those of the out-of-plane beams (BWoPP1–BWoPP4). This demonstrates that the change in geometry creates eccentric load transfer, which weakens the section's ability to resist composite forces.
- The multi-segment models showed that increasing the number of connected sections resulted in a gradual decrease in beam stiffness of up to 15% for each additional section. This was due to increased stress concentration at the contact points and the emergence of restricted torsional deformations.
- Comparing the models partially strengthened with steel plates at the contact points only with the un-strengthened models, a significant improvement in yield loads was observed, ranging from 10–14%. This reflects the effectiveness of local strengthening in reducing initial crack accumulation and delaying shear-flexural failure.
- The models strengthened along the entire length of the beam exhibited a different behavior, with the failure load increasing by 28–32% compared to the partially strengthened models, with the failure mode shifting from brittle shear-flexural failure to a more ductile flexural failure. This change reflects the pivotal role of the structural integration between concrete and steel in stress redistribution.
- The ductility index showed a marked difference; the unreinforced beams decreased by approximately 20% compared to the reference, while the partially strengthened models increased by 12% and the fully strengthened ones by more than 25%. These results confirm that strengthening not only increases resistance but also enhances the ability to absorb strains before failure.
- Regarding energy absorption, the tensile strength of the un-strengthened models decreased by 18–21% compared to the reference model, while the partially strengthened models increased by about 15%, and the fully strengthened models increased by more than 35%, making them more suitable for applications subjected to repetitive or dynamic loading.
- The numerical stress distribution within the sections showed that the force paths in BWOPP beams do not travel in straight lines as in conventional beams, but rather take a curved and sinuous form resembling a strut-and-tie mechanism, generating local concentrations at the interior corners of the out-of-plane sections. This distribution explains the reduced stiffness and the emergence of complex failure modes combining bending, shear, and torsion.
- Compared to previous studies such as Hsu & Mo (2010) and Mander et al. (1988), the present results are consistent with theories showing that the presence of geometric or eccentric discontinuities in loads generates additional moments due to restrained torsion, but the addition of steel plates - either partially or fully - redistributes stresses and reduces the effect of torsional deformations, which makes this technique an effective means for improving the design of composite beams with out-of-plane members.
- The present study is limited to the investigated beam geometry, material properties, loading conditions, and strengthening configurations. Therefore, the obtained results should not be directly generalized to other geometries or strengthening arrangements without further experimental and numerical investigation.

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